TIGHT BINDING BOOK

ALTERNATING CURRENT
ELECTRICAL ENGINEERING

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Alternating Current Electrical Engineering

BY

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PREFACE

An attempt has been made in the present volume to produce a text-book which covers in a general way the main ground included in the title without going into too great detail in any one particular branch.

It is hoped that the work will be found useful to engineers and students and has been designed to cover the Syllabus for the Grade II. (A.C.) Paper of the City and Guilds Examination. It may also be used as a preparation for the B.Sc. Examinations in Electrical Engineering, but will, of course, have to be supplemented by more specialised treatises.

The author has endeavoured to combine theory with practice, since a certain amount of the former is necessary to understand properly the application to practical work. In this connection a plea is made for a higher standard of mathematical attainment than is commonly met with at present, as without this the proper study of Alternating Currents is rendered very difficult. In a number of cases proofs by means of the Calculus have been included on account of their relative simplicity, but these have been relegated, usually, to footnotes, so that the uninitiated reader may pass them by if necessary. The notation used is, in the main, that recommended by the International Electrotechnical Commission, and the vectors have been made to rotate throughout in an anti-clockwise direction.

The great majority of the illustrations have been specially prepared for this book, and the author takes this opportunity

of thanking Messrs. Bruce Peebles & Co., Ltd., The Cambridge Scientific Instrument Company, Ltd., Everett Edgcumbe & Co., Ltd., R. W. Paul, H. Tinsley & Co., the *Electrician*, and the Institution of Electrical Engineers for their kindness in supplying the blocks from which the remaining illustrations have been taken.

The author's thanks are also due to the Institution of Electrical Engineers for permission to reproduce on p. 112 portions of a paper by him, entitled "A Practical Method of Harmonic Analysis," which has been accepted for publication.

PHILIP KEMP.

POLYTECHNIC, REGENT STREET,
LONDON, W.1.
June, 1918.

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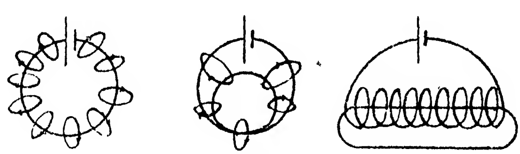
ALTERNATING CURRENT ELECTRICAL ENGINEERING

CHAPTER I

GENERAL CONSIDERATIONS OF ALTERNATING E.M.F. AND CURRENT

Production of Electromotive Force.—When an electrical conductor cuts a magnetic field there is an E.M.F. induced in the former, the magnitude of which is proportional to the rate of cutting lines of force. Since both the paths of the electric current and the magnetic flux are closed loops, this is equivalent to saying that when the number of linkages is changed an E.M.F. is induced in the electric circuit, a linkage meaning one line of force linked with one turn. In a number of problems it will be more convenient to speak of a change of linkages rather than the cutting of a number of lines of force, since the latter may be linked with more than one



(a) (b) (c)

FIG. 1.—Linkages.

turn. Thus ten linkages may be produced by ten lines of force linked with one turn as in Fig. 1 (*a*), or by five lines of force linked with two turns as in Fig. 1 (*b*), or by one line of force linked with ten turns as in Fig. 1 (*c*), or by any such combination.

The idea of linkages certainly gives a truer idea than that of lines of force cut by a conductor, since, in the latter case, one has to be careful to remember the return circuit both of the electric current and the magnetic flux. Notwithstanding this disadvantage, it will still be found expedient to consider a large number of problems from the point of view of cutting lines of force, but the student should

make himself familiar with both conceptions. In the case where an electrical conductor not forming part of a complete circuit moves across a magnetic field, it will have an E.M.F. induced in it, but no current will flow and no work will be done on the conductor, which will consequently exert no reaction on the magnetic field. If the circuit be closed, current will flow which will endeavour to set up its own magnetic field, producing a reaction upon the main field. This effect is seen in the distortion of the magnetic flux in continuous current dynamos and motors, and is particularly important in alternating current work.

Law of a Simple Alternating E.M.F.—When a conductor rotates with a constant angular velocity in a uniform magnetic field, as indicated in Fig. 2, it cuts the lines of force at varying rates depending upon the instantaneous direction of motion. The induced E.M.F. will therefore be of a varying character as well. When the conductor is passing the centre of the pole, it is moving momentarily at right angles to the field and is cutting the lines of force at its maximum rate; its maximum E.M.F. therefore occurs at this point. When the conductor has advanced 90° from this point, it is, for a moment, moving parallel to the field, and has no E.M.F. induced in it. To obtain the value of the induced E.M.F. at any point throughout the complete revolution, it is necessary to determine that component of the velocity which is at right angles to the magnetic field. A knowledge of the direction in which the magnetic field is being cut is also necessary, as this determines the direction of the induced E.M.F. If the conductor is rotating with

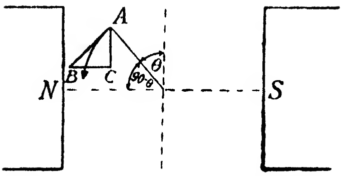


FIG. 2.—Generation of Alternating E.M.F.

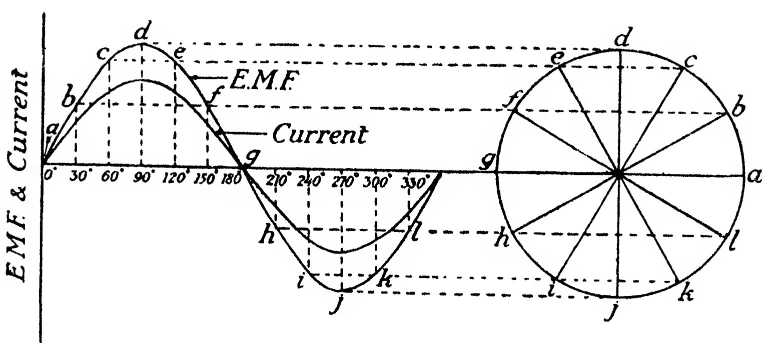


FIG. 3.—Graphical Construction of Sine Wave.

a constant peripheral velocity of v cm. per second, represented in Fig. 2 by AB, then AC will represent the useful component and will be equal to $v.\sin\theta$, θ being the angle moved through taking the

vertical line as the starting point. Let l equal the length of the conductor in cm. and B the density of the magnetic field in lines per square cm., then the E.M.F. induced in the conductor at any instant is equal to $Blv \sin \theta \times 10^{-8}$ volts. The only quantity which varies throughout the revolution is $\sin \theta$, and therefore the induced E.M.F. will obey a simple sine law as represented in Fig. 3. This diagram also shows a simple graphical method of constructing a sine wave. A circle is constructed as shown, and radiating lines are drawn every 30°. From the points a, b, c, d, etc., horizontal lines are drawn until they intersect the ordinates erected at 0°, 30°, 60°, 90°, etc. The intersections give points on the required curve.

Law of a Simple Alternating Current.—If the ends of the above active conductor are connected to the ends of a simple resistance, a current will flow the value of which at any instant is equal to the resultant E.M.F. divided by the resistance. If the resistance is constant and no other E.M.F. acts upon the circuit, the current will be exactly proportional to the E.M.F. and will follow the same law.

Alternating Magnetic Field.—Whenever a current flows in a circuit, it always sets up a magnetic field linking with the electric circuit. The magnitude of this field is proportional to the strength of the current, assuming no iron to be present, for when the current is zero the field which it sets up is zero, and when the current reaches its maximum value the field does likewise. Thus an alternating field is produced by an alternating current, both following the same law, and the number of linkages of lines with turns is continually changing. But a line of force is imagined to grow from point size by gradually swelling, and thus getting larger. It certainly does not grow after the fashion of a broken thread, gradually encircling the conductor by moving end on, for a line of force is always a closed loop. Bearing this fact in mind, it is easy to realise that during the production of a line of force it must cut the conductor which produces it at some time during its generation. The result of this action is to generate an E.M.F. in the conductor. This can be summarised by saying that when the current changes the field also changes, and thus the conductor is cut by its own lines of force, resulting in a self-induced E.M.F. proportional to the change of linkages per second. The direction of this self-induced E.M.F. will be such as to oppose the change. This effect can be seen in a C.C. circuit, particularly in the shunt field circuit of a dynamo or motor where, when the E.M.F. is first applied, the induced voltage opposes the rise of the current, thus retarding its growth; similarly, when the circuit is opened, the induced voltage tries to keep the current flowing and produces a vicious spark at the opening contacts of the switch.

If the magnetic circuit consists of non-magnetic materials, the field which is set up is proportional to the current and obeys the same law, whether this be a simple sine law or not; and if the

resistance of the circuit is constant, the resultant E.M.F. producing the current will also obey the same law. But the resultant E.M.F. is obtained from a combination of the applied voltage and the self-induced back voltage due to the conductor cutting its own magnetic field. There is, however, no guarantee that this latter voltage will obey the same law as the current, and consequently it cannot be said that the applied voltage will follow the same law as the current. The presence of iron in the magnetic circuit exerts a further disturbing influence. As a matter of fact, there is very often a considerable difference in practice in the laws followed by the current and the applied voltage. It should, however, be borne in mind that the resultant E.M.F. acting upon the circuit, the current which it produces when the resistance is constant and the magnetic field which is set up when no iron is present, all obey the same law, whatever it may be.

Frequency.—For a particular circuit, the instantaneous value of the current, or the voltage, may be plotted against time as a base. This curve may be an ideal sine wave, or it may take some

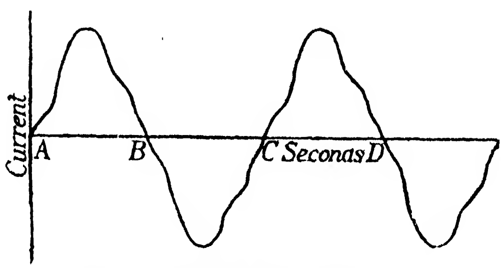


FIG. 4.—Non-sinusoidal Wave Form.

such form as is indicated in Fig. 4. It is said to be periodic, *i.e.* it keeps on repeating itself over and over again. The whole series of events included from a given point to the next similar point in the same direction is called a *period* or a *cycle*. For example, the series of events which occurs between *A* and *C* or between *B* and *D* is a complete cycle, whilst the time taken to accomplish this is called the *periodic time*. The number of complete cycles per second is called the *frequency* or *periodicity*. The frequencies most commonly used in this country at the present time are 50 and 25 cycles per second. The frequency of both current and voltage is, of course, the same. The wave form, as it is called, may vary considerably from the ideal sine wave, and in old-fashioned alternating current machinery it often was very far indeed from the desired shape, but in modern machinery, owing to the advance in methods of design, a good approximation to the ideal sine wave is usually achieved, although it may not be exactly correct in a mathematical sense. With the ordinary forms of alternating current dynamos both halves of the wave must be identical, *i.e.* any two

points situated 180° apart must have the same value. The reason for this is that as many lines of force enter the south pole as issue from the north pole (see Fig. 2), and the two halves of each turn of the armature winding are situated a pole pitch apart. With a uniform conductor velocity, therefore, the half-wave is repeated exactly in alternate directions.

Non-sinusoidal Wave Form.—In order that the wave form shall be sinusoidal, *i.e.* follow a sine law, it is necessary that the conductor should cut the magnetic field at a rate proportional to sin θ, the speed being constant. Now consider the elementary alternating current dynamo shown in Fig. 5, where there is one turn mounted on an iron core in the field of an electromagnet excited by means of a continuous current. The magnetic field in this case may be considered to be radial across the air gap, and as the conductor rotates with constant speed it cuts the lines of force at a constant rate whilst under the influence of the pole. Throughout the remainder of the revolution the conductors are not in the magnetic field at all, thus generating no voltage during these periods. A rectangular wave form of E.M.F. would be obtained as shown in Fig. 6. In between this and the original sine wave

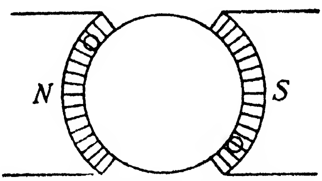


FIG. 5.—Conductors Rotating in Uniform Field.

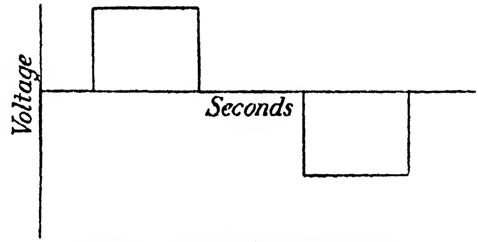


FIG. 6.—Rectangular Wave Form.

obtained from the arrangement shown in Fig. 2 there are an infinite number of possibilities, and it is from these that the wave forms obtained in practice are built up.

In the majority of problems it will be necessary to make the calculations on the assumption that the quantities concerned follow a sine law, since the mathematics would otherwise be complicated to an undesired extent. Nevertheless, it must not be forgotten that the actual wave forms dealt with nearly always depart, to a more or less extent, from the ideal wave form. Unfortunately, a number of writers insist too much, in the author's opinion, on the sinusoidal wave form and neglect to instil into the minds of the readers the fact that it is only an approximation made for the purpose of simplifying the calculations

CHAPTER II

MAXIMUM, R.M.S., AND AVERAGE VALUES

Variation of an Alternating Current.—In dealing with alternating currents, it is at once necessary to fix upon a unit of current which must obviously have some differences from the ampere as used in continuous current work. The current is varying at a very rapid rate, and although its value at any one instant can be compared with the continuous current ampere, yet it is not self-evident what is the resultant effect. Take, for example, a circuit the frequency of which is 50 cycles per second, the current obeying a sine law and rising up to an instantaneous maximum of 10 amperes. The current rises from zero to its maximum value in the time taken to execute one-quarter of a cycle, that is in $\frac{1}{200}$th second. Starting from an instant when the current is zero, the value of the current is 5, 7·07 and 8·66 amperes at times equal to $\frac{1}{600}$th, $\frac{1}{400}$th and $\frac{1}{300}$th second respectively. Moreover, after $\frac{1}{100}$th second has elapsed, the current has reversed and is following the same procedure in the opposite direction for $\frac{1}{100}$th second. If, then, a moving coil ammeter is placed in circuit, with the object of measuring the current, it will endeavour to indicate all these different values of the current from instant to instant, including those where the current is flowing in the opposite direction. But, owing to the mechanical inertia of the moving system of the instrument, it cannot follow out all the rapid variations of the current, with the result that the pointer merely vibrates about the mean of the positions that it would like to take up. Since, however, there is as much current flowing in one direction as in the other, it follows that this mean position is the zero of the scale, independent of the actual value of the current. Thus an ordinary moving coil ammeter is useless for the purpose of measuring an alternating current, nor is it desired, at this present juncture, that the various instantaneous values should be recorded.

Maximum Value.—Since it would be extremely inconvenient to have to designate the value of an alternating current by stating its various values from instant to instant, apart from the practical difficulty of carrying this out, it is necessary to choose one representative value to specify the strength of the current. The first

that suggests itself for this purpose is the maximum value, but there are obvious objections to this immediately currents of different wave forms are considered. Take, for example, the two currents of different wave form indicated in Fig. 7. They have the same maximum value, but there is obviously more current flowing in

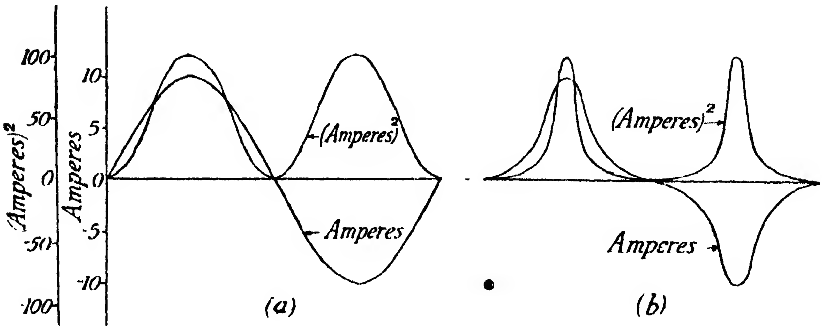


FIG. 7.—Values of (Current)^2.

the first case than in the second, so that it is not desirable to represent these two currents by the same number. Therefore, the maximum value of the current is not a suitable measure of it unless the wave form is known, and this latter condition puts it out of court as far as practical purposes are concerned.

Average Value.—The next representative value that suggests itself is the average or mean value. Since both the positive and negative[1] halves of the wave are equal, the average value taken over a complete cycle is zero, as was previously seen when discussing the action of the moving coil ammeter. This will not do, but it would be possible to consider the average value taken over a half-period, since this would have a quite definite numerical meaning. Following out this idea, suppose that the current represented in Fig. 7 (b) is passed through a simple resistance, the instantaneous value of the power being represented by i^2r, where i is the instantaneous value of the current and r is the resistance. This power will be varying at a very rapid rate, since, in a circuit the frequency of which is 50, the power rises from zero to a maximum value in $\frac{1}{200}$th second. In the great majority of cases, it will be the average value of the power over a given time which is desired, and consequently the average value of i^2r over a complete cycle must be determined. Since r is a constant, the power at any instant is proportional to i^2. The curve of i^2 is shown in Fig. 7 (b), having been obtained by squaring all the ordinates of the current curve. The average power is equal to (average value of i^2) $\times$ r, and on measuring the average value of i^2 from the curve this is found to

[1] The term "negative" is used merely to indicate the fact that the current is flowing in the opposite direction, and has no absolute meaning.

be 50, whilst the average value of i is found to be 5·30. Thus if r be 1 ohm, the true value of the average power is 50 watts, whilst the value of the expression

$$(\text{average value of current})^2 \times r = 5{\cdot}30^2 \times 1 = 28{\cdot}1.$$[1]

From this it follows that if the average value of the current, taken over a half period, is to be the representative value chosen, then the power in watts must be represented by an expression of the type

$$\text{watts} = k \times I^2 r,$$

where k is some constant. Moreover, if different wave forms be considered, it will be found that the value of k changes, so that again it is necessary to know the wave form in order to determine the power in the circuit, rendering the average value of the current also impracticable as a representative value.

Root-Mean-Square Value.—From the above it is seen that the value of the current desired is such that the power expended in a simple resistance shall be given by the expression I^2r independent of wave form. This means that the average value of the squares of the various instantaneous values of the current is to be multiplied by the resistance.

In C.C. work the watts are given by the expression I_c^2r where I_c is the continuous current.

In A.C. work, the average watts are to be given by an expression

$$\text{average value of } (i^2) \times r,$$

where i is the instantaneous current. If r be given the same value in each case, then in order that the power shall be the same in both cases it follows that

$$I_c^2 = \text{average value of } (i^2).$$

It is now seen that the representative value of the alternating current, which has the same effect as a continuous current in producing power in a circuit, is given by the expression

$$\sqrt{(\text{average value of } i^2)},$$

and when this expression has a certain value it means that the current has the same power effect, or heating effect, as a continuous current of the same value, whilst the watts are given by the expression I^2r without the introduction of any constant, I being the

$$\underline{\sqrt{(\text{average value of } i^2)}}.$$ R.M.S.

The foregoing argument has made no assumptions as to wave form and, indeed, is true for all periodic wave forms, so that it is

[1] The student is warned to distinguish carefully between the (average value of current)^2 and the average value of (current)^2.

not necessary to know the various instantaneous values in order to estimate the resultant value of any current.

This value of the current is known as the "Root-Mean-Square" or "R.M.S." value, the name expressing its meaning. It is also sometimes known as the effective value and as the virtual value, although the first name is by far the most commonly used.

Current indicated by Ammeters.—At first sight the above chosen representative value appears somewhat complex, but it is only an appearance. This is seen when the action of various ammeters is considered (see page 132 *et seq.*). Take, for example, the hot wire ammeter; the heating at any instant is proportional to the square of the current, and it is to the average value of this quantity that the indications of the instrument are proportional. When such an ammeter is connected in an alternating current circuit it would register an amount proportional to the mean square of the current and the square root of the deflection would be proportional to the R.M.S. current. Consequently the hot wire ammeter indicates that value of the current which it is desired to know. In the actual instrument the scale is not exactly a square scale, being somewhat modified by the construction and control, but this does not affect the fact that it is the mean square of the current to which the deflection is proportional.

Similarly, when considering a moving iron ammeter, it is again seen that the torque at any moment is proportional to the square of the instantaneous current. The current induces a certain pole strength in the moving iron which is proportional to the current provided the iron is not saturated, whilst the solenoid itself can be replaced by an equivalent magnet having a pole strength proportional to the current. The force of attraction or repulsion is proportional to the product of these pole strengths and, consequently, to the square of the current. Modifying factors are again introduced in practice, due to the construction and method of control, but the main fact still remains, viz., that the instrument pointer will take up a mean position indicating the average value of the square of the current. Thus a moving iron ammeter will indicate the R.M.S. current.

A dynamometer type instrument also will give indications which are proportional to the products of the currents in the moving coil and in the fixed coil and will register accordingly the R.M.S. current.

Thus it is seen that, instead of the R.M.S. value of the current being a somewhat obscure quantity, it is actually the value recorded by the ordinary types of ammeters.

Representative Value of Voltage.—The question of settling a unit of E.M.F. can now be dealt with in the same way as for current. Imagine an alternating voltage applied to a simple resistance. The instantaneous value of the power will be $\frac{e^2}{r}$, where e is the

instantaneous value of the voltage and r is the resistance. The average value of the power will be the average value of $\frac{e^2}{r}$. Suppose that a continuous pressure of E_c be applied to a similar resistance of r ohms. In this case the power will be $\frac{E_c^2}{r}$, and it is desired that the power in both cases shall be the same. From this it follows that

$$\frac{E_c^2}{r} = \frac{\text{average value of } e^2}{r}$$

or that
$$E_c = \sqrt{(\text{average value of } e^2)}.$$

Thus it is seen that the R.M.S. value of the voltage ($= E$) is also the representative value which will enable the expression

$$\text{average watts} = \frac{(\text{R.M.S. volts})^2}{\text{Resistance}} = \frac{E^2}{r}$$

to remain true without the addition of any multiplying constant. No assumption with regard to wave form has been made, and consequently the above expression holds good for any periodic wave form.

Voltage indicated by Voltmeters.—With regard to the indications given by various voltmeters, it should be remembered that in all electromagnetic instruments it is the current which produces the deflection, and since the instruments previously discussed indicate the R.M.S. current they will also indicate the R.M.S. voltage. There is one particular type of voltmeter which does not come in the above classification, and this is the electrostatic voltmeter (see page 138), but it will be shown later that this instrument also indicates the R.M.S. voltage.

Ohm's Law for Alternating Current Circuit.—It has been shown that the average power is given by both I^2r and $\frac{E^2}{r}$, and therefore

$$I^2r = \frac{E^2}{r},$$

$$I^2 = \frac{E^2}{r^2},$$

and
$$I = \frac{E}{r}.$$

Thus Ohm's Law holds good for an alternating current circuit where the R.M.S. values of the current and voltage are employed. It obviously holds good when the maximum values are considered and also in the case of the average values, because if the circuit consists of a simple resistance the current will be directly proportional to the voltage at any instant and the two wave forms will

No. of Ordinate.	Value of Ordinate.	Square of Ordinate.
1	1	1
2	2	4
3	2	4
4	2	4
5	2	4
6	1	1
Sum total ...	10	18
Average	1·67	3
R.M.S.	—	$\sqrt{3} = 1·73$

$$\text{Form factor} = \frac{\text{R.M.S.}}{\text{average}} = \frac{1·73}{1·67} = 1·04.$$

As a general rule, the wave may be assumed to be flatter than a sine wave if the form factor is less than 1·11 and more peaked if this ratio is greater than 1·11.

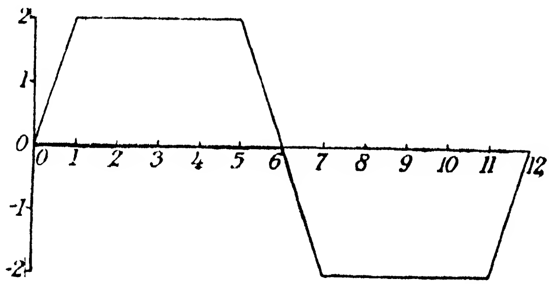


FIG. 8.—Example of Wave Form.

Crest Factor.—The use of the term "Crest Factor" has been introduced by Prof. Kapp to indicate the ratio

$$\frac{\text{Maximum value}}{\text{R.M.S. value}},$$

which in the case of a sine wave has the value $\sqrt{2} = 1·414$. This ratio is mostly used in connection with voltage waves, where it gives an indication of the amount by which the insulation is strained. For a given R.M.S. voltage, the greater the crest value the greater the strain on the insulation.

Polar Curve.—When the curves of E.M.F. and current depart from the simple sine law, it becomes a tedious matter to measure all the ordinates, square them, take the mean, and determine the square root in order to obtain the R.M.S. value. A simpler method,

involving the use of a polar diagram, has been devised by Prof. Fleming. A polar curve is a graphical method of representing a function, only, instead of using two co-ordinates mutually at right angles for axes, a line called a radius vector is made to rotate round a fixed point. The angle through which the radius vector has moved, measured from some starting point, corresponds to the horizontal axis of the ordinary graph, whilst the length of the radius vector corresponds to the vertical axis. A curve is then traced out through the points obtained in this manner. The curve is really the locus of the radius vector.

This method of graphical representation lends itself to the study of periodic curves, for one revolution of the radius vector can be made to correspond to one complete period. The polar curve for a sine wave is a complete circle as shown in Fig. 9. OP is the radius vector, having moved through an angle θ from the commencement. But if the curve OPQ is a true circle and OQ its vertical diameter, the angle OQP will also be equal to θ, and hence

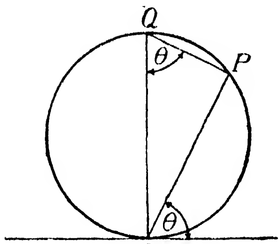


FIG. 9.—Polar Curve of Sine Wave.

$$\frac{OP}{OQ} = \sin \theta.$$

Since OQ is constant, OP is proportional to $\sin \theta$, OQ representing the maximum value. Therefore, the polar curve for a sine wave is a circle. At first sight it might appear that a second circle below the horizontal is needed to represent the second half of the wave, but this is not so. After 180° has been swept out, the radius vector certainly falls below the zero line, but the instantaneous value is now negative, and consequently the radius vector must be projected back to the other side of the origin. The net result of this is that, for the second half of the period, the curve repeats itself. This must always be the case when the two halves of the wave are equal, whether the simple sine law is obeyed or not.

Graphical Determination of R.M.S. Value.—Having thus briefly explained the principles of the polar diagram, they can be applied to the study of non-sinusoidal waves.

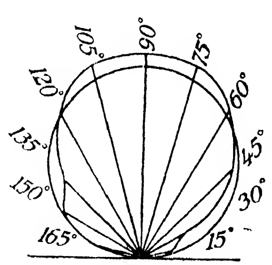


FIG. 10.—Polar Curve of Non-sinusoidal Wave.

Fig. 10 shows the polar curve of the wave represented in the ordinary manner in Fig. 4. In order to determine the R.M.S. value of the wave, the area of the polar curve is required. This can be measured by means of a planimeter, but in cases where such an instrument is not available a circle of the same area as the

polar curve can be estimated by eye. Usually this will be justifiable, since most of the waves met with in practice do not depart very much from the true sine law. The diameter of this circle represents the maximum value of the equivalent sine wave, and the R.M.S. value can be obtained by multiplying by $\frac{1}{\sqrt{2}}$ or 0·707.

CHAPTER III

INDUCTANCE, REACTANCE AND IMPEDANCE

Back E.M.F. set up due to Alternating Current.—When a varying current flows in a circuit it sets up a varying field which is in phase with the current, *i.e.* goes through similar phases at the same instant, such as, for example, the maximum value. The form of the circuit can be so arranged that the field is negligible in strength although it can never be done away with absolutely. If a portion of a circuit be wound in the form of a coil, then the lines set up by each turn, due to the passage of the current, can be made to link with all the other turns and thus set up a considerable number of linkages. This effect is increased considerably if an iron core is placed in the coil, which is then known as a choking coil.

The alternating current causes the field to alternate with it, and consequently the linkages vary from instant to instant. This sets up an E.M.F. in the coil the direction of which is such as to oppose the change, the effect being somewhat akin to inertia. If the flux set up is proportional to the current, then the back E.M.F. which is generated is proportional to the rate at which the current is changing and is numerically equal to

$$\frac{\text{linkages set up per ampere} \times \text{change of amperes per second}}{10^8} \text{ volts.}$$

Inductance.—The linkages set up per ampere is a definite quantity for a given coil, and if the core be made of a non-magnetic material it is strictly a constant, but if an iron core be introduced this quantity varies to some extent, depending upon the saturation of the iron. For example, if the current be increased to ten times its former value, the total linkages may only be increased seven times, and thus the linkages per ampere will only be 0·7 times their former value. In the majority of elementary calculations, however, this quantity is assumed to be a constant for a given coil and is known as the *inductance* or the *co-efficient of self-induction*, the former term being the one most used at the present time.

On the practical system of units, the unit of inductance is the

Henry, which is the inductance of a coil setting up 10^8 linkages per ampere. If, therefore, the current varies at the rate of one ampere per second in such a coil, there will be an E.M.F. of one volt set up. The inductance of any other coil, measured in henries, is

$$L = \frac{\text{number of linkages set up per ampere}}{10^8}.$$

As an example, the flux will be determined in a choking coil having 100 turns and an inductance of 0·2 henry, when a current of 5 amperes is passing.

$$\begin{aligned}
\text{Linkages per ampere} &= L \times 10^8 \\
&= 0{\cdot}2 \times 10^8 \\
\text{Total linkages} &= 5 \times 0{\cdot}2 \times 10^8 \\
\text{Total flux} &= \frac{5 \times 0{\cdot}2 \times 10^8}{100} \\
&= 10^6 \text{ lines or 1 megaline.}
\end{aligned}$$

Rate of Change of Current.—The next thing to determine is the rate of change of the current in the circuit. This will depend upon

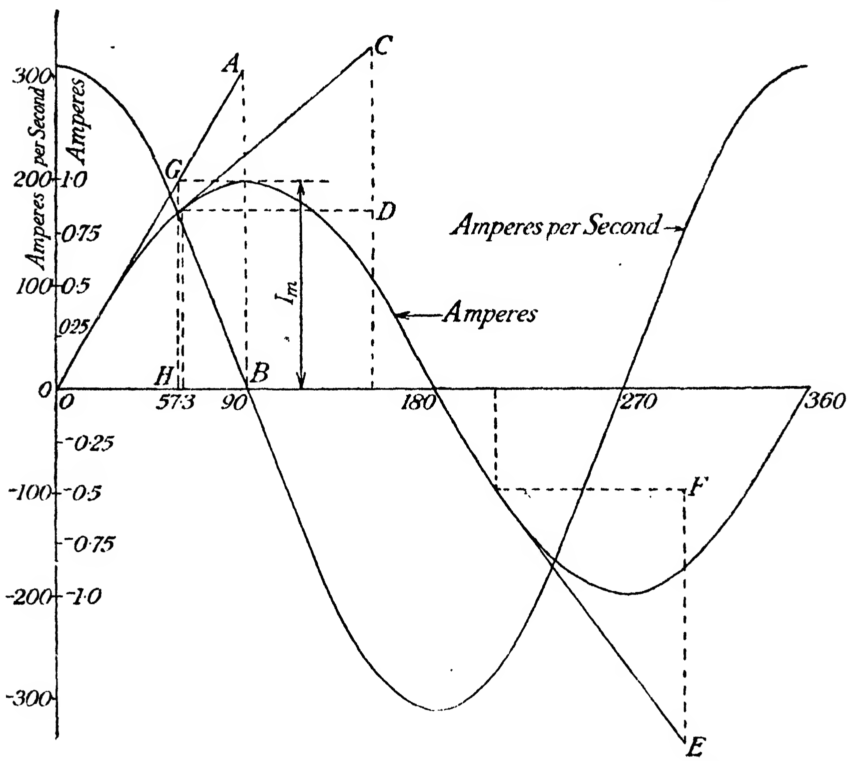


FIG. 11.—Relation between Current and Rate of Change of Current.

three things, the maximum value of the current (I_m), the wave form and the frequency (f). Fig. 11 represents the current curve where the wave form is sinusoidal. The rate of change of the current at any instant is measured by the slope of the curve at that particular point. At 0°, this is equal to AB amperes per 90°. Scaling this off the diagram, it is found to be $1{\cdot}57 \times I_m$ amperes per 90°, or $6{\cdot}28 \times I_m$ amperes per period, or $314 \times I_m$ amperes per second, assuming a frequency of 50.

At 60, the slope of the curve is CD amperes per 90°. Scaling this off the diagram, it is found to be equal to $0{\cdot}785 \times I_m$ amperes per 90°, or $157 \times I_m$ amperes per second. Repeating this again at 210°, the slope is found to be $4 \times 50 \times EF \times I_m$ ($= 272$) amperes per second, but this time the current is changing in the reverse direction, so that this statement means that the current is decreasing at the rate of 272 amperes per second. Plotting a number of values obtained in this manner, the curve of change of current in amperes per second is obtained. On examination, this curve is seen to be a sine curve also, but is 90° ahead of the current curve. In reality it is a cosine curve. This establishes the fact that if the current varies according to a sine law, the rate of change of current also obeys a sine law. The maximum value of the rate of change curve occurs when the actual value of the current is zero. If a horizontal line is drawn through the vertex of the current curve, it is seen that the maximum slope is GH amperes in OH degrees. On scaling off the diagram, OH will be found to be one radian or 57·3° approximately. Therefore, the maximum value of the rate of change of current is I_m amperes per radian, or $2\pi I_m$ amperes per period, since there are 2π radians in 360°. Measured in amperes per second, this is equal to $2\pi f I_m$, where f is the frequency.[1]

Reactance.—It has been shown that a voltage is set up in an alternating current circuit equal to

$$-\frac{\text{linkages set up per ampere}}{10^8} \times (\text{change of amperes per second}) \text{ volts.}$$

The minus sign denotes the fact that it is a back voltage. The maximum value of this voltage is given by the expression

$$E'_m = - L \times 2\pi f I_m \text{ volts}$$

[1] This can be determined by the calculus as follows :—

$$i = I_m \sin \theta$$

$$\frac{di}{dt} = \frac{d\,(I_m \sin \theta)}{dt}$$

$$= I_m \times \frac{d\,(\sin \theta)}{d\theta} \times \frac{d\theta}{dt}$$

$$= I_m \cos \theta \times 2\pi f.$$

The maximum value of $\frac{di}{dt} = 2\pi f I_m$.

and the value at any other instant by

$$e' = -2\pi f L I_m \cos\theta \text{ volts}$$
$$= -2\pi f L I_m \sin(\theta + 90°) \text{ volts.}$$

This back voltage has to be overcome by the application of an equal and opposite forward voltage having a value

$$e = 2\pi f L I_m \sin(\theta + 90°) \text{ volts.}$$

This voltage is seen to vary according to a sine law also, but it *is not in phase with the current.* When θ is 0°, the current is zero; but this voltage has its maximum value at this instant, since $\sin(\theta + 90°)$ is equal to unity. This voltage is said to *lead* the current by 90°, or the current is said to *lag* behind the voltage by 90°. The R.M.S. value of the applied voltage is obviously equal to $E = 2\pi f L I$, where I is the R.M.S. value of the current.

When the circuit contains nothing but resistance, the voltage is given by the expression

$$E = R \times I.$$

If the circuit has no resistance, but is linked with a magnetic field, the applied voltage is given by the expression

$$E = 2\pi f L \times I.$$

Thus the quantity $2\pi f L$ can be considered as being in some respects analogous to resistance, inasmuch as the current has to be multiplied by it in order to obtain the voltage. Strictly speaking, it is more in the nature of an E.M.F. than a resistance, but owing to the similarity of the equations $E = RI$ and $E = 2\pi f L I$, it is usually considered as a kind of resistance.

The quantity L is called the *Inductance* and is measured in henries, whilst the quantity $2\pi f L$ is called the *Reactance* [1] and is measured in apparent ohms. They are not true ohms, because they do not result in the production of heat in the circuit, but have the appearance of being ohms because they are the ratio of the volts to the amperes. If the circuit contains nothing but reactance, the current is given by the expression

$$I = \frac{E}{2\pi f L},$$

and the way in which the current varies with each of these quantities is shown by the curves in Fig. 12.

[1] When a circuit possesses capacity as well as inductance, another term must be added to the expression for the reactance as shown on page 59.

Very often the inductance is given in milli-henries, *i.e.* thousandths of a henry, in order to avoid dealing with very small numbers. For

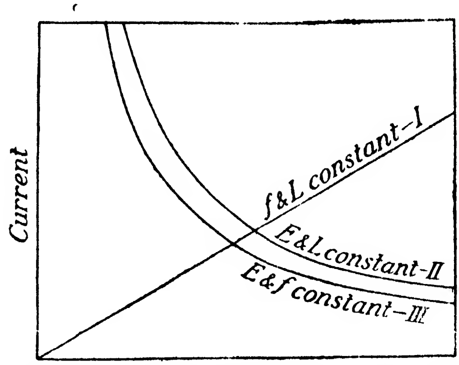


FIG. 12.—Dependence of Current upon Voltage, Frequency and Inductance.

example, if $L = 10$ milli-henries and $f = 50$ cycles per second, the reactance is equal to

$$\begin{aligned} X &= 2\pi \times 50 \times \tfrac{10}{1000} \\ &= 3{\cdot}14 \text{ apparent ohms.} \end{aligned}$$

Circuit containing Resistance and Reactance.—The previous example has not taken into consideration the effect of resistance, but it must be borne in mind that it is theoretically impossible to have a circuit without resistance and also without reactance, since wherever there is a current of electricity there must always be a magnetic field set up linking with the circuit. The reactance may be negligible in a large number of cases, but in very few circuits will the effect of resistance be negligible in comparison with the reactance. It is therefore necessary to study the circuit when both are present in appreciable proportions.

Consider a circuit consisting of an ordinary choking coil having a resistance R and an inductance L. In order to force an alternating current of I amperes through this circuit, a voltage must be applied in order to overcome the resistance (RI volts), and, in addition, a further voltage ($2\pi fLI$ volts) must be applied in order to overcome the reactance. The former voltage is in phase with the current, whilst the latter leads it by 90°. At any instant represented by the point P (Fig. 13) there will be required a voltage PQ to overcome the resistance and a voltage PR to overcome the reactance. The total voltage required at this instant is given by $PS = PQ + PR$, the current being PT. Repeating this for all other points throughout the cycle, a curve representing the resultant applied voltage is obtained as shown. The effect is just as if the reactance were concentrated at one part of the circuit and the resistance at another, the two being placed in series. The resultant voltage also follows

a sine law, since it is the sum of two sine waves. The maximum value is less than the sum of the maximum values of the two components, but is greater than either of them taken separately. Another point to notice is that the resultant voltage is neither exactly in phase with the current, nor does it lead by 90°, being

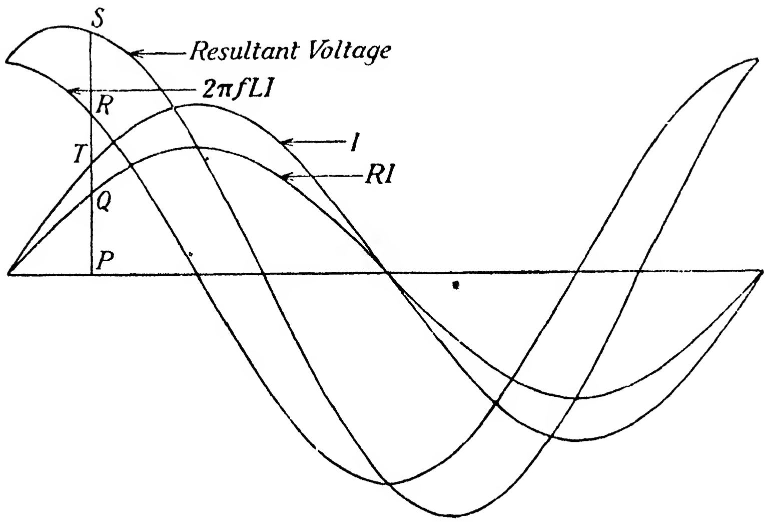


FIG. 13.—Current and Voltage Curves in a Circuit containing Resistance and Inductance.

somewhere in between these two limiting values, the angle of lead of the voltage becoming greater the more the reactance predominates over the resistance, and approaching more and more nearly to zero as the resistance is increased. If the resistance and reactance were equal, the current would lag behind the voltage by 45°.

Phase Difference between Current and Voltage.—The exact angle of phase difference between the current and voltage can be determined in general terms by the application of a little trigonometry. The total voltage required to overcome the effects of resistance and reactance combined is equal to

$$RI_m \sin\theta + XI_m \cos\theta$$

where $$X = 2\pi fL.$$

Multiplying and dividing by $\sqrt{R^2 + X^2}$ we get

Voltage required

$$= (\sqrt{R^2 + X^2})\left(\frac{R}{\sqrt{R^2 + X^2}} I_m \sin\theta + \frac{X}{\sqrt{R^2 + X^2}} I_m \cos\theta\right).$$

Let $$\frac{R}{\sqrt{R^2 + X^2}} = \cos a.$$

Then $R = (\sqrt{R^2 + X^2}) \cos a$

and $R^2 = (R^2 + X^2) \cos^2 a$

$= (R^2 + X^2) - (R^2 + X^2) \sin^2 a.$

Therefore $(R^2 + X^2) \sin^2 a = X^2,$

$$\sin^2 a = \frac{X^2}{R^2 + X^2}$$

and $$\sin a = \frac{X}{\sqrt{R^2 + X^2}}.$$

Thus the voltage required

$$= (\sqrt{R^2 + X^2})(I_m \sin\theta \cos a + I_m \cos\theta \sin a)$$

$$= (\sqrt{R^2 + X^2}) \times I_m \sin(\theta + a).$$

Obviously the curve representing this quantity is a sine curve, not in phase with the current, but leading it by an angle a the value of which can be determined in the following manner.

$$\sin a = \frac{X}{\sqrt{R^2 + X^2}}$$

and $$\cos a = \frac{R}{\sqrt{R^2 + X^2}}.$$

Therefore $$\tan a = \frac{\sin a}{\cos a}$$

$$= \frac{X}{\sqrt{R^2 + X^2}} \times \frac{\sqrt{R^2 + X^2}}{R}$$

$$= \frac{X}{R}$$

and $$a = \tan^{-1}\frac{X}{R}.$$

Thus a can be calculated for every value of X and R and will always lie between the limits of 0° and 90°.

Instead of saying that the voltage leads the current, it is more usual to say that the current lags behind the voltage, the two statements being synonymous.

Impedance.—The expression representing the current in a simple series circuit is

$$i = I_m \sin\theta.$$

If nothing but resistance is present the voltage is

$$e = RI_m \sin\theta.$$

If reactance only is present the voltage is

$$e = XI_m \sin(\theta + 90°).$$

If both resistance and reactance are present the voltage is

$$e = (\sqrt{R^2 + X^2}) I_m \sin(\theta \pm \alpha).$$

The quantity $\sqrt{R^2 + X^2}$ is called the *Impedance* and obviously depends on both the resistance and reactance of the circuit, although it must be borne in mind that it is not the arithmetic sum of the two. The impedance is the ratio of volts to amperes and will be denoted by the symbol Z.

$$\frac{E}{I} = Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (2\pi f L)^2}.$$

Fig. 14 shows the way in which the impedance varies with the resistance, frequency and inductance for a particular circuit, and also how the angle of lag depends upon these quantities.

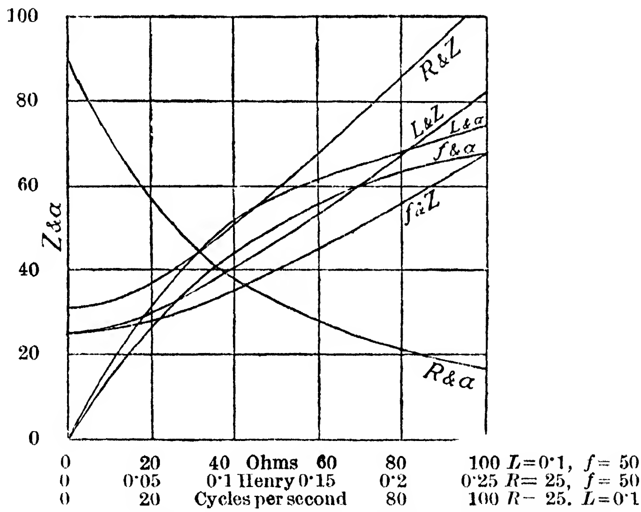


FIG. 14.—Effect of Resistance, Frequency and Inductance upon Impedance.

Variable Inductance.—In a number of cases the inductance of a circuit will not remain constant, but will depend to some extent upon the value of the current flowing. This can easily be shown by means of a choking coil having an iron core. At first, as the current is gradually increased, the flux will be proportional to the current and the linkages per ampere will remain constant. When the iron becomes saturated, however, the flux increases at a slower and slower rate compared with the current and the *linkages per ampere* become less. Thus the inductance actually is reduced when the iron gets saturated.

The true ohmic resistance is also considerably greater with alternating than with continuous currents in a number of cases due to what is called the skin effect.

Mutual Inductance.—Two circuits are frequently placed so that when a current is passed through one of them the flux set up by it becomes linked with the other. This alternating flux induces a voltage in the second circuit in the same way that a back E.M.F. is induced in an ordinary inductance. In the case of the two circuits, a voltage is induced in either one when the current changes in the other and the circuits are said to possess *Mutual Inductance.* This quantity is comparable with inductance, sometimes called *Self-Inductance* to distinguish it, and is measured in the same units.

Two coils are said to possess a mutual inductance of one henry if 10^8 linkages are set up in one coil due to a current of one ampere in the other. If the two coils have the same number of turns, it is clear that the same current in either coil will set up the same number of linkages with the other coil. If the two coils have different numbers of turns, the flux set up by one ampere in the coil having the smaller number of turns will be less than the flux set up by one ampere in the other. But this smaller flux will link with a proportionately larger number of turns in the second coil, so that the linkages set up per ampere will be the same, no matter which is the inducing coil. In considering the mutual inductance of a pair of coils, it is therefore immaterial which coil is supplied with current and which has the voltage induced in it, or, in other words, which is the primary and which the secondary.

In an ordinary inductance the induced voltage is 90° out of phase with the current, so that in a pure reactance the applied voltage leads the current by 90°, but in a circuit which possesses mutual inductance with respect to a second circuit, the voltage is 90° out of phase with the *current in the second circuit.* In a general case, the latter may have any phase whatsoever with respect to the current in the first, so that the voltage set up by mutual inductance may have any phase, and in certain cases it may therefore tend to neutralise the effects of self-inductance.

Skin Effect.—When an alternating current passes through a conductor it does not distribute itself uniformly throughout the cross-section, but tends to concentrate itself in those portions of the conductor which are situated nearest the surface. This is called the *skin effect,* and in certain cases may be of very appreciable magnitude. In order to see the reason for this effect, consider the case of a solid conductor of circular cross-section, and imagine that it is replaced by a large number of small conductors in parallel, the total cross-sectional area being unchanged. This bundle of small conductors must be bunched together so that they occupy the same space as the original conductor, each one carrying a small fraction of the total current. The lines of force set up by a surface element of current will link with the whole conductor, but some of the lines of force set up by an internal element of current will not extend to the surface of the conductor. Thus the surface portions

f the conductor will be linked with less flux than the more central ›ortions and will have less inductance per unit of cross-sectional ›rea. This naturally leads to an unequal distribution of the urrent, the density of which gradually diminishes as the distance rom the surface of the conductor increases. However, the watts ost in a particular conductor for a given current are always greater or a non-uniform than for a uniform distribution of current. To lemonstrate this, take the case of a conductor having a resistance ›f R ohms and carrying a current of I amperes and suppose that his conductor be divided up into n equal parallel filaments. The esistance of one of these filaments is nR ohms, and for a uniform listribution of the current the total watts lost would be

$$n \times \left(\frac{I}{n}\right)^2 nR = I^2R \text{ watts.}$$

Next, consider the case of a non-uniform distribution where each ›ne of half the filaments carries a current of $\left(\frac{I}{n} + I'\right)$ amperes, vhilst each of the other half carries a current of $\left(\frac{I}{n} - I'\right)$ amperes. The total current is I amperes the same as before, but the total vatts dissipated in the non-uniform case would be

$$\frac{n}{2} \times \left(\frac{I}{n} + I'\right)^2 nR + \frac{n}{2}\left(\frac{I}{n} - I'\right)^2 nR$$

$$= \frac{n^2R}{2}\left(2\frac{I^2}{n^2} + 2I'^2\right)$$

$$= I^2R + n^2I'^2R \text{ watts.}$$

Thus the watts lost in a conductor due to ohmic resistance depend ıpon the distribution of the current and are a minimum when the listribution is uniform. If no disturbing factors entered into the ›ase the current would naturally distribute itself uniformly, but vhen this condition is destroyed an increased power is necessary o drive the current through a given conductor. But if the watts ıre still considered as being equal to I^2R, then R must be given a ıigher value, due to the lack of constancy of the current density. This effect is dependent on the frequency, the effective resistance ncreasing as the current goes up. It is very marked in the case of iron and steel conductors, because a greater proportion of the lux set up actually confines itself to the conductor due to its magnetic properties. This is the reason why the voltage drop in steel rails is so much larger with alternating than with continuous currents.

Analogy of Inductance and Inertia.—A self-induced E.M.F. always opposes any change in the current which produces it. It is therefore impossible for an instantaneous change to take place in the

strength of the current, all such changes being necessarily of a gradual character. It takes a definite time for the current to rise after the application of an E.M.F., and a definite time for it to die away after the E.M.F. has ceased to act. The analogy of the inductance of a circuit to the mechanical inertia of a body is suggested. A choking coil may be likened to a flywheel. Neglecting resistance on the one hand and friction on the other, it is seen that a certain amount of energy is required to start the current or the rotation as the case might be. When the full current is flowing or when full speed is attained, no further supply of energy is required to maintain the conditions. If a small amount of resistance or friction be present, a certain amount of power must be supplied to continually overcome these losses. When the flywheel stops, it gives out the whole of its kinetic energy. Similarly, when the current in the choking coil dies down to zero, it gives out all the energy which had previously been supplied to it when the current was started. This effect is studied in detail on page 40.

Starting a Continuous Current.—Suppose a circuit has a resistance of R ohms and an inductance of L henries, and that a continuous E.M.F. of E volts is suddenly applied. The current will rise up to a final value equal to $\frac{E}{R}$; this will not occur instantly, but will take a quite definite time to accomplish. This time will usually be a small fraction of a second.

The applied E.M.F. has to overcome the resistance and also the back E.M.F. set up due to the fact that the current is changing in an inductive circuit. The applied E.M.F. can be expressed as

$$E = Ri + L\frac{di}{dt},\text{[1]}$$

where i is the instantaneous value of the current.

Therefore $$(E - iR)dt = L\,di$$

and $$dt = \frac{L}{E - iR}\,di.$$

Integrating both sides we get

$$\int dt = \int \frac{L}{E - iR}\,di$$

$$t = -\frac{L}{R}\log_\epsilon(E - iR) + K.$$

When $$t = 0, \quad i = 0.$$

[1] Students who are unable to follow the mathematical reasoning given are requested to memorise the result.

Therefore
$$K = \frac{L}{R}\log_\epsilon E$$

and
$$t = -\frac{L}{R}\log_\epsilon (E - iR) + \frac{L}{R}\log_\epsilon E$$
$$= \frac{L}{R}\log_\epsilon \left(\frac{E}{E - iR}\right)$$
$$\frac{Rt}{L} = \log_\epsilon \left(\frac{E}{E - iR}\right)$$
$$\epsilon^{\frac{Rt}{L}} = \frac{E}{E - iR}$$
$$(E - iR)\,\epsilon^{\frac{Rt}{L}} = E$$
$$E - iR = E\epsilon^{-\frac{Rt}{L}}$$
$$iR = E\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$$
$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right).$$

The value of ϵ is approximately 2·718.

As an example, take a circuit having a resistance of 10 ohms and an inductance of 1 henry. If the applied voltage is 100, the expression for the current becomes
$$i = \tfrac{100}{10}\left(1 - \epsilon^{-\frac{10t}{1}}\right)$$
$$= 10\left(1 - \epsilon^{-10t}\right).$$

At the end of 0·1 second after the E.M.F. has been applied the value of the current becomes
$$i = 10\left(1 - 2{\cdot}718^{-1}\right)$$
$$= 6{\cdot}32 \text{ amperes.}$$

At the end of 1 second the value of the current becomes
$$i = 10\left(1 - 2{\cdot}718^{-10}\right)$$
$$= 9{\cdot}999 \text{ amperes.}$$

The ratio $\frac{L}{R}$ is called the *time constant*, and the larger this ratio is, the greater is the time taken by the current in rising to its final value. Theoretically it takes an infinite time to rise to the value given by the ratio $\frac{E}{R}$, but as far as practical measurements are concerned the final value is attained very soon. For instance, in the example quoted above, the current has risen to within 0·01 per cent. of its final value at the end of one second.

Stopping a Continuous Current.—The usual way of stopping a current is by the opening of a switch. What exactly goes on in the circuit during this operation is rather complicated, but considering the problem from the simplest point of view, it may be considered that an enormous resistance is very rapidly introduced into the circuit, this resistance being the air gap between the fixed and the moving contacts of the opening switch. The result of this is to bring the current rapidly to zero, although the presence of inductance in the circuit retards the fall of the current to some extent. This effect is seen when the field winding of a dynamo or motor is suddenly open circuited. The spark which ensues is much more vicious than would be the case if there were no inductance in the circuit. It must be remembered that inductance can be present in a continuous current circuit, just as in an alternating one, since inductance is due to the linkage of magnetic lines of force with ampere-turns. The effects of inductance, however, are only noticeable when the current is varying. Another way of looking at this question of the broken field winding is to consider the large E.M.F. which is suddenly introduced into the circuit at the moment of opening the switch, this E.M.F. being due to the rapid change of linkages in the circuit. This large E.M.F. will cause a spark to persist across the retreating switch contacts for a longer time than would be the case if only the normal E.M.F. of the circuit were acting.

A particular case of the stoppage of a current is where the source of E.M.F. is suddenly removed without opening the circuit. Problems of this character arise in the study of the commutation of contin ous current machines, coils carrying current, but without any E.M.F. being induced in them, being suddenly cut off from the E.M.F. of the remainder of the circuit by the short circuiting action of the brushes. Using the same symbols as in the case where the starting of a current was considered, the equation for the circuit can be written down

$$Ri + L\frac{di}{dt} = 0,$$ [1]

since the applied E.M.F. is zero.

Developing this equation in the same manner as before, we get

$$dt = -\frac{L}{R}\frac{di}{i}.$$

Integrating both sides as before we get

$$\int dt = -\frac{L}{R}\int\frac{di}{i}$$

and

$$t = -\frac{L}{R}\log_e i + K.$$

[1] See footnote, page 26.

When $t = 0$, $i = I$ the initial value of the current.

Therefore $$0 = -\frac{L}{R}\log_\epsilon I + K$$

and $$K = \frac{L}{R}\log_\epsilon I$$

$$t = \frac{L}{R}\log_\epsilon I - \frac{L}{R}\log_\epsilon i$$

$$= \frac{L}{R}\log_\epsilon \frac{I}{i}$$

$$\frac{Rt}{L} = \log_\epsilon \frac{I}{i}$$

$$\epsilon^{\frac{Rt}{L}} = \frac{I}{i}$$

$$i = I\epsilon^{-\frac{Rt}{L}}.$$

The shape of the curve showing the relation between current and time is the same as that for starting a current, except that the

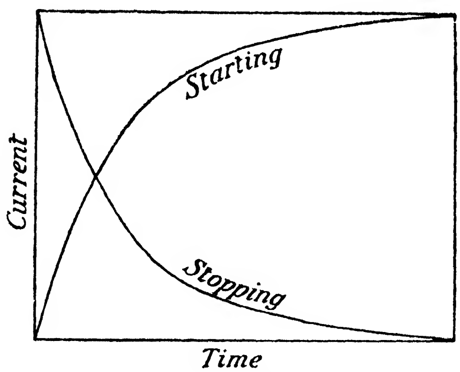


FIG. 15.—Current at Starting and Stopping.

curve appears the other way up. Examples of these two curves are shown in Fig. 15.

CHAPTER IV

VECTORS

Vector Representation.—Imagine a point P moving with a circular motion around a point O called the origin, the distance OP

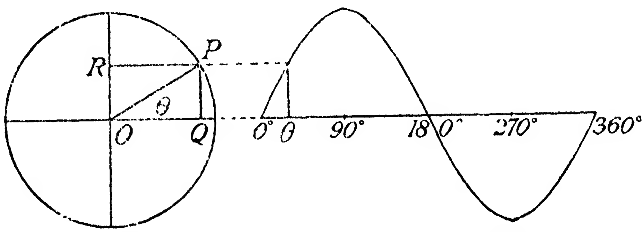


FIG. 16.—Vector Representation of Sine Wave.

being constant. Let $PQ = RO$ be the vertical height of the point P after an angle θ has been swept out. Then

$$\frac{PQ}{OP} = \sin\theta,$$

and since OP is constant, PQ is proportional to $\sin\theta$ and

$$PQ = OP\sin\theta.$$

Thus if OP is made equal to the maximum value of some quantity which is varying according to a sine law, PQ will represent its instantaneous value.

In order to represent a sinusoidal quantity, it is only necessary to draw OP to a scale equal to the maximum value. Then as OP rotates round O as centre, the vertical height of the point P above the horizontal axis will trace out all the instantaneous values throughout the cycle for one complete revolution of OP. Thus in the case of an alternating voltage obeying the law

$$e = E\sin\theta,$$

E is made equal to OP, θ is the angle made by OP with the horizontal, and e is the instantaneous voltage equal to PQ. The line OP is called a *vector*, and voltage is called a vector quantity. By means

of this method of representation all that is necessary to specify a voltage fully is a line representing to scale the maximum value. There is no necessity even to draw in the vertical and horizontal axes unless it be desired. Now imagine the vector to rotate in a counter-clockwise direction at such a speed that it makes one complete revolution for every cycle. The vertical projection of *OP* at any instant of time will represent the value of the voltage at the same instant, provided that the zero position of *OP* corresponds to the zero value of the voltage. If the whole diagram be imagined to rotate in the reverse direction with a speed of one revolution per cycle, or, if it be preferred, if the observer be imagined to rotate forward with the same speed, the line *OP* will appear to be fixed in position, whilst the axes will appear to be travelling in a clockwise direction at the rate of one revolution per cycle. Although *OP* is now apparently fixed in space, its actual position may be anywhere depending upon the relative times when *OP* and the axis pass the same point. The position of *OP* at any instant with respect to its zero position is called its *phase.*

Any quantity which varies according to a sine law can be represented in this way and is called a vector quantity, but there are only four of these with which the student need concern himself at present, viz., alternating voltage, current, M.M.F. and magnetic flux.

Vector Diagram.—When the various quantities in a circuit are represented after this manner in one diagram, the latter is called a *vector diagram*, or, by some writers, a *clock diagram.* Since these are frequently drawn without the axes being put in, it is necessary to indicate which is the moving end and which the origin. For this purpose it is the convention to draw an arrow head on the moving end, which is always made to rotate in a counter-clockwise direction. In order to differentiate further between voltage, current, ampere-turn and flux vectors, different types of arrow head will be used throughout this book as indicated in Fig. 17.

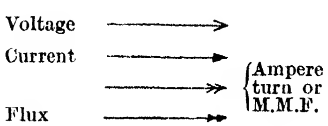


FIG. 17.—Convention of Arrow Heads.

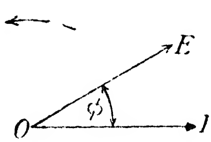


FIG. 18.—Simple Vector Diagram.

Fig. 18 shows an example of a simple vector diagram where the current and voltage are represented simultaneously, the current lagging behind the voltage by an angle ϕ which remains constant. *OI* is drawn to a scale of amperes and *OE* to a scale of volts.

Vector Sum of Two Quantities.—Suppose that there are, in a particular instance, two alternating voltages acting in series, one leading the other by a fixed angle a [see Fig. 19 (a)]. When the

second vector has moved through θ° from the start, the first will have moved through $(\theta + a)^\circ$. At this moment the instantaneous value of E_1 is the vertical projection ON, that of E_2 being the

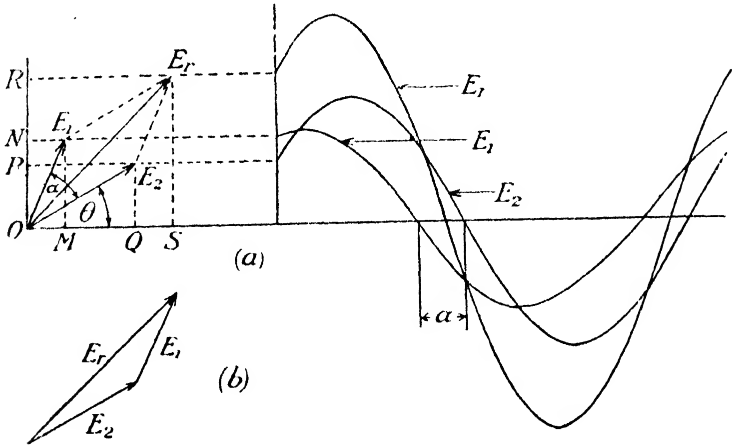


FIG. 19.—Vector Sum of Two Voltages.

vertical projection OP. The resultant instantaneous value is $ON + OP = OR$. Similarly, the resultant horizontal projection is $OM + OQ = OS$. The vector giving OR and OS as its vertical and horizontal projections at this instant is OE_r, which is obviously the diagonal of the parallelogram the sides of which consist of OE_1 and OE_2. The corresponding sine curves are drawn to the right of Fig. 19 (*a*) and serve to emphasise the relationship existing between the vector representation and the graphical representation. A vector diagram of this kind is frequently drawn as shown in Fig. 19 (*b*) for the sake of convenience, just as in drawing a diagram for the parallelogram of forces. Thus voltages, and, of course, currents and fluxes, can be added vectorially just the same as forces or other vector quantities.

Resolution of Vectors.—Vectors can be resolved along any two axes in a similar way to which they are compounded, the majority of cases where this is done requiring a resolution into two components mutually at right angles. Take, for example, the case

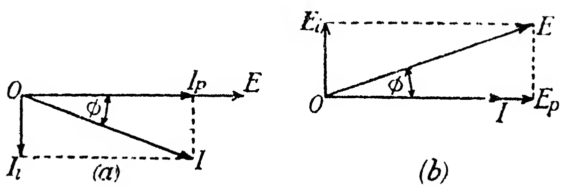


FIG. 20.—Resolution of Vectors.

shown in Fig. 20, where (*a*) the current and (*b*) the voltage is resolved into two components, one in phase with the voltage and current respectively and the other 90° out of phase. The component in

phase is called the power component, whilst that out of phase by 90° is called the idle component because, as will be shown later, there is no net transference of power associated with it. All the power in the circuit must be associated, therefore, with the other component, which is consequently called the power component. Thus both currents and voltages can be split up into power and idle currents and voltages. Take, for example, the case shown in Fig. 20 (a). The total current is split up into a power current of magnitude $I \cos \phi$ and an idle current of magnitude $I \sin \phi$, the angle ϕ being fixed. It must be borne in mind that according to the definitions already given the expressions $I \cos \phi$ and $I \sin \phi$ are the maximum values of the respective components. At any angle θ from the start the values of the two components are $I \cos \phi \sin \theta$ and $I \sin \phi \sin (\theta - 90°)$. The resultant instantaneous value is therefore

$$I \cos \phi \sin \theta + I \sin \phi \sin (\theta - 90°) = I \sin (\theta - \phi),$$

and this is represented by the vector OI. Thus the phase of the resultant can be determined by a knowledge of the magnitude of the two components.

Since the scales to which the vectors are drawn are arbitrarily chosen, the lengths of the vectors may be made to represent the R.M.S. values of the current and voltage, instead of the maximum values; it merely means a different scale.

Vector Diagrams of Simple Series Circuit.—In the case of a circuit consisting of a simple resistance, the vector diagram would be as shown in Fig. 21 (a), the current being in phase with the voltage, whilst in the case of a simple reactance the vector diagram would be as shown in Fig. 21 (b), the current lagging 90° behind the voltage. The actual inclination of the lines does not matter; it is the angle between the various components which is definite. It must be borne in mind that the voltage here represented is only the voltage overcoming resistance or reactance, as the case may be, and does not in any way refer to the total E.M.F. acting in the circuit.

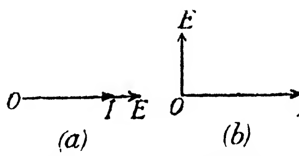


FIG. 21.—Vector Diagram of Simple Series Circuit.

In the case of a circuit containing resistance and reactance in series, the voltage over the combination must be sufficient to overcome both effects, *i.e.* the vectorial sum of RI and $2\pi fLI$ as shown in Fig. 22. From this diagram it is obvious that

$$\begin{aligned} E_r &= \sqrt{(RI)^2 + (2\pi fLI)^2} \\ &= I\sqrt{R^2 + (2\pi fL)^2} \\ &= IZ, \end{aligned}$$

where Z is the impedance and is equal to $\sqrt{R^2 + (2\pi fL)^2}$.

Impedance Diagram.—In Fig. 22 we have a right-angled triangle the sides of which are equal to RI, XI and ZI, X being the reactance and equal to $2\pi fL$. The value of I is the same in all three expressions, since it is a series circuit, and thus another triangle can be

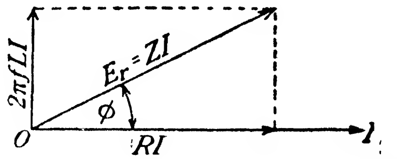


FIG. 22.—Vector Diagram of Circuit containing Resistance and Reactance in Series.

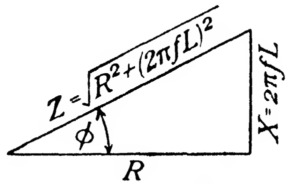


FIG. 23.—Impedance Diagram.

constructed, similar to the first, by dividing each side by the current I. This new triangle is shown in Fig. 23 and is called the impedance diagram. It is *not* a vector diagram, since the lines do not represent vector quantities, but is called a *scalar* diagram. Since it is similar in shape to the vector diagram of voltages and currents, the angle ϕ also represents the angle of lag in the circuit. Thus $\frac{R}{Z} = \cos\phi$ and $\frac{X}{R} = \tan\phi$. The angle of lag of the current behind the voltage is therefore given by $\tan^{-1}\frac{X}{R}$.

Two Impedances in Series.—In the case where there are two impedances in series, each consisting of resistance and reactance in different proportions, the vector diagram takes the form shown in Fig. 24, the resultant voltage being given by Z_rI. If every line in the diagram be divided by the current, a scalar impedance diagram is obtained, similar in every geometrical respect to the vector diagram. In order to determine the value of two impedances in series, it is necessary to know the relative amounts of resistance and reactance in each. The resultant impedance can also be calcu-

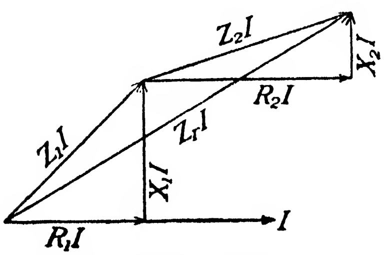


FIG. 24.—Vector Diagram of Two Impedances in Series.

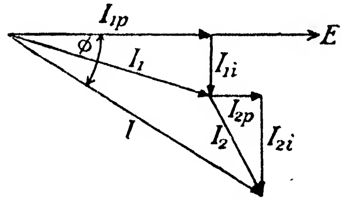


FIG. 25.—Vector Diagram of Two Impedances in Parallel.

lated by means of the various trigonometrical relations, instead of using the graphical method adopted above.

Two Impedances in Parallel.—In calculating the resultant value of two impedances in parallel, a vector diagram of currents is

drawn with reference to the applied voltage. In Fig. 25 each current is shown split up into its power and idle components, I_{1p} and I_{2p} being in phase with the voltage, whilst I_{1i} and I_{2i} are lagging by 90°. The resultant current is given by I, the total power current by $I_{1p} + I_{2p}$, and the total idle current by $I_{1i} + I_{2i}$. The angle of lag of the total current is given by

$$\phi = \tan^{-1}\left(\frac{I_{1i} + I_{2i}}{I_{1p} + I_{2p}}\right).$$

The resultant impedance is given by $\frac{E}{I}$, where I is the resultant current.

The particular case of a resistance and a reactance in parallel is of interest. Referring to Fig. 26, it is seen that the resultant current is $I = \sqrt{I_R^2 + I_X^2}$ and the joint impedance is therefore

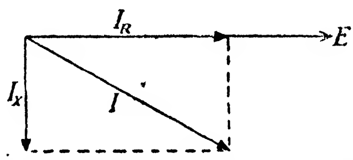


FIG. 26.—Vector Diagram of Resistance and Reactance in Parallel.

$$\begin{aligned} Z &= \frac{E}{\sqrt{I_R^2 + I_X^2}} \\ &= \frac{E}{\sqrt{\left(\frac{E}{R}\right)^2 + \left(\frac{E}{X}\right)^2}} \\ &= \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X}\right)^2}} \\ &= \frac{RX}{\sqrt{R^2 + X^2}}. \end{aligned}$$

Admittance.—The reciprocal of impedance is called admittance and hence

$$\text{Current} = \text{Admittance} \times \text{Voltage}.$$

When dealing with a number of circuits in parallel, it is usually best to work from the point of view of the various admittances, adding them together to get the total admittance. This addition must be performed vectorially, bearing in mind, however, that admittance is not, strictly speaking, a vector, but only a scalar quantity. The rule for addition is, however, the same. Having

determined the total admittance of the circuit, the joint impedance is found by taking the reciprocal.

Summarising the above, in determining the joint impedance of a circuit, the impedances are added for a series and the admittances for a parallel circuit.

Numerical Example.—An example, containing both impedances in series and in parallel, is illustrated in Fig. 27. The impedance of the $R = 20$, $X = 30$ branch is 36·0 apparent ohms, and its admittance is consequently $\frac{1}{36 \cdot 0} = 0 \cdot 0278$ whilst $\tan \alpha = \frac{30}{20}$ and $\alpha = 56 \cdot 3°$. The impedance of the $R = 5$, $X = 60$ branch is 60·2 apparent ohms and its admittance is

$$\frac{1}{60 \cdot 2} = 0 \cdot 0166.$$

$\operatorname{Tan} \beta = \frac{60}{5}$ and $\beta = 85 \cdot 2°$. The resultant horizontal component is

$$0 \cdot 0278 \cos 56 \cdot 3° + 0 \cdot 0166 \cos 85 \cdot 2° = 0 \cdot 0167.$$

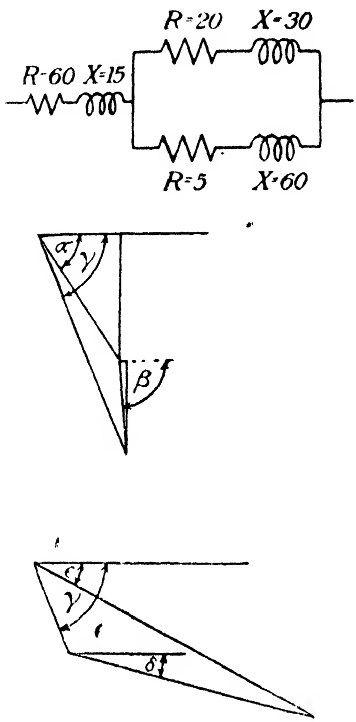


FIG. 27.—Admittance and Impedance Diagrams.

The resultant vertical component is

$$0 \cdot 0278 \sin 56 \cdot 3° + 0 \cdot 0166 \sin 85 \cdot 2° = 0 \cdot 0397.$$

The joint admittance is therefore

$$\sqrt{0 \cdot 0167^2 + 0 \cdot 0397^2} = 0 \cdot 0430,$$

corresponding to an impedance of $\frac{1}{0 \cdot 0430} = 23 \cdot 3$ apparent ohms, whilst the angle γ is

$$\tan^{-1} \frac{0 \cdot 0397}{0 \cdot 0167} = 67 \cdot 2°.$$

Next consider the impedance diagram, adding the joint impedance already obtained to that in series with it. A line is drawn at 67·2° below the horizontal to represent the impedance of 23·3 apparent ohms. The impedance line for the $R = 60$, $X = 15$ branch is

inclined at an angle δ to the horizontal such that $\tan\delta = \frac{15}{60}$; thus $\delta = 14{\cdot}0°$. The resultant horizontal component in the impedance diagram is

$$\begin{aligned} &23{\cdot}3 \cos 67{\cdot}2° + 60 \\ &= 69{\cdot}0. \end{aligned}$$

The resultant vertical component is

$$\begin{aligned} &23{\cdot}3 \sin 67{\cdot}2° + 15 \\ &= 36{\cdot}5. \end{aligned}$$

The joint impedance is therefore

$$\begin{aligned} &\sqrt{69{\cdot}0^2 + 36{\cdot}5^2} \\ &= 78{\cdot}1 \text{ apparent ohms.} \end{aligned}$$

The angle ϵ is

$$\begin{aligned} &\tan^{-1}\frac{36{\cdot}5}{69{\cdot}0} \\ &= 27{\cdot}9°. \end{aligned}$$

The equivalent resistance of the whole circuit is

$$\begin{aligned} &78{\cdot}1 \cos 27{\cdot}9° \\ &= 69{\cdot}0 \text{ ohms,} \end{aligned}$$

whilst the equivalent reactance is

$$\begin{aligned} &78{\cdot}1 \sin 27{\cdot}9° \\ &= 36{\cdot}5 \text{ apparent ohms.} \end{aligned}$$

These results could, of course, have been obtained by scaling off the diagram, but unless great care is taken as to accuracy it is preferable to work out the figures by calculation.

In most cases it is easier to work out problems by means of vector diagrams rather than by the more tedious method of drawing the various sine curves. It must be remembered, however, that the vector diagram is based upon the assumption that the various quantities concerned obey the simple sine law.

CHAPTER V

POWER AND POWER FACTOR

Power in a Circuit.—The power developed in a circuit at a given instant of time is equal to the product of the instantaneous values of the current and voltage. If the current and voltage are obeying a simple sine law, it follows that the magnitude of the power developed varies from instant to instant. The simplest case to consider is that of a circuit containing nothing but resistance, the current being in phase with the voltage. The power curve (see Fig. 28) is obtained by multiplying the instantaneous values of current and voltage throughout the cycle. It will be noticed that this power curve never falls below the zero line, although it touches it at the moment when the current and voltage are zero. Furthermore, the watt curve obeys a sine law with a displaced axis, the frequency being double that of the current or voltage. This can be shown mathematically as follows. Let the voltage and current at any instant be represented by $E_m \sin \theta$ and $I_m \sin \theta$ respectively. The expression for the watts is therefore

$$E_m I_m \sin^2 \theta = E_m I_m (\tfrac{1}{2} - \tfrac{1}{2} \cos 2\theta).$$

Thus the power consists of two components, viz., $\frac{1}{2} E_m I_m$ and $-\frac{1}{2} E_m I_m \cos 2\theta$. The former component is independent of the particular instant of time, whilst the latter obviously obeys a sine law of double frequency. The watt curve is really a sine wave displaced from the original axis by an amount $\frac{1}{2} E_m I_m$. Since the average value of a sine or a cosine taken throughout a complete period is zero, the average value of the power is equal to $\frac{1}{2} E_m I_m$, E_m and I_m being the maximum values of the voltage and current respectively, and since the maximum values are equal to $\sqrt{2}$ times the R.M.S. values, the average power becomes

$$\tfrac{1}{2} (\sqrt{2}E \times \sqrt{2}I) = EI,$$

thus obtaining the same expression as with continuous currents.

Power in a Reactive Circuit.—When a circuit contains both resistance and reactance, the current lags behind the voltage by

an angle ϕ, the value of which depends upon the relative magnitudes of the resistance and reactance. Figs. 29, 30 and 31 illustrate these conditions, the angle ϕ having values of 30°, 60° and 90° respectively. The watt curves are obtained in the same way as before, viz., by multiplying the instantaneous values of current and voltage. Each of these curves cuts the zero line four times in every cycle, the points occurring when either the current or voltage is zero. Also, during those portions of the cycle where the current and applied voltage are acting in opposite directions the power is negative, which means that during these intervals the circuit actually is sending energy back to the source of supply.

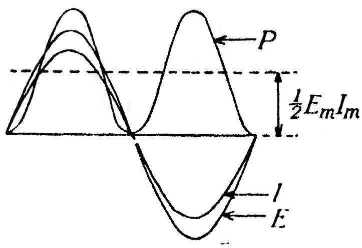


FIG. 28.—Power Curve, $\phi=0°$.

The expression for the power is given by

$$E_m \sin\theta \times I_m \sin(\theta - \phi).$$

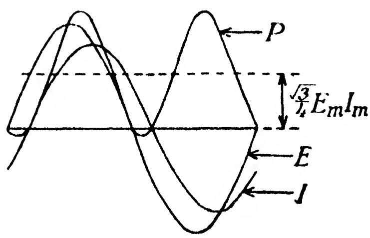


FIG. 29.—Power Curve, $\phi=30$.

Expanding this expression it becomes

$$E_mI_m \sin\theta\,[\sin\theta\cos\phi - \cos\theta\sin\phi]$$
$$= E_mI_m[\sin^2\theta\cos\phi - \sin\theta\cos\theta\sin\phi]$$
$$= E_mI_m[\tfrac{1}{2}\cos\phi - \tfrac{1}{2}\cos 2\theta\cos\phi - \tfrac{1}{2}\sin 2\theta\sin\phi]$$
$$= E_mI_m[\tfrac{1}{2}\cos\phi - \tfrac{1}{2}\cos(2\theta - \phi)].$$

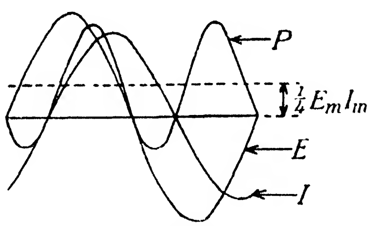


FIG. 30.—Power Curve, $\phi=60°$.

Since the average value of a cosine, taken throughout a complete period, is zero, the average value of the above expression is

$$\tfrac{1}{2}E_mI_m\cos\phi,$$

or

$$EI\cos\phi,$$

considering R.M.S. values.

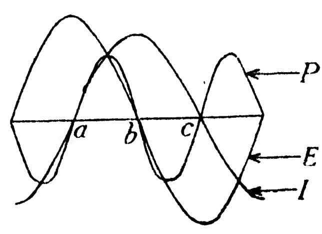


FIG. 31.—Power Curve, $\phi=90°$.

Power Factor.—Since the value of $\cos\phi$ can never be greater than unity, it follows that the power developed in a circuit can never be greater than EI, although it may be less. Obviously the amount of power developed, when the current and voltage are fixed, depends upon the angle of phase difference between the current and voltage. The factor by which the volt-amperes must be multiplied in order

to arrive at the watts is called the *power factor*, the value of which may be anything from unity to zero. Also, it is equal, numerically, to $\cos\phi$ in the case where the simple sine law is obeyed, but where this law is not obeyed it is impossible to speak of the angle of lag with any definite meaning, since it may be different at various parts of the cycle. However, the ratio $\frac{\text{watts}}{\text{volt-amperes}}$ still is called the power factor, and an equivalent sine wave may be substituted for the actual wave, the angle of lag, ϕ, being made such that $\cos\phi$ is equal to the power factor. The product of volts and amperes is termed the apparent power which is measured in volt-amperes.

Power Curves for a Reactive Circuit.—Figs. 28–31 show the power curves for four circuits where the current lags behind the voltage by 0°, 30°, 60° and 90° respectively, the maximum values of current and voltage being the same in each case, and the sine law obeyed throughout. It will be noticed that as the angle of lag increases the proportion of the watt curve below the zero line increases, indicating that the circuit is returning to the source of supply a larger fraction of the energy supplied to it. It follows, therefore, that the net power absorbed by the circuit is less for the same volts and amperes. This is consistent with the fact that the power factors in the four cases are cos 0°, cos 30°, cos 60° and cos 90°, or, numerically, 1·000, 0·866, 0·500 and zero. The particular case of Fig. 31 is worthy of note. The current here is a purely idle one, lagging by 90°, the power factor is zero and the net amount of power supplied is zero. This does not mean that the value of the power at any instant is zero, but means that the circuit gives back to the source of supply as much energy as it receives, and therefore the net transference of power is zero. This is indicated in the watt curve by the portions below the zero line being equal to the portions above the zero line.

The watt curve is the same shape and size for all values of ϕ, but the amount by which the axis of the curve is displaced from the true zero line depends solely upon the value of ϕ. For example, when ϕ is zero and the power factor consequently unity, the watt curve is wholly on one side of the true zero line, just touching it twice every cycle. When ϕ is 60° and the power factor 0·5, the average height of the power curve is

$$\begin{aligned}&\tfrac{1}{2}E_mI_m\cos 60^\circ\\ &=\tfrac{1}{4}E_mI_m,\end{aligned}$$

this being the amount by which the axis of the curve is displaced from the true zero line. The maximum height of the curve in this case is therefore

$$\begin{aligned}&\tfrac{1}{4}E_mI_m+\tfrac{1}{2}E_mI_m\\ &=\tfrac{3}{4}E_mI_m\end{aligned}$$

and the curve will fall below the true zero line by an amount $\frac{1}{4} E_m I_m$.

Power Curves for Non-sinusoidal Wave Form.—When the wave forms of the current and voltage are not sinusoidal the shape of the power curve reflects these irregularities, being obtained as before by the multiplication of the instantaneous values of current and voltage. The average height of this curve represents the average value of the power and can be obtained by drawing a horizontal line halfway between the highest and lowest points on the curve. When the voltage wave form contains irregularities the current wave form will usually differ in shape, as shown in Fig. 32, which illustrates a typical non-sinusoidal case.

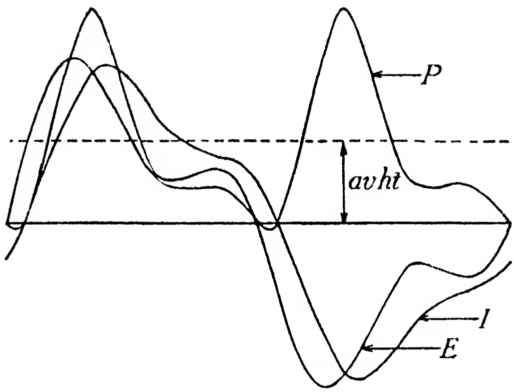


FIG. 32.—Power Curve for Non-sinusoidal Wave Form.

Energy supplied throughout the Cycle.—It is interesting to study the amount of energy which is supplied during a complete cycle. The total amount of energy supplied per cycle is given by the expression

$$\frac{EI \cos \phi}{f} \text{ joules,}$$

where f is the frequency. This energy is not, however, supplied at a constant rate. Taking the instant when the current is zero as the starting point, divide the base line of the power curve into a number of equal divisions. For example, assuming a frequency of 50 cycles per second, the base line might be divided into twenty equal divisions, each representing 0·001 second. Measure the average power in watts during the first interval and multiply by 0·001 to obtain the joules. This gives the total energy supplied up to the end of the first 0·001 second. Repeat this for the second interval of time and add the energy thus obtained to that already supplied, thus getting the total energy transferred up to the end of 0·002 second. Repeat this until the complete cycle has been gone

through. Fig. 33 shows the curves thus obtained in the cases where ϕ is 0°, 30°, 60° and 90°. These curves emphasise the fact that when a certain amount of reactance is present the circuit actually delivers energy back to the source of supply during a certain portion of the cycle. It is as well to inquire into the physical meaning of this. The alternating current sets up an alternating field in phase with it. When the field is being built up energy has to be supplied, this energy being transferred to the magnetic field and stored up in it. But the field is being created in the interval between the instants when the current is zero and when it is a maximum, *i.e.* in the interval between the points *a* and *b* in Fig. 31. During this time the power curve is solely on the positive side of the zero line, indicating that power is being supplied to the field throughout. Referring to the curve for a pure reactance (Fig. 31), it is seen that the amount of energy supplied to the circuit reaches a maximum at the end of a quarter of a period. During the next quarter of a period, represented in Fig. 31 by the interval between

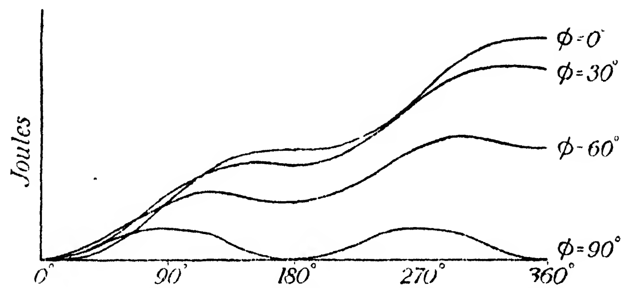


FIG. 33.—Energy supplied throughout a Cycle.

the points *b* and *c*, the current is decreasing from a maximum value down to zero. Since the current and voltage are now acting in opposite directions, the power is negative, which means that the circuit is sending energy back to the source of supply. Hence the power curve in Fig. 31 lies below the zero line between *b* and *c*, whilst the energy curve in Fig. 33 drops from its maximum value down to zero again. This means that the field gives out just as much energy whilst it is being destroyed as was supplied to it whilst it was being built up. Thus at the end of half a cycle the net amount of energy delivered is zero. During the next half-cycle the same process is repeated, the only difference being that the current is flowing in the opposite direction.

When both resistance and reactance are present, the energy curve is the resultant of two components. One of these consists of the energy supplied to heat up the resistance, there being no feeding back to the supply at any portion of the cycle. The heat is, of course, produced at a rate proportional to the square of the

current. The other component is the energy associated with the reactance which is constantly being supplied to the circuit and then given back again to the source of supply.

Measurements of Power by Means of a Wattmeter.—The simplest method of measuring power is by means of a wattmeter (see page 140). A wattmeter has two elements, one of which acts like, and is connected as, an ammeter, being called the *current coil*, whilst the other acts like, and is connected as, a voltmeter, being called the *volt coil* or *pressure coil*. Fig. 34 illustrates the method of connecting a wattmeter so as to measure the power in a circuit.

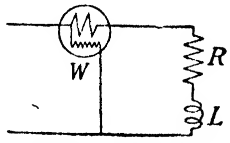


FIG. 34.—Wattmeter Connections.

The wattmeter, used in conjunction with an ammeter and a voltmeter, presents the simplest method of measuring the power factor of a circuit, for

$$\text{power factor} = \frac{\text{watts}}{\text{volts} \times \text{amperes}}.$$

Three Voltmeter Method of Measuring Power.—This is an instructive method of measuring the power in a circuit without the use of a wattmeter. Suppose that it is required to measure the power absorbed by a partially inductive resistance. A non-inductive resistance of somewhere about the same value is chosen, connected in series with the unknown inductive resistance, and an alternating E.M.F. applied to the circuit. The voltage drop over each component part and over the whole circuit must be measured, and also the current flowing. Fig. 35 shows the diagram of connections of

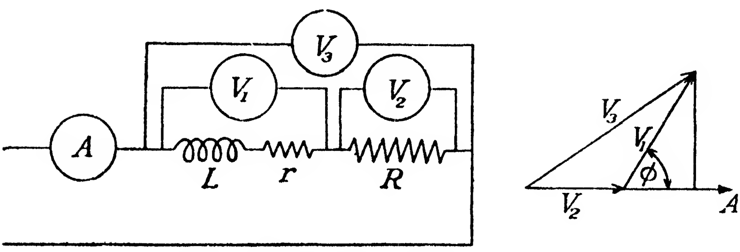


FIG. 35.—Three Voltmeter Method of Measuring Power.

the circuit and explains the symbols used. It also shows a vector diagram of the circuit. The power absorbed in the unknown inductive resistance is obviously

$$P_1 = A V_1 \cos \phi,$$

ϕ being the angle of lag in the inductive resistance.

From the geometry of the figure we have

$$V_3^2 = V_2^2 + V_1^2 + 2V_2V_1 \cos\phi$$

$$= V_2^2 + V_1^2 + \frac{2V_2P_1}{A}.$$

Therefore $$P_1 = \frac{A}{2V_2}(V_3^2 - V_2^2 - V_1^2)$$

$$= \frac{1}{2R}(V_3^2 - V_2^2 - V_1^2).$$

If the value of the resistance R is known there is no necessity for the ammeter A, although usually it is desirable to have it in circuit to avoid overheating. The reason why the resistance R should be chosen so as to be of the same order of magnitude as the unknown resistance is to get the vectors V_1 and V_2 approximating to one another in magnitude and thereby getting the best experimental accuracy with given instruments.

With the addition of a wattmeter to measure directly the power consumed by the unknown inductive resistance, this forms a very valuable experiment for students to perform in the laboratory. By varying the value of the resistance R and adjusting the applied voltage so as to keep the current constant a number of observations of the power absorbed by the unknown inductive resistance can be obtained. These should, of course, all agree with one another. The resistance of the unknown inductive resistance can be obtained by dividing the power absorbed by the square of the current. The value of the resistance obtained in this manner may be larger than the true ohmic resistance, since there may be an appreciable power expended due to iron loss consisting of hysteresis and eddy currents. The reactance can also be determined from the observations already made. Referring to the vector diagram in Fig. 35, it is seen that the voltage overcoming the reactance is given by

$$AX = \sqrt{V_1^2 - (Ar)^2},$$

where r is the equivalent resistance determined as shown above. The reactance can therefore be determined as follows:

$$X = \frac{\sqrt{V_1^2 - (Ar)^2}}{A}$$

$$= \sqrt{\left(\frac{V_1}{A}\right)^2 - r^2}.$$

The power factor of the inductive circuit can be obtained without

the use of the ammeter at all, and without knowing the value of the resistance R, for

$$\text{power factor} = \frac{P_1}{AV_1}$$

$$= \frac{A}{2V_2}(V_3^2 - V_2^2 - V_1^2) \times \frac{1}{AV_1}$$

$$= \frac{(V_3^2 - V_2^2 - V_1^2)}{2V_1V_2}.$$

Three Ammeter Method of Measuring Power.—The principle involved in this method of measuring power is very similar to that in the three voltmeter method, the difference being that, instead of having two component voltages and their vector sum, there are two component currents and their vector sum. For this purpose it is necessary to have two parallel circuits and to measure the currents in the two branches as well as the total current. Fig. 36 shows a

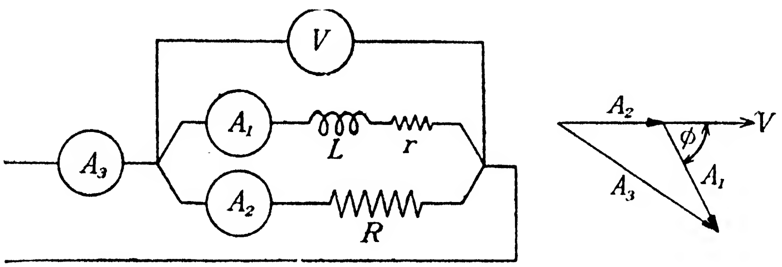


FIG. 36.—Three Ammeter Method of Measuring Power.

diagram of the circuit and the necessary instruments, together with a vector diagram showing how the various quantities are related together. As before, L, r is an unknown inductive resistance, whilst R is a non-inductive resistance, which, if its value is known, renders the voltmeter V unnecessary; otherwise R must be determined from observations of V and A_2.

From the geometry of the vector diagram we get

$$A_3^2 = A_2^2 + A_1^2 + 2A_2A_1 \cos\phi,$$

ϕ being the angle of lag in the branch inductive circuit. The power in the inductive resistance is given by

$$P_1 = A_1V\cos\phi.$$

Therefore $$A_3^2 = A_2^2 + A_1^2 + \frac{2A_2P_1}{V}$$

and $$P_1 = \frac{V}{2A_2}(A_3^2 - A_2^2 - A_1^2)$$

$$= \frac{R}{2}(A_3^2 - A_2^2 - A_1^2),$$

whilst the power factor of the inductive circuit is given by

$$\frac{P_1}{A_1 V} = \frac{V}{2A_2}(A_3^2 - A_2^2 - A_1^2) \times \frac{1}{A_1 V}$$

$$= \frac{(A_3^2 - A_2^2 - A_1^2)}{2A_1 A_2}$$

and may be determined without the use of the voltmeter at all.

As in the previous case, it is desirable to have the magnitudes of A_1 and A_2 more or less of the same order. Knowing the watts, volts and amperes in the inductive resistance, it is a simple matter to determine the reactance and the equivalent resistance.

The student can check this reasoning experimentally by inserting a wattmeter in the circuit so as to measure the current through and the volts across the inductive resistance. By keeping the voltage constant and varying the value of R, a number of observations can be made, all of which should, of course, give the same value for the power absorbed in the partially inductive resistance. If the wattmeter be connected so as to measure the power absorbed by the whole circuit, which would be the case if the current through A_3 were measured instead of that through A_1, this also can be made to serve as a check on the voltmeter and ammeter readings. The total power in both branches is given by

$$P_1 + P_2 = A_1 V \cos \phi + A_2 V$$

$$= \frac{R}{2}(A_3^2 - A_2^2 - A_1^2) + A_2^2 R$$

$$= \frac{R}{2}(A_3^2 + A_2^2 - A_1^2).$$

The same reasoning can be applied to the case of the three voltmeters.

A point of practical importance in carrying out either of these tests is that the instrument losses must be negligible, *i.e.* the voltmeter must take a negligible current and the ammeters must absorb a negligible voltage. A disadvantage of both methods is that the results depend upon the differences of the squares of the observed quantities and this tends to magnify the experimental errors.

Wattful and Wattless Components.—Very frequently it is desirable to consider the current as split up into two components, one in phase with the voltage and the other 90° out of phase, or, as it is sometimes termed, in *quadrature* with the voltage. In certain cases it is the voltage which is thus resolved, but the method of treatment is just the same.

Fig. 37 shows the vector diagram of a circuit where the current is lagging by an angle ϕ behind the voltage. The current is resolved

into two components as indicated above. The magnitude of that component of the current in phase with the voltage is $I_p = I \cos \phi$, whilst the magnitude of the component in quadrature with the voltage is $I_i = I \sin \phi$. When the former component is multiplied by the voltage it gives the total power in the circuit and is called by various writers the "energy," "power," or "wattful" component, because all the watts in the circuit are associated with it. There is no net power associated with that component of the current which is in quadrature with the voltage, for the expression for the instantaneous value of the power, as far as this component is concerned, is,

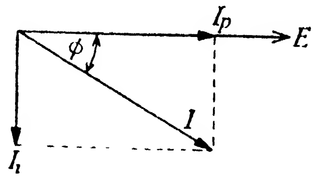


FIG. 37.—Wattful and Wattless Currents.

$$E_m \sin \theta \times I_m \sin \phi \sin (\theta - 90^\circ)$$
$$= -\tfrac{1}{2} E_m I_m \sin 2\theta \sin \phi.$$

Since the average value of $\sin 2\theta$ is zero, there is no net power. This component of the current is therefore called the "idle" or "wattless" component.

Energy Stored in the Magnetic Medium.—Whilst there is no continued expenditure of energy involved in the passage of a current through an inductance, yet there is a quite definite, though small, amount of energy expended in building up the field, the whole of which is returned to the circuit when the field is destroyed. This energy is stored up in the magnetic medium and can be calculated in the following way.

Consider a theoretical circuit having an inductance of L henries and absolutely no resistance, and that an E.M.F. is applied to it in such a manner that the current increases at a constant rate. Then

$$E = LI'$$

where E is the instantaneous value of the applied E.M.F. and I' is the rate of growth of the current measured in amperes per second (see page 17). Since both L and I' are assumed constant, the applied E.M.F. must also be constant. In other words, a constant voltage suddenly applied to a circuit containing only inductance would produce a uniformly increasing current, this going on indefinitely. Since the resistance cannot be eliminated in actual practice, the current tends towards a limiting maximum value $\frac{E}{R}$, as shown on page 26. Returning to the theoretical case with no resistance, suppose that the voltage be applied until the current reaches the value of I amperes. The time necessary for this is

$\frac{I}{I'}$ seconds and the average power expended during this interval of time is

$$E \times \tfrac{1}{2}I,$$

since E is constant throughout and $\frac{1}{2}I$ is the average current.

The total energy supplied is

$$\tfrac{1}{2}EI \times \frac{I}{I'}$$

$$= \tfrac{1}{2}LI'I \times \frac{I}{I'}$$

$$= \tfrac{1}{2}LI^2 \text{ joules.}$$ [1]

The total energy stored up in the magnetic field is independent of the rate at which the field was produced, and so it does not matter whether it was produced at a uniform rate or not.

As an example, the energy stored up in the magnetic field of a circuit having an inductance of 0·2 henry, when a current of 5 amperes is flowing, is

$$\tfrac{1}{2} \times 0{\cdot}2 \times 5^2 = 2{\cdot}5 \text{ joules.}$$

The whole of this energy is restored to the circuit when the magnetic field is destroyed.

If a sinusoidal E.M.F. be applied to a pure reactance having an inductance of L henries the resulting current will lag by 90°. The magnetic field is completely built up in the interval between the times when the current is zero and at its maximum value of I_m amperes. During this interval the voltage is decreasing from the maximum value of E_m volts to zero. The expression for the power, taking the time when the current is zero as a starting point, is

$$E_m \sin(\theta + 90°) \times I_m \sin\theta$$

$$= 2\pi fLI_m{}^2 \sin\theta \sin(\theta + 90°)$$

$$= \pi fLI_m{}^2 \sin 2\theta.$$

The maximum value of the instantaneous power is, therefore,

$$\pi fLI_m{}^2 \text{ watts,}$$

[1] This can be determined quite simply by the aid of the calculus, as follows :—

$$E = L\frac{di}{dt}.$$

$$EI = P = LI\frac{di}{dt}.$$

$$\int Pdt = \int LI\,di.$$

Integrating between the limits 0 and I :—

$$\text{Energy} = \tfrac{1}{2}LI^2.$$

and hence the average value of the power during the interval taken is

$$\frac{2}{\pi} \times \pi f L I_m^2 \text{ watts (see page 11)}$$
$$= 2fLI_m^2 \text{ watts.}$$

Since the duration of this interval of time is $\frac{1}{4f}$ seconds, the total energy supplied in building up the field is therefore

$$2fLI_m^2 \times \frac{1}{4f}$$
$$= \tfrac{1}{2}LI_m^2 \text{ joules.}$$

CHAPTER VI

CAPACITY AND CONDENSERS

Condensers.—When two conducting bodies are separated by a dielectric they are said to possess *capacity* and the combination is called a *condenser*. If a difference of potential be applied to the two conducting bodies, no current actually flows *through* the condenser, unless the insulation be broken down, but the conducting plates become charged. This means that a certain definite quantity of electricity is stored up on the plates, one of which is positively and the other negatively charged. The amount of charge which a condenser will take in given conditions depends upon its dimensions and the material of the dielectric, being proportional to the area of the conducting plates and inversely proportional to the thickness of the separating dielectric. If a number of condensers be taken, having equal dimensions but with various materials as the dielectric, it will be found that they take different charges for the same potential difference. This is due to a property of the dielectric known as its *specific inductive capacity*, or *Dielectric Constant*, corresponding in some ways to specific resistance in the case of a conductor. The specific inductive capacity of air is taken as unity and the following table gives the relative specific inductive capacities of various other dielectrics.

TABLE OF SPECIFIC INDUCTIVE CAPACITIES.

Glass	6—10
Paraffin wax	1·7—2·3
Indiarubber, pure	2·1
,, vulcanised	2·7
Shellac	3·1
Mica	5
Ebonite	2·2—3·2

Thus a condenser having mica as its dielectric would take five times as much charge for a given potential difference as it would if air were substituted for the mica.

Commercial Forms of Condensers.—From what has been shown above it will readily be seen that, in commercial forms of condensers, dielectrics such as mica, paraffin wax, and oiled paper will take the place of air, in so far as the use of such materials will reduce sub-

stantially the size of the condenser for a given duty. Furthermore, as such materials will stand greater electrical pressures for a given thickness, it enables condensers to be built for higher voltages without unreasonably adding to their dimensions.

On account of the high cost of mica sheet suitable for this work, mica condensers are but little used. They will, however, stand a higher voltage than paper condensers for a given thickness of dielectric. Mica sheets about 2 mils thick are used in building up condensers of this type, care being taken to exclude all air-bubbles when assembling the sheets.

Paper insulated condensers are now largely used, a familiar example of this type being the *Mansbridge* condenser. The latter is made of paper coated on one side with a thin metal layer like the so-called silver paper, and has the great advantage that if it is broken down by the application of too large a voltage the fault automatically heals itself. This is due to the fact that since the film of metal is exceedingly thin, it is readily disintegrated by the heating effect of a spark. Thus the tin coating is made discontinuous in the near neighbourhood of the puncture and the short circuit is removed.

Some condensers with an adjustable range have a small plug-board outside the case, arranged so as to connect the various sections of the condenser in parallel when so desired. Others have a layer of tin foil and paper which can be wound and unwound on a drum. In this manner, alternate layers of tin foil and paper are obtained on this drum, these being separately coiled up on other drums when they are unwound.

A form of glass condenser which has a considerable application in the protection of high voltage overhead transmission lines is the *Moscicki* condenser illustrated in Fig. 38. This consists of a specially formed glass tube closed at one end and coated inside and out with chemically deposited silver, which is protected by a further deposit of copper. The glass tube is thickened in the neighbourhood of the neck and edge, experience having shown this to be necessary, as most of the breakdowns occur near the edge. This arrangement is immersed in a mixture of glycerine and water contained in an outer metal tube. An insulator is mounted upon the neck of the glass tube, and carries the terminal which is connected to the inner coating of the condenser.

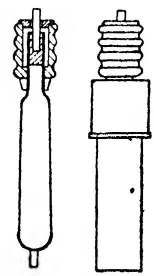


FIG. 38.—Moscicki Condenser.

Unit of Capacity.—The charge which a given condenser will store up is proportional to the charging voltage. Since the charge consists of a quantity of electricity, it is measured in coulombs on the practical system of units. Thus we have

$$Q = CE,$$

where Q is the charge measured in coulombs, E is the charging voltage, and C is some constant. If a particular condenser takes a charge of one coulomb when a potential difference of one volt is applied, the value of the constant C is also unity. Such a condenser is said to possess unit capacity, and the name of the unit is the *farad*. Thus the capacity of any condenser is given by the expression

$$C = \frac{Q}{E},$$

or, expressed in words, the capacity of a condenser in farads is equal to the coulombs per volt. Since it would require a condenser of enormous size to take a charge of one coulomb for a difference of potential of one volt, a very much smaller unit must be devised for practical purposes. Such a unit is the micro-farad, which is one-millionth of a farad—

$$1 \text{ micro-farad} = 10^{-6} \text{ farad.}$$

The condensers which are met with in practice usually range from anything up to, say, 10 mfds. (micro-farads).

Analogy between Condenser and Gas Globe.—A condenser may be considered analogous to a globe filled with gas, the gas in the globe corresponding to the charge in the condenser. The weight of gas which can be stored in the globe is proportional to the gas pressure : similarly, the amount of charge in the condenser is proportional to the electrical pressure. If the gas be changed from, say, hydrogen to oxygen, the weight of gas which can be stored in the globe at a given pressure is increased : similarly, if the dielectric in the condenser be changed from, say, air to mica, the charge for a given difference of potential is increased. A change in the material of the globe does not affect the weight of gas stored up ; similarly, a change in the material of the plates of the condenser does not affect the electrical charge.

Analogy of Rubber Diaphragm in a Tube.—Another analogy may be drawn between a condenser and a tube in the middle of which a rubber diaphragm separates the two ends. Water cannot flow straight through the tube because there is no through passage : similarly, a current cannot flow straight through the condenser because of the layer of insulation between the two plates. If a difference of water pressure be set up between the two ends of the tube, the rubber diaphragm will be distended in one direction. One side of the tube will admit a little water ; the other side will lose a little. When an electrical pressure is applied to a condenser, one plate receives a little extra electricity, *i.e.* becomes positively charged, whilst the other plate loses a little electricity, *i.e.* becomes negatively charged. If the water pressure be continually increased, there comes a time when the rubber diaphragm bursts and allows

a through passage of the water. If the electrical pressure be continually increased, there comes a time when the insulation breaks down and forms a conducting path right through the condenser. If an alternating water pressure be applied to the tube, the diaphragm is distended in each direction alternately and there is a continual slight flow of water into and out of the tube. If an alternating electrical pressure be applied to the condenser, there is a continual flow of electricity into and out of the condenser which makes it appear as if an alternating current were flowing *through* the condenser.

Condenser on Alternating Voltage.—Since the coulombs stored up in the condenser are proportional to the charging voltage, any change in the latter produces a corresponding change in the former. This means that some coulombs must flow into or out of the condenser and corresponds to a current the magnitude of which is proportional to the rate of change of the voltage. For example if a condenser of one farad capacity be connected to a difference of potential of 10 volts, there will be 10 coulombs stored up in the plates. If the voltage is uniformly decreased to 4 volts in one second, the voltage is changing at the rate of 6 volts per second, resulting in a change of charge of 6 coulombs per second. It will appear, as far as an ammeter is concerned, as if a current of 6 amperes were flowing through the condenser, but it must be remembered that there is no passage of the current from one plate to another of opposite polarity, although it is usually spoken of as a current in the ordinary sense of the word. Strictly speaking, it is simply a case of the charge flowing into or out of the condenser as the case may be.

The effect of an alternating E.M.F. on a condenser will now be discussed, a sinusoidal wave form being assumed.

Let the applied E.M.F. be represented by

$$E_m \sin \theta.$$

As shown on page 17, the maximum rate of change of voltage is E_m volts per radian or $2\pi f E_m$ volts per second, and is positive when the actual value of the voltage is zero and increasing. But since

$$Q = CE,$$

$$\begin{aligned}\text{Current} &= \text{rate of change of } Q\\ &= C \times \text{rate of change of } E.\end{aligned}$$

Therefore

$$\begin{aligned}I_m &= C \times \text{maximum rate of change of } E\\ &= C \times 2\pi f E_m.\end{aligned}$$

Considering R.M.S. values

$$I = 2\pi fCE.$$[1]

On examining the curves in Fig. 39, which represents the conditions in a capacity circuit, it is seen that the charging current leads the voltage by 90°, being exactly opposite in this respect to an inductance. The reason for this is that, in an inductance, the applied voltage is equal and opposite to the back voltage set up, this being proportional to the rate of change of the current, whilst, in a condenser, the current is proportional to the rate of change of voltage.

Physical Meaning of Charging Current.—At the commencement of the cycle, when the value of the voltage is zero, the rate at which

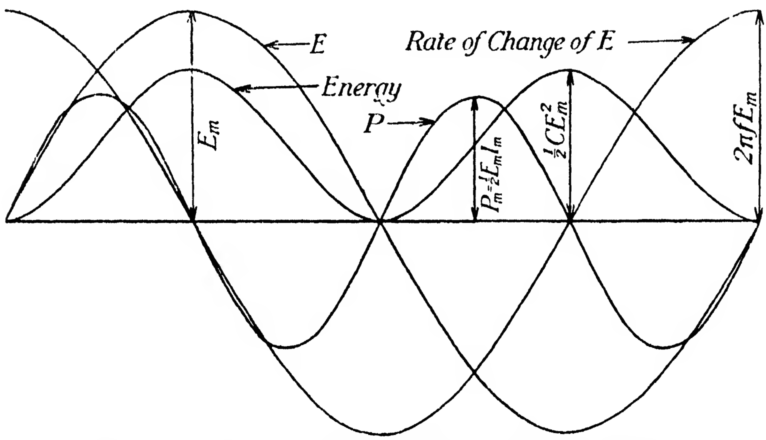


FIG. 39.—Power and Energy Curves for a Condenser Circuit.

the voltage is increasing is a maximum, and consequently the rate at which the charge is increasing is a maximum. The current flowing into the positive plate is, therefore, a maximum at this instant. During the progress of the next quarter of a cycle the

[1] The current can be calculated by means of the calculus, thus :—

$$\begin{aligned} I &= \frac{dQ}{dt} \\ &= C\frac{dE}{dt} \\ &= C\frac{d(E_m \sin\theta)}{dt} \\ &= CE_m \times \frac{d(\sin\theta)}{d\theta} \times \frac{d\theta}{dt} \\ &= CE_m \cos\theta \times 2\pi f \\ &= 2\pi fCE_m \sin(\theta + 90°). \end{aligned}$$

This shows that the current leads the voltage by 90°.

rate at which the voltage is increasing continually diminishes and the current flowing into the positive plate continually decreases until, at the moment when the voltage is no longer increasing, the current falls to zero. All this time current has been flowing into the positive plate of the condenser which contains its maximum charge at the moment when the current ceases to flow in. Thus, at the moment of maximum voltage, the charge on the plates is a maximum, but the current is zero. As the voltage begins to decrease, the charge also begins to decrease, and this results in an apparent current flowing the other way through the condenser. Really it is the charge flowing out of the plates. The voltage continues to decrease at a greater and greater rate and, consequently, the current continues to increase. This goes on until the voltage has fallen to zero and the condenser is completely discharged. The voltage now begins to rise in the other direction and the complete chain of events is repeated in the next half-cycle. In a complete cycle, the condenser is twice charged and discharged, once in each direction.

Power and Power Factor of Capacity Circuit.—It has been shown that the current leads the voltage by 90°, and if the voltage is represented by the expression

$$e = E_m \sin\theta$$

the current will be represented by the expression

$$i = I_m \sin(\theta + 90^\circ).$$

The instantaneous value of the power will be represented, therefore, by the expression

$$\begin{aligned} ei &= E_m I_m \sin\theta \sin(\theta + 90^\circ) \\ &= \tfrac{1}{2} E_m I_m \sin 2\theta. \end{aligned}$$

Since the average value of $\sin 2\theta$, taken throughout a complete cycle, is zero, it follows that the average power supplied to the condenser is zero. The power factor of a circuit containing only capacity is, therefore, also zero. This can be realised when it is remembered that no heat is produced and no work done. As a matter of fact, in commercial condensers, there is a slight energy loss in the dielectric, but the mechanism of this action is somewhat complicated and lies outside the scope of this book. There is also a minute I^2R loss, due to the fact that the current has to flow over the plates which have a definite ohmic resistance. The result of these two actions is to raise the power factor from zero to a value which usually lies between 0·003 and 0·01. For ordinary purposes, the power factor can be taken as zero.

As in the case of an inductance, although the average power supplied is zero, yet the power at a given instant may have quite a definite value. The power curve of a condenser is shown in Fig. 39.

It is seen that the condenser is receiving charge up to the moment when the voltage attains a maximum and that it is being supplied with energy during this interval. During the next quarter of a period, the voltage falls to zero and the condenser is discharged. The power curve shows that during this interval the condenser delivers back to the line the same amount of energy that it previously received, and this chain of events is continually repeated. The fourth curve in Fig. 39 shows the amount of energy stored up in the condenser at different moments throughout the cycle.

Energy of Charge.—In order to determine the energy associated with the charge, assume that a uniformly increasing voltage is applied at the rate of E' volts per second for a period of t seconds. The total voltage applied, E, is equal to $E't$. If the capacity of the condenser be C farads, then the charge on the plates is equal to CE coulombs. Since the rate of change of voltage is constant the current will be constant and equal to

$$I = \frac{Q}{t} = \frac{CE}{t} \text{ amperes.}$$

The average power supplied is

$$\begin{aligned} &\tfrac{1}{2} E \times I \\ &= \tfrac{1}{2} E \times \frac{CE}{t} \text{ watts.} \end{aligned}$$

The total energy supplied is

$$\begin{aligned} &\tfrac{1}{2} E \times \frac{CE}{t} \times t \\ &= \tfrac{1}{2} CE^2 \text{ joules.[1]} \end{aligned}$$

In Fig. 39 the height of the energy curve at the end of a quarter of a period is equal to $\frac{1}{2}CE_m{}^2$ joules. It is immaterial whether the

[1] Taking the case when a sinusoidal E.M.F. is applied, this can be proved by means of the calculus as follows:—

Instantaneous power $= ei = \frac{1}{2}E_m I_m \sin 2\theta$
$= \frac{1}{2}E_m I_m \sin 2pt$

where $p = 2\pi f$ and $t =$ time in seconds.

Energy supplied during a quarter of a cycle

$$= \int_0^{\frac{\pi}{2p}} ei\,dt = \int_0^{\frac{\pi}{2p}} \tfrac{1}{2}E_m I_m \sin 2pt\,dt$$

$$= \left[-\frac{1}{4p} E_m I_m \cos 2pt\right]_0^{\frac{\pi}{2p}}$$

$$= \frac{E_m I_m}{2p}.$$

But $I_m = 2\pi f C E_m = pCE_m.$

Therefore energy $= \frac{1}{2}CE_m{}^2.$

voltage is applied at a uniform rate or not, the energy associated with a given charge being dependent only on the magnitude of the applied voltage and independent of the rate at which it is applied.

When the applied voltage obeys a sine law the instantaneous value of the power is given by

$$\begin{aligned} ei &= \tfrac{1}{2} E_m I_m \sin 2\theta \\ &= \tfrac{1}{2} E_m \times 2\pi f C E_m \sin 2\theta \\ &= \pi f C E_m^2 \sin 2\theta \text{ watts.} \end{aligned}$$

The maximum value of the power is, therefore,

$$\pi f C E_m^2 \text{ watts}$$

and the average value, taken over a quarter-period, is

$$\frac{2}{\pi} \times \pi f C E_m^2 \text{ watts.}$$

The total energy supplied in a quarter of a cycle is

$$\begin{aligned} &\frac{2}{\pi} \times \pi f C E_m^2 \times \frac{1}{4f} \\ &= \tfrac{1}{2} C E_m^2 \text{ joules.} \end{aligned}$$

Dependence of Current on Voltage, Capacity and Frequency.—If any two of these quantities be kept constant and the third one varied, the current will be directly proportional to the quantity which is varied. Fig. 40 shows the relations which exist in a particular circuit, the data being given in the diagram.

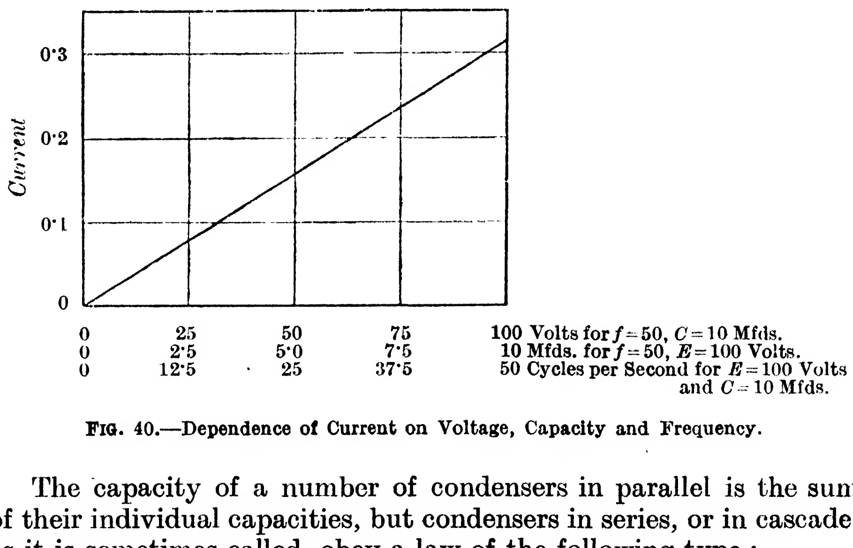


FIG. 40.—Dependence of Current on Voltage, Capacity and Frequency.

The capacity of a number of condensers in parallel is the sum of their individual capacities, but condensers in series, or in cascade, as it is sometimes called, obey a law of the following type :—

$$\frac{1}{\text{joint capacity}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Impedance of a Capacity Circuit.—The relation between current and voltage in a capacity circuit is given by

$$I = 2\pi fCE,$$

and since the impedance of a circuit is given by the ratio of volts to amperes, it follows that

$$Z = \frac{E}{I} = \frac{1}{2\pi fC}.$$

The formula is similar to the corresponding one for a purely inductive circuit with the important exception that $2\pi fC$ comes in the *denominator* whilst $2\pi fL$ comes in the *numerator* of the respective expressions. The quantity $\frac{1}{2\pi fC}$ is called the *capacity reactance.*

This relationship enables us to calculate the capacity of the circuit if the voltage, current and frequency are known; for accurate measurements, however, this method is not to be commended, as the presence of a condenser in the circuit has the effect of magnifying any distortion of the current wave form, and the factor 2π is only correct in the case of a sine wave.

Circuit containing Resistance and Capacity.—In order to force a current through a circuit containing both resistance and capacity

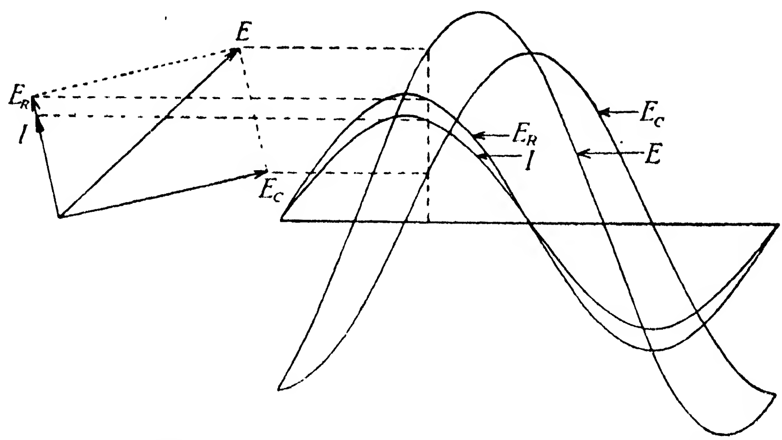


FIG. 41.—Voltage Curves for Circuit containing Resistance and Capacity.

in series, a voltage must be applied capable of overcoming both the ohmic resistance and the impedance of the condenser. The magnitude of the former component is IR volts and is in phase with the current, whilst the magnitude of the latter component is $\frac{1}{2\pi fC} \times I$ and lags behind the current by 90°. The resultant voltage which has to be applied is the vector sum of these two components and will lag behind the current by some angle less than 90°.

Fig. 41 shows graphs of these various quantities and also a vector diagram of the circuit. It is seen that the resultant voltage E is equal to

$$\sqrt{E_R{}^2 + E_C{}^2}$$

$$= \sqrt{(RI)^2 + \left(\frac{1}{2\pi fC} \times I\right)^2}$$

$$= I\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}.$$

The impedance Z is equal to

$$\frac{E}{I} = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}.$$

As an example, take the case of a circuit consisting of a resistance of 40 ohms in series with an adjustable condenser, the voltage being 100 and the frequency 50. When the value of the condenser is 60 mfds. the current is

$$I = \frac{100}{\sqrt{40^2 + \left(\frac{10^6}{2\pi \times 50 \times 60}\right)^2}}$$

$$= 1{\cdot}507 \text{ amperes.}$$

The voltage across the resistance is

$$E_R = 40 \times 1{\cdot}507$$

$$= 60{\cdot}3 \text{ volts.}$$

The voltage across the condenser is

$$E_C = \frac{10^6}{2\pi \times 50 \times 60} \times 1{\cdot}507$$

$$= 79{\cdot}9 \text{ volts.}$$

The values of the current and impedance for various values of the capacity are shown graphically in Fig. 42.

Circuit containing Resistance, Inductance and Capacity.—When a circuit contains all these three quantities, peculiar conditions are set up due to the various phase relationships. In the case of an inductance the current lags by 90° behind the voltage, whilst in the case of a capacity the current leads by 90°. If these two are in series the current must have the same phase throughout, and consequently the voltage must have a phase difference of 180° over these two portions of the circuit. This means that the two voltages are diametrically opposite and the combined voltage over the two is their arithmetic difference. It is quite possible for

each of these component voltages to be considerably in excess of the impressed E.M.F. Fig. 43 shows the vector diagram and also

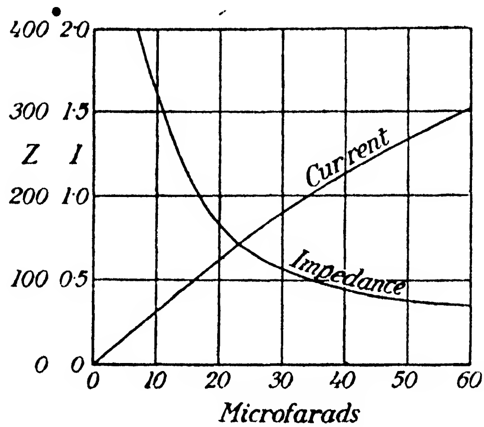


FIG. 42.—Variation of Current and Impedance with Capacity.

the various sine curves for a simple series circuit containing resistance, inductance and capacity. The resultant voltage may lead or lag

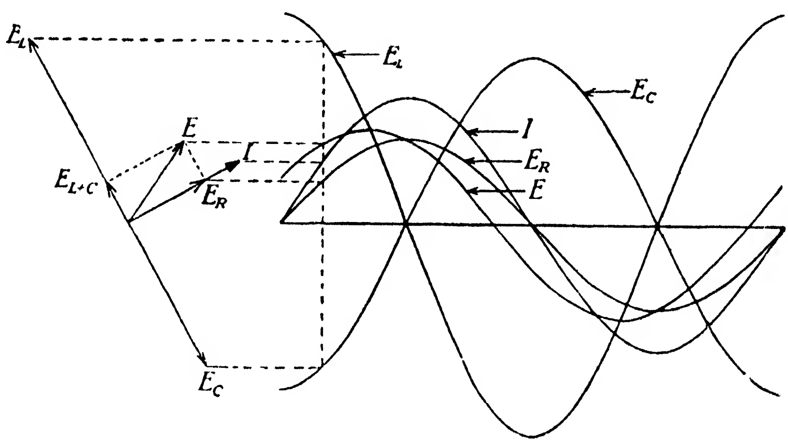


FIG. 43.—Voltage Curves for Circuit containing Resistance, Inductance and Capacity.

behind the current, this depending on whether the inductance or the capacity predominates. The magnitude of the voltage absorbed by the inductance plus the capacity is given by

$$E_L - E_C = 2\pi f L I - \frac{1}{2\pi f C} \times I$$

$$= I\left(2\pi f L - \frac{1}{2\pi f C}\right).$$

Combining this with the voltage absorbed by the resistance, we get, for the total applied voltage,

$$E = \sqrt{I^2R^2 + I^2\left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= I \times \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}.$$

The impedance of the circuit is therefore given by

$$Z = \frac{E}{I} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}.$$

If $\frac{1}{2\pi fC}$ is greater than $2\pi fL$, the quantity inside the bracket is negative and this signifies a leading current. However, the square of this quantity is always positive, so that the impedance can never be less than R, no matter what may be the values of $2\pi fL$ and $\frac{1}{2\pi fC}$.

Resonance.—When a circuit contains both inductance and capacity it is said to possess *resonance.* The effects may be likened to those produced on a pendulum in which considerable vibrations can be set up by the successive application of very small blows, providing these blows are timed correctly. The frequency of the blows corresponds to the frequency of the applied voltage. In the case of the pendulum, if the blows are applied at a different rate the resulting swing will be considerably less, even though the impulses may be of greater magnitude. Similarly, in the electrical case, if the frequency be changed, the current will be considerably reduced.

It is obvious from the expression for the impedance that its minimum value occurs when

$$2\pi fL - \frac{1}{2\pi fC} = 0.$$

Of course, a decrease in the resistance will always lower the impedance, and *vice versa*. The resonance is only affected by the inductive and capacity portions of the circuit. With a given amount of inductance and capacity there is one particular value of the frequency which will give maximum resonance. This is obtained when

$$2\pi fL - \frac{1}{2\pi fC} = 0.$$

Then
$$2\pi fL = \frac{1}{2\pi fC},$$
$$(2\pi f)^2 = \frac{1}{LC}$$
and
$$f = \frac{1}{2\pi\sqrt{LC}}.$$

Resonance of a lesser amount will occur at other frequencies.

If a circuit contains a fixed amount of inductance and the frequency is kept constant, then maximum resonance can be obtained by varying the capacity in the circuit. The necessary conditions are obtained when
$$C = \frac{1}{(2\pi f)^2 L}.$$

As an example, take the case of a series circuit containing a capacity of 50 mfds., an inductance of 0·2 henry and a resistance of 5 ohms. The frequency for maximum resonance is
$$f = \frac{1}{2\pi\sqrt{0{\cdot}2 \times 50 \times 10^{-6}}}$$
$$= 50{\cdot}33 \text{ cycles per second.}$$

At this frequency the impedance is exactly 5 ohms, so that if the voltage were 100 the current would be 20 amperes.

The voltage across the condenser is given by
$$E_C = \frac{I}{2\pi fC}$$
$$= \frac{20 \times 10^6}{2\pi \times 50{\cdot}33 \times 50}$$
$$= 1268 \text{ volts}$$

The voltage across the inductance is given by
$$E_L = 2\pi fLI$$
$$= 2\pi \times 50{\cdot}33 \times 0{\cdot}2 \times 20$$
$$= 1268 \text{ volts}$$

If the frequency falls to 49, then the impedance would be
$$Z = \sqrt{5^2 + \left(2\pi \times 49 \times 0{\cdot}2 - \frac{10^6}{2\pi \times 49 \times 50}\right)^2}$$
$$= 6{\cdot}02 \text{ apparent ohms.}$$

The current would be

$$I = \frac{E}{Z} = \frac{100}{6 \cdot 02} = 16 \cdot 6 \text{ amperes}$$

and would lead the voltage since $\frac{1}{2\pi f C}$ is now greater than $2\pi f L$.

The voltage now across the condenser is

$$E_C = \frac{16 \cdot 6 \times 10^6}{2\pi \times 49 \times 50}$$
$$= 1078 \text{ volts,}$$

and the voltage across the inductance is

$$E_L = 2\pi \times 49 \times 0 \cdot 2 \times 16 \cdot 6$$
$$= 1022 \text{ volts.}$$

The voltage across the two combined is

$$1078 - 1022$$
$$= 56 \text{ volts.}$$

The voltage across the resistance is $16 \cdot 6 \times 5 = 83$ volts.
If the frequency were raised to 51 the impedance would be

$$Z = \sqrt{5^2 \times \left(2\pi \times 51 \times 0 \cdot 2 - \frac{10^6}{2\pi \times 51 \times 50}\right)^2}$$
$$= 5 \cdot 28 \text{ apparent ohms.}$$

The current would be $\frac{100}{5 \cdot 28} = 18 \cdot 94$ amperes.

A variation of the frequency in either direction brings about a

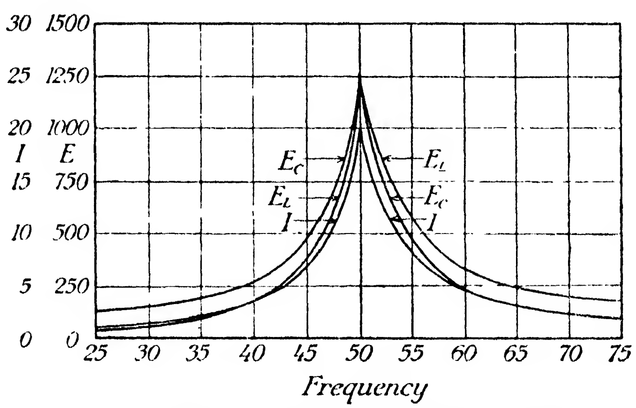


FIG. 44.—Effect of Frequency in Resonating Circuit.

reduction in the current. Fig. 44 shows the current at various frequencies and also the voltages across the condenser and inductance. A remarkable point about these curves is the sharpness of the peaks

exhibited, this being accentuated by the fact that the resistance is low. If the resistance had been only 1 ohm instead of 5 ohms, the current would rise to a maximum of 100 amperes and the maximum voltage across the condenser or the choking coil would be 6340 volts, thus making the peak more pronounced. The term "maximum" used above does not refer to the peak value of the wave, but to the maximum value of the R.M.S. voltage obtained by varying the frequency.

Current Resonance.—The above example serves as an illustration of voltage resonance in a series circuit. When, however, a condenser is placed in parallel with a choking coil, with a fixed voltage across each, a local circulating current is set up producing what is known as "current resonance." Fig. 45 illustrates such a circuit. A

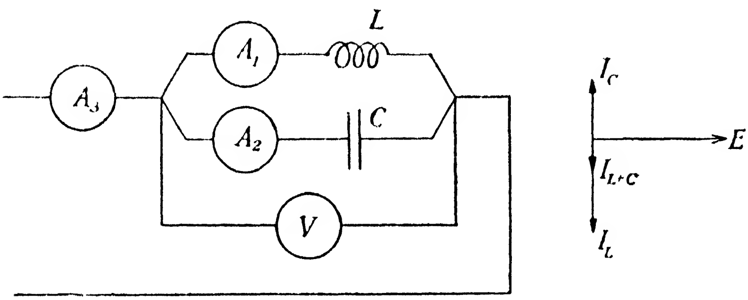


FIG. 45.—Circuit possessing Current Resonance.

voltage, E, applied to the circuit produces, through the inductance, L, a current

$$I_L = \frac{E}{2\pi fL},$$

lagging 90° behind the voltage, and, through the condenser, C, a current

$$I_C = 2\pi fCE,$$

leading the voltage by 90°. The resultant current is the numerical difference of these two components and is given by

$$\begin{aligned} I &= I_L - I_C \\ &= E\left(\frac{1}{2\pi fL} - 2\pi fC\right). \end{aligned}$$

If perfect resonance takes place in an ideal resistanceless circuit, then $I = 0$.

Therefore $$\frac{1}{2\pi fL} - 2\pi fC = 0$$

and $$f = \frac{1}{2\pi\sqrt{LC}}$$

as before.

If only partial resonance takes place, *i.e.* if the above equation is not fulfilled, then the current supplied from the source is the difference of the currents in the two branches. Assuming that the current in the inductive branch is the larger, then the difference between this and the main current is

$$I_L - I = I_C.$$

The conditions correspond to a circulating current equal to that in the weaker branch, together with a current from the supply to make up the current in the stronger branch. This external current will lead or lag by 90°; the former if the current in the condenser predominates, and the latter if the current in the choking coil predominates.

In practice there will be usually an appreciable amount of resistance in the inductive circuit, and an example will be worked out in order to illustrate the procedure in this case. The circuit in the example on series resonance will be chosen for this purpose, the condenser being in one branch and the inductance and resistance in series in the other branch. At a frequency of 50·33 the current through the condenser is

$$\begin{aligned} I_C &= 2\pi \times 50{\cdot}33 \times 50 \times 10^{-6} \times 100 \\ &= 1{\cdot}581 \text{ amperes.} \end{aligned}$$

The current through the inductive circuit is

$$\begin{aligned} I_{L+R} &= \frac{100}{\sqrt{5^2 + (2\pi \times 50{\cdot}33 \times 0{\cdot}2)^2}} \\ &= 1{\cdot}576 \text{ amperes.} \end{aligned}$$

But the voltage over the resistance is

$$\frac{R}{Z} \times E = \frac{R}{\sqrt{R^2 + X^2}} E.$$

The power wasted in the resistance is

$$\frac{R}{\sqrt{R^2 + X^2}} \times EI_{L+R}.$$

The same result would be attained if the current were split up into a power and an idle component. The magnitude of the former is

$$\begin{aligned} &\frac{R}{\sqrt{R^2 + X^2}} \times I_{L+R} \\ &= \frac{5}{\sqrt{5^2 + (2\pi \times 50{\cdot}33 \times 0{\cdot}2)^2}} \times 1{\cdot}576 \\ &= 0{\cdot}124 \text{ ampere.} \end{aligned}$$

The idle component of the current is

$$\sqrt{1{\cdot}576^2 - 0{\cdot}124^2}$$
$$= 1{\cdot}571 \text{ amperes.}$$

This balances an equivalent current in the condenser, the latter also taking an additional current of 1·581 — 1·571 = 0·010 ampere from the supply. The total current flowing through the main ammeter is

$$\sqrt{0{\cdot}124^2 + 0{\cdot}010^2}$$
$$= 0{\cdot}124 \text{ ampere}$$

and leads the voltage by an angle

$$\tan^{-1}\frac{0{\cdot}010}{0{\cdot}124}$$
$$= 4{\cdot}6^\circ.$$

Physical Meaning of Resonance.—At first sight it appears as if the phenomenon of resonance destroys the principle of the conservation of energy, but this is not so. Considering the case of current resonance, the condenser contains its maximum charge and has the maximum amount of energy stored up in it at the instant when the voltage is a maximum. But at this moment the current in the inductance is zero, and hence the magnetic field is zero and has no energy stored up in it. A quarter of a period later the field has reached its maximum value and has its maximum amount of energy stored up in it. But now the voltage across the condenser has dropped to zero and it is discharged. If perfect resonance is taking place, these two maximum amounts of energy are equal, for

$$2\pi f L = \frac{1}{2\pi f C}.$$

Multiplying each side by $I_m{}^2$,

$$2\pi f L I_m{}^2 = \frac{I_m{}^2}{2\pi f C}$$
$$= \frac{(2\pi f C E_m)^2}{2\pi f C}$$
$$= 2\pi f C E_m{}^2.$$

Therefore $$\tfrac{1}{2} L I_m{}^2 = \tfrac{1}{2} C E_m{}^2.$$

At intermediate points in the cycle there is energy present in both magnetic and electrostatic form, but the total quantity is always the same. For instance, when the voltage has the instantaneous value of

$$e = E_m \sin\theta,$$

the energy stored up in the condenser is

$$\tfrac{1}{2} C e^2 = \tfrac{1}{2} C E_m{}^2 \sin^2\theta.$$

The current in the inductance is

$$i = I_m \sin(\theta - 90°),$$

and the energy stored up in the inductance is

$$\tfrac{1}{2} Li^2 = \tfrac{1}{2} LI_m{}^2 \sin^2(\theta - 90°)$$
$$= \tfrac{1}{2} LI_m{}^2 \cos^2 \theta.$$

But since $\tfrac{1}{2} LI_m{}^2 = \tfrac{1}{2} CE_m{}^2$

$$\tfrac{1}{2} Li^2 = \tfrac{1}{2} CE_m{}^2 \cos^2 \theta,$$

and the total energy stored up in both condenser and inductance is, therefore,

$$\tfrac{1}{2} CE_m{}^2 \sin^2 \theta + \tfrac{1}{2} CE_m{}^2 \cos^2 \theta$$
$$= \tfrac{1}{2} CE_m{}^2 (\sin^2 \theta + \cos^2 \theta)$$
$$= \tfrac{1}{2} CE_m{}^2.$$

After the currents have been once started there is no further supply of power necessary. The condenser in discharging liberates sufficient energy to build up the magnetic field, and this in turn, on its destruction, provides the necessary energy to charge the condenser. If resistance is present in the local circuit, energy is being continually frittered away due to the I^2R loss, and this has to be made up from the external source of supply.

Resonance with Harmonics.—In cases where the wave form of the applied E.M.F. is not a simple sine wave it can be resolved into a number of sine waves of various frequencies, and it may be possible for one of the higher harmonics, as they are termed, to produce resonance in a very marked degree, resulting in an increased distortion of the current wave form. This effect is dealt with in greater detail on page 102.

Distributed Capacity.—When a pair of insulated mains are laid side by side and there is a difference of potential between them, a condenser action takes place. The system may be represented diagrammatically in the way shown in Fig. 46. Due to the appreciable

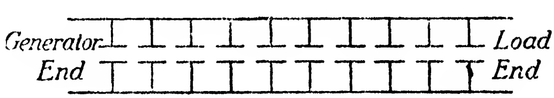


FIG. 46.—Distributed Capacity.

resistance and inductive reactance of the line, there is not a uniform voltage across all these hypothetical condensers, but as a first approximation they may be considered as being supplied with the full generator voltage.

The capacity of a concentric cable is given by the formula

$$C = \frac{0{\cdot}0388kl}{\log_{10}\left(\frac{D}{d}\right)} \text{ mfd},$$

where k is the specific inductive capacity,
l is the length in miles,
D is the inner diameter of the outer conductor,
and d is the outer diameter of the inner conductor.

The capacity of two parallel bare conductors is given by the formula

$$C = \frac{0{\cdot}0194 \times l}{\log_{10}\left(\frac{2D}{d}\right)} \text{ mfd},$$

in this case D being the distance between the centres of the conductors and d being the diameter of the wires. The unit used is immaterial, since it is only the ratio which is required. The ratio $\frac{D}{d}$ must be at least 10, otherwise the formula will not give correct results.

In addition to the capacity current which the cables take, there is also a small power current due to the leakage through the insulation, this being quite a distinct effect.

Various types of windings on alternating current apparatus also exhibit this condenser effect, since insulated conductors at different potentials are separated by dielectrics.

Uses of Condensers.—Condensers are very largely used in wireless telegraphy, where they are employed to "tune" the circuits, *i.e.* to bring them into perfect resonance, for which purpose variable condensers are used.

Condensers are also occasionally employed to protect high tension overhead transmission lines, one set of plates being connected to the line whilst the other set is earthed.

It has been suggested that condensers be put at the receiving end of transmission lines for the purpose of improving the power factor and thereby reducing the current in the mains. That this is so is seen from the fact that most transmission schemes operate with a lagging current due to motors and other apparatus which is connected to the line. The condensers take a leading current and tend to neutralise the wattless component of the line current.

It has been suggested also that condensers may be used as regulators in series with lamps on alternating current circuits. Instead of the usual parallel arrangement, the series system is adopted, each circuit consisting of a number of lamps all in series with each other and with a condenser. The lamps are put out by means of short-

circuiting switches and are lit when the switch is opened. Fig. 47 shows a diagram of connections of such a circuit. Not more than 35 to 40 per cent. of the supply voltage should be used on the lamps, otherwise the voltage regulation on the lamps becomes poor. Although a very large percentage of the supply voltage is absorbed by the condenser, yet the power loss is negligible, since the power factor of the condenser is practically zero.

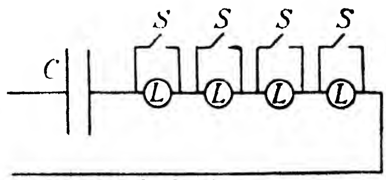


FIG. 47.—Condensers in Lighting Circuit.

As an example, consider the case illustrated in Fig. 47. The supply voltage is 220 at a frequency of 50, and there are four 20 volt lamps, each having a resistance of 20 ohms. If the lamps are each to have 20 volts when they are all in, the voltage across the condenser is

$$\sqrt{220^2 - (4 \times 20)^2}$$
$$= 204 \cdot 9 \text{ volts.}$$

The current being one ampere, the capacity of the condenser is

$$C = \frac{I}{2\pi f E}$$
$$= \frac{1}{2\pi \times 50 \times 204 \cdot 9} \text{ farad}$$
$$= 15 \cdot 53 \text{ micro-farads.}$$

When only one lamp is lit the current is

$$I = \frac{220}{\sqrt{20^2 + \left(\frac{10^6}{2\pi \times 50 \times 15 \cdot 53}\right)^2}}$$
$$= 1 \cdot 07 \text{ amperes.}$$

The voltage across the single lamp is now 21·4 volts. The maximum variation of voltage in the extreme cases of all the lamps lit and only one lit is only 7 per cent., and this figure is reduced if a smaller percentage of the supply voltage is used actually on the lamps.

Two important points in connection with this scheme are (1) the simplification of the wiring, and (2) the fact that the condenser acts as a current limiting device in the event of a short circuit.

CHAPTER VII

CIRCLE DIAGRAMS

Simple Circle Diagram.—If a constant voltage be applied to a circuit consisting of a resistance and a reactance in series, the voltages over the two component parts of the circuit will always be 90° out of phase with each other, this being independent of the magnitudes of the resistance and reactance. Fig. 48 shows a vector diagram of the circuit, the applied voltage, AB, being constant. A semicircle is erected on AB, this passing through the point C, since the angle in a semicircle is a right angle. If the relative values of the resistance and reactance be altered, the angle ϕ is altered, but the angle ACB remains a right angle. If, however, the diagram be drawn as shown in Fig. 48 (b), AB is fixed in position, whilst the

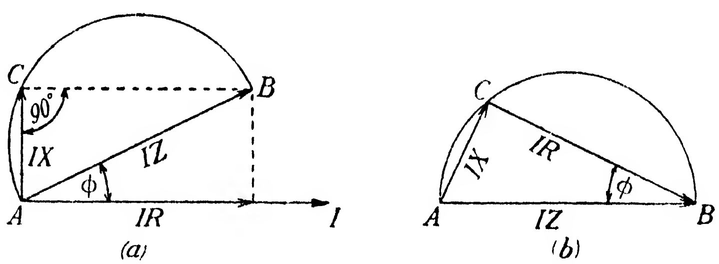


Fig. 48.—Simple Circle Diagram.

point C moves round the semicircle. The angle of lag of the current still is given by the angle ABC, since AB is the phase of the applied voltage and CB is the phase of the current.

Circuit with Constant Reactance.—If a circuit consists of a constant reactance and a variable resistance, the magnitude of the current will be proportional to the voltage across the reactance, whilst the phase of the current will be the phase of the voltage across the resistance. Fig. 49 illustrates this circuit, AC being proportional to the voltage across the reactance and to the current, to a scale of volts and amperes respectively. The power absorbed by the circuit is

$$P = EI \cos \phi,$$

and this is represented, to scale, by

$$AB \times AC \times \cos\phi.$$

But the angle ACD is also equal to ϕ, CD being a vertical line dropped from C on to AB. Thus

$$AC \times \cos\phi = CD$$

and the power becomes proportional to

$$AB \times CD.$$

Since AB is constant, the power absorbed by the circuit is given by the vertical line CD to a suitable scale of watts. This scale having been determined experimentally for one value of the resistance, the power absorbed for all other values of the resistance can be scaled off from the diagram. This power becomes a maximum

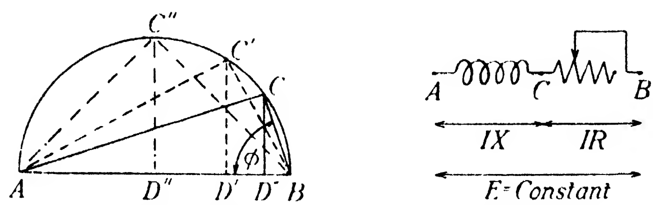


FIG. 49.—Circuit with Constant Reactance.

when the point D is situated at D'' midway along AB, the height $C''D''$ being equal to the radius of the circle. Moreover, at this point the angle of lag is 45° and the power factor is, therefore, $\frac{1}{\sqrt{2}}$ or 0·707. If the current and the voltage be measured under these conditions the power will be given by

$$EI \times 0{\cdot}707,$$

and this will enable the scale of watts to be settled. The power factor is given by $\frac{CB}{AB}$, and since AB is constant, CB is proportional to the power factor. The scale can be ascertained from the fact that at C'' the angle of lag is 45°, and consequently $C''B$ is 0·707. The total impedance is equal to $\frac{E}{I}$, and hence is proportional to $\frac{1}{AC}$.

If a wattmeter be included in the circuit, and the value of the resistance gradually reduced, the power will first be seen to increase, then attain a maximum value, and finally decrease again, even though the current is increasing throughout. The reason for this

is that as the resistance is decreased the current goes up, but the power factor is reduced since the ratio of resistance to reactance is getting less. At first, the current increases at a greater rate than the power factor decreases, the power increasing, whilst beyond a certain point this is reversed and the power begins to decrease again. At the commencement, the current is small, but the power factor is high, whilst later on the current is large, but the power factor is low. Looking at it from the point of view that the power is equal to I^2R, R is reduced whilst I goes up, so that I^2R may go up or down as R is varied, depending upon which alteration predominates.

One point worthy of note is that when the power has attained its maximum value, the angle of lag of the current is 45° and the resistance is equal to the reactance. This forms a method of measuring the reactance by means of a wattmeter and a known variable resistance. The wattmeter is connected so as to measure the total power in the circuit and the resistance is gradually cut out. When the wattmeter arrives at its maximum deflection, the resistance is equal to the reactance.

Effect of Resistance in Choking Coil.—In most choking coils met with in practice the ohmic resistance is not negligible, and in this case the circle diagram assumes the form shown in Fig. 50. The resistance and reactance of the choking coil bear a constant ratio

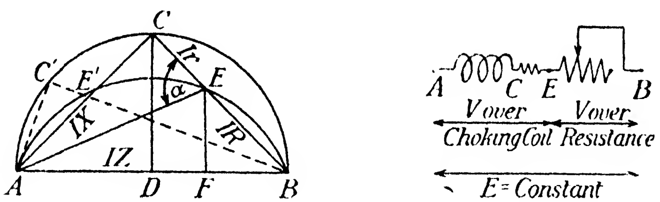


FIG. 50.—Effect of Resistance in Choking Coil.

to each other and consequently the angles AEC and AEB remain constant, independent of the position of C. The triangle AEC varies in size, but remains similar in shape throughout. Thus, wherever the point C may be on the semicircle the angle AEB is fixed and the locus of E lies on an arc of a circle of which AB is a chord. The centre of this circle will be on the diameter of the original circle at right angles to AB. The power consumed by the resistance external to the choking coil is measured by EF and the power wasted in the choking coil itself is given by $CD - EF$. It will be observed that the maximum power in the external resistance does not occur at the same point as the maximum power in the whole circuit, but occurs shortly after the total power has begun to decrease. In measuring the magnitude of the reactance by the method shown in the last paragraph, a modification is necessary.

When the circuit is absorbing the maximum power, the reactance is equal to the external resistance plus the equivalent resistance of the choking coil itself, or

$$AC = CE + EB.$$

But $$AC = AE \sin \alpha$$

and $$CE = AE \cos \alpha.$$

Therefore $$AE \sin \alpha = AE \cos \alpha + EB$$

and $$\sin \alpha - \cos \alpha = \frac{EB}{AE}.$$

Again, $$AC = AE \cos \alpha + EB$$

$$= EB \times \frac{\cos \alpha}{\sin \alpha - \cos \alpha} + EB$$

$$= EB\left(\frac{\cos \alpha}{\sin \alpha - \cos \alpha} + 1\right)$$

$$= EB \times \frac{\sin \alpha}{\sin \alpha - \cos \alpha}$$

$$= EB \times k,$$

where $$k = \frac{\sin \alpha}{\sin \alpha - \cos \alpha} = \frac{AC}{EB}.$$

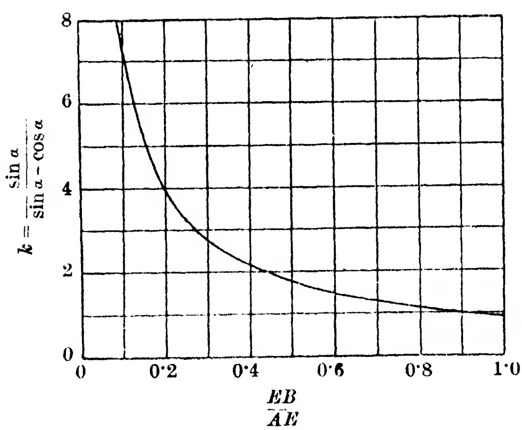


FIG. 51.—Values of $k = \frac{\sin \alpha}{\sin \alpha - \cos \alpha}$.

The quantity $k = \left(\frac{\sin \alpha}{\sin \alpha - \cos \alpha}\right)$ can be determined from the voltage ratio $\frac{EB}{AE}$, and this relationship is shown in Fig. 51. The reactance of the choking coil, therefore, is equal to the external

resistance multiplied by the constant k which depends upon the ratio of the voltages over the resistance and choking coil. The observations must, of course, be taken at the point where the wattmeter reading is a maximum.

Circuit containing Capacity and Resistance.—The circle diagram in this case is similar to the previous ones, with the exception that the voltage over the resistance leads the applied voltage and consequently is drawn from the left end of the semicircle as shown in Fig. 52. Otherwise, this case can be treated in the same way as a pure inductance, since the energy losses in the condenser will be negligible.

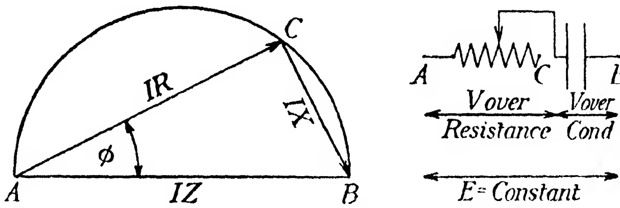


FIG. 52.—Circuit containing Capacity and Resistance.

Two Impedances in Series.—If a constant impedance be connected in series with a variable impedance, the latter being varied by means of its resistance only, the example resolves itself into a case of two constant reactances, a constant resistance and a variable resistance in series. The circle diagram for this circuit is illustrated in Fig. 53. AB, on which the semicircle ACB is erected

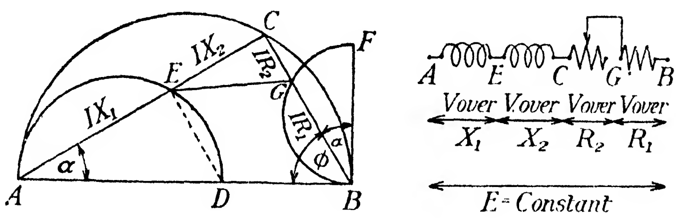


FIG. 53.—Two Impedances in Series.

represents the constant applied voltage. The base AB is divided at D so that $\frac{AD}{DB}=\frac{X_1}{X_2}$, where X_1 and X_2 are the reactances of the two impedances, and a semicircle AED is erected on the base AD. Now any line drawn from A, such as AC, will be divided at E so that $\frac{AE}{EC}=\frac{X_1}{X_2}$. This can be seen from a consideration of the similar triangles AED and ACB. A vertical line BF is next erected at the point B, its length being such that $\frac{BF}{AD}=\frac{R_1}{X_1}$, where

R_1 is the constant resistance of the first impedance. A semicircle BGF is erected on BF intersecting the line CB at G. Since the angle $GBF = a$, therefore

$$GB = BF \cos a$$

and $$AE = AD \cos a.$$

Therefore $$\frac{GB}{AE} = \frac{BF}{AD} = \frac{R_1}{X_1}.$$

Since it is a series circuit and AE represents the voltage across X_1, GB will represent the voltage across R_1, and consequently CG will represent the voltage across the variable resistance R_2. The voltage across the variable impedance is given by EG and the voltage across the constant impedance by the vector sum of AE and GB.

Circle Diagram for Two Parallel Circuits.—If a circuit be built up of two parallel branches, each consisting of a resistance and a

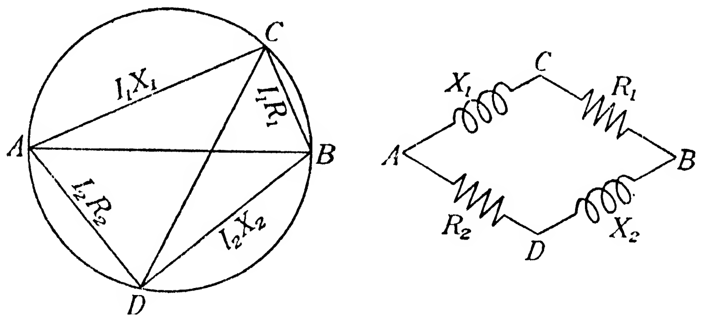


FIG. 54.—Circle Diagram for Two Parallel Circuits.

reactance, but connected so that the resistance of one branch is opposite to the reactance of the other, then a complete circle diagram can be drawn as shown in Fig. 54.

The voltage across the two points C and D is given by the vector CD, its maximum value being equal to the applied voltage and occurring whenever C and D are at opposite ends of a diameter. The conditions necessary to make the voltage CD a maximum are that

$$I_1R_1 = I_2R_2$$

and $$I_1X_1 = I_2X_2$$

or $$\frac{R_1}{R_2} = \frac{X_1}{X_2}.$$

The phase difference between the voltage CD and the applied voltage can be adjusted to any value by regulating the amounts of resistance and reactance in the circuit. In order to obtain a phase

difference of 90°, the resistance of each branch must be made equal to the corresponding reactance.

A similar circle diagram can be constructed for a circuit in which the choking coils have been replaced by condensers, the only difference being that the idle voltage is drawn to the right in the top half of the diagram, and to the left in the bottom half, since the current leads the voltage in a condenser.

A further case of parallel circuits which may be considered is that wherein one branch consists of a choking coil and a resistance whilst the other consists of a condenser and a resistance. The circle diagram for such a circuit is illustrated in Fig. 55. A point to be noted is that this time the two resistances are opposite to one another, instead of being opposite to the choking coils. In this

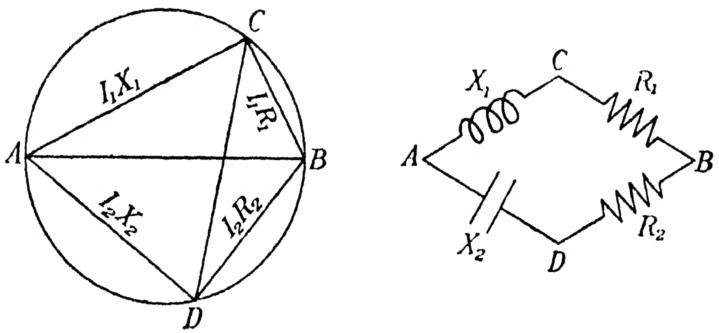


FIG. 55.—Circle Diagram for Two other Parallel Circuits

case, the conditions necessary to make the voltage CD a maximum are that

$$I_1R_1 = I_2X_2$$

and

$$I_1X_1 = I_2R_2$$

or

$$\frac{R_1}{X_1} = \frac{X_2}{R_2}.$$

If, in addition, $R_1 = X_1$ and, consequently, $R_2 = X_2$, the figure $ACBD$ becomes a square and the voltage across CD is 90° out of phase with that across AB.

Again, if X_1 and X_2 are made equal to $\sqrt{3} \times R_1$ and $\sqrt{3} \times R_2$ respectively, the angles CAB and DAB will each be 30°, the triangle ACD will become equilateral, and the three voltages AC, CD and DA will all be 120° out of phase with each other.

If, however, additional circuits are connected to these various points and current taken off, the relative phases of the voltages will be affected and the above relations will no longer hold good.

CHAPTER VIII

MAGNETIC PROPERTIES OF IRON

Magnetic Induction.—When a specimen of iron is magnetised by means of an electric current, a certain magnetic flux is set up, which increases with the current, although not proportionally. Since the cross section of the iron remains constant, the flux density B is proportional to the total flux Φ, whilst the magnetic force H is proportional to the ampere-turns of the solenoid enclosing the specimen. The relationship between B and H is a most important one, since it tells us what magnetic force is necessary to produce a given flux density in a certain specimen, and the curve connecting these two quantities is a characteristic of that particular brand of iron or steel.

The ratio

$$\frac{\text{Flux density}}{\text{Magnetic force}} = \frac{B}{H}$$

is termed the *permeability* and is denoted by μ. It is therefore evident that a good brand of iron from the magnetic point of view must have a high value for the permeability, although this is not the only criterion, as will be seen later. Different brands of iron and steel have different B—H curves, the shapes of which are affected by the composition and treatment of the material, such as the degree of hardness. A number of typical examples are shown in Fig. 56. At first, the flux density is roughly proportional to the magnetic force, but after a time this proportionality falls off and the iron is said to be *saturated*, that portion of the curve where the bend occurs being called the *knee*. It is, however, important to note that this knee is not a definite point, since it will appear at quite a different place if the curve is drawn to a different scale, so that the term saturation is only a relative one. When a B—H curve is plotted so as to give information at high flux densities, the knee appears further up the curve, as illustrated in Fig. 57, which curve is useful when calculations on the flux density in armature teeth are required.

Hysteresis.—The induction density which results, in a specimen of iron, from the application of a given magnetic force depends upon the previous magnetic history of the specimen. The B—H curve lies higher when taken with descending than with ascending values of H, and if the magnetic force be reversed the induction density appears to lag behind the magnetic force. In fact, if the

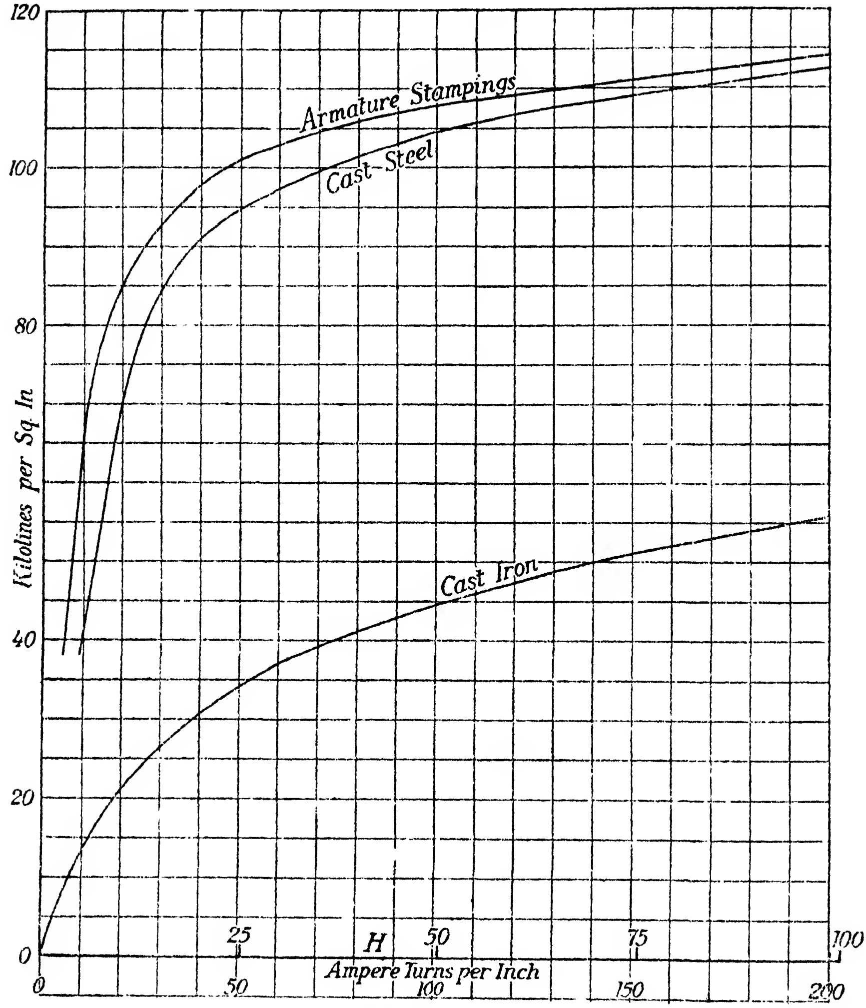


FIG. 56.—Magnetisation Curves.

magnetic force be carried through some cycle, finally returning to the starting point, the curve connecting the induction density with the magnetic force will describe a figure which may or may not be a closed loop. If the specimen be taken through the same cycle of events a number of times it will eventually come into what is

termed a *cyclical* state, and the complete *B—H* curve will then be a closed figure known as the *Hysteresis loop.* It is usually determined by magnetising the iron to equal extents in either direction

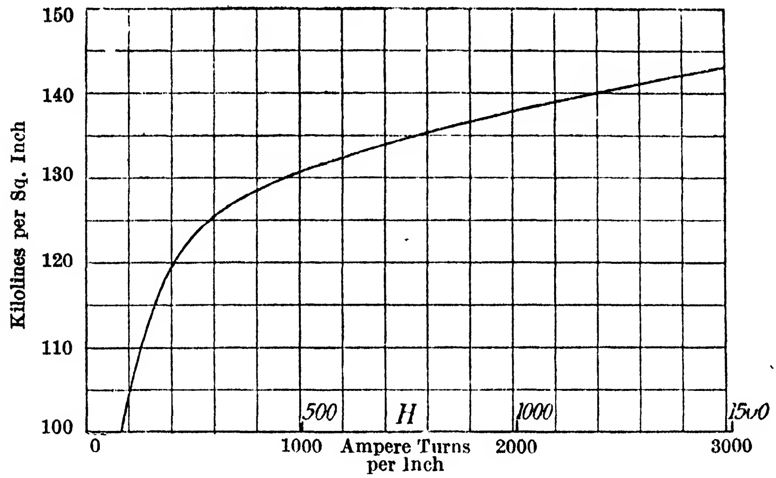


FIG. 57.—Magnetisation Curve for Armature Stampings at High Flux Densities.

so that the hysteresis loop is symmetrical about the origin of the curve. This really amounts to an alternating flux with the same maximum in either direction.

In changing the state of magnetisation of the iron, some of the molecular magnets are displaced relatively to one another, and this results in a kind of molecular friction, heat being engendered. Hence energy must be supplied to the iron to provide for this wastage. It has been shown that a certain definite quantity of energy must be supplied in building up the magnetic field, but in the case of materials exhibiting hysteresis the whole of this energy is not given back to the source of supply, for when the magnetising current has dropped to zero some magnetic flux still remains, retaining a certain amount of energy. Hence, during the complete cycle there is a net amount of energy supplied to the specimen, this energy being converted into heat, and it can be shown that the amount thus lost is proportional to the area of the hysteresis loop.

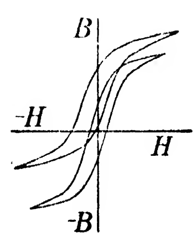


FIG. 58.—Hysteresis Loops.

In order to get the iron into a cyclical state so as to obtain a normal hysteresis loop, it is necessary to take the specimen through the complete cycle about fifty times, always taking care to travel round the loop in the same direction. Fig. 58 shows a typical example of a hysteresis loop when the specimen has been carried through a symmetrical and an

unsymmetrical cycle. The hysteresis loss increases appreciably when the cycle is changed from a symmetrical to an unsymmetrical one, indicated by an increase in the area of the loop, although the maximum vertical height remains unaltered.

Steinmetz's Law.—Steinmetz enunciated the law, based on experimental observation, that the area of the hysteresis loop is proportional to the 1·6th power of the maximum induction density, although more modern determinations of this index give the value as being more nearly 1·7. This figure is known as the *Steinmetz index*. For very accurate work, a slight correction has to be made, since the hysteresis loop has no appreciable area if the maximum induction density is not carried beyond about 100 lines per sq. cm., and the corrected expression takes the form $(B - 100)^x$, the index x being very slightly modified. The value of this correction becomes apparent when measurements at low induction densities are made, although for most practical purposes it may be neglected. This variation is sometimes allowed for by using different values of the index for different maximum values of the flux density.

The hysteresis loss per cycle is proportional to the volume of the iron or steel used, but it has been found that over very wide ranges the loss of energy per cycle is unaffected by change of frequency, from which it follows that the power loss in watts is directly proportional to the frequency. The final expression for the power wasted in hysteresis is

$$P_h = h \times v \times f \times B^{1 \cdot 7} \times 10^{-7} \text{ watts},$$

where h is a constant known as the *hysteretic constant* and usually has a value ranging from 0·0005 to 0·001 in the case of soft annealed plates. The volume v is measured in c.c.

In determining the area of the hysteresis loop, a first approximation can be made by replacing the loop with a rectangle of equal width and height to the actual loop, as shown in Fig. 59. This is allowable, since the horizontal width of the loop is approximately constant throughout. The area of this rectangle is $2Bw$, and is proportional to $B^{1 \cdot 7}$, from which it follows that w is proportional to $B^{0 \cdot 7}$. This is useful when it is desired to predetermine a particular loop, the loop at another induction density being known. For example, if the maximum value of B be increased by 10 per cent. the horizontal width of the loop is increased in the ratio $1 \cdot 1^{0 \cdot 7} = 1 \cdot 07$, or by 7 per cent.

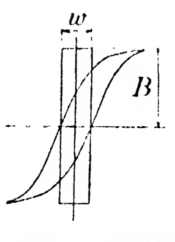


FIG. 59.—Approximate Area of Hysteresis Loop.

The hysteresis loss measured in watts per c.c. is shown graphically in Fig. 60, the bounding lines of the shaded portion representing good and bad specimens of soft annealed iron plates. In order to

determine the loss at other frequencies, the values obtained from the curve must be changed in the direct ratio of the frequencies.

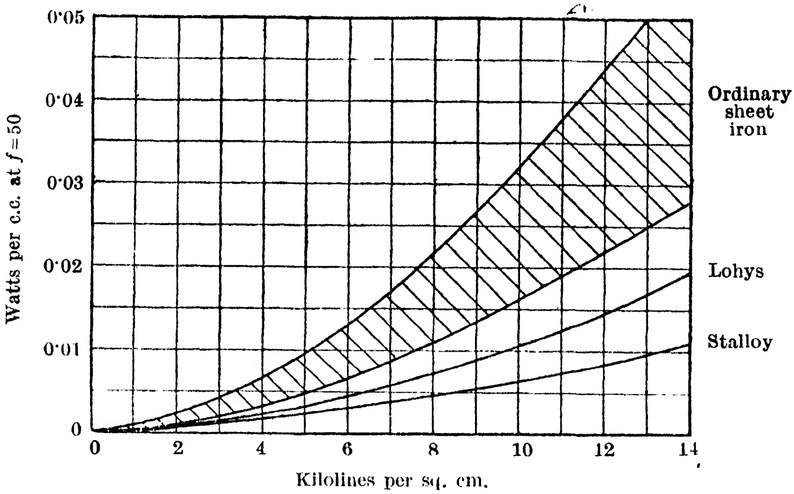


FIG. 60.—Hysteresis Loss in Iron.

Rotating Hysteresis.—When the induced flux in the iron is of a rotating instead of an alternating character, the curves of hysteresis are modified to a very considerable extent as illustrated in Fig. 61. A striking peculiarity about rotating hysteresis is that as the induction density is gradually raised the hysteresis suddenly disappears in

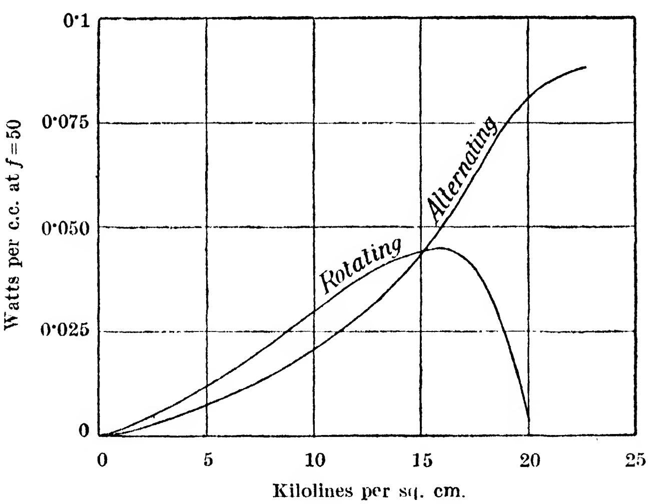


FIG. 61.—Alternating and Rotating Hysteresis.

the neighbourhood of $B = 20{,}000$. In certain cases it may therefore be desirable actually to increase the flux density in order to bring

about a reduction of the hysteresis loss, although the exciting ampere-turns may be considerably increased thereby.

Ageing of Iron.—It has been determined experimentally that the hysteresis loss in annealed iron stampings increases with lapse of time, this increase being as much as 50 per cent. in some cases of very old iron. The temperature at which the iron is worked is a very important factor, since this ageing, as it is called, only becomes of such a magnitude if the working temperature is allowed to exceed about 85° C. Consequently, in modern apparatus efforts are made to avoid higher temperatures than this. A suggested comparison of different brands of iron with respect to their ageing properties is the percentage increase of hysteresis loss caused by keeping the sample at a temperature of 100° C. for a period of 600 hours. With ordinary iron stampings, the ageing, measured on this basis, should be below 15 per cent. It can be reduced by suddenly cooling the plates from a red heat, whilst unannealed iron plates show scarcely any signs of ageing at all.

A special silicon–iron alloy containing about $3\frac{1}{2}$ per cent. of silicon, known as *stalloy*, has a very much lower hysteresis loss than pure iron, but its high cost militates against its more general adoption.

It has been observed that mechanical pressure also increases the hysteresis loss to an appreciable extent, so that it is desirable that excessive pressures should not be used in the building up of iron cores, although a certain amount is absolutely indispensable from the constructional point of view.

Eddy Currents.—When an alternating magnetic flux follows an iron path, small but definite E.M.F.'s are set up in the iron itself, due to the fact that it is an electrical conductor being cut by lines of force. These E.M.F.'s act upon closed electrical circuits in the iron, and set up local circulating currents known as *Foucault* or *Eddy Currents* and give rise to a loss of power, since these currents flow through paths of definite resistance. The energy thus dissipated reappears as heat in the same way as the hysteresis loss, but must be supplied from an external source in the first place.

The lamination of the iron should be carried out in such a plane that the thickness of the plate corresponds to the length of the conductor which is having an E.M.F. induced in it, this being proportional to the thickness of the laminations. But the resistance of the mean path of the eddy currents is inversely proportional to the thickness of the plates as well, so that the watts lost per lamination, $\left(\frac{E^2}{R}\right)$, are proportional to the cube of the thickness. However, the total number of laminations in a given volume of iron is inversely proportional to the thickness, so that finally the watts per c.c. are proportional to the square of the thickness. They are obviously proportional also to the total volume of iron used.

The induced E.M.F. is proportional to the maximum value of the flux density, and since the watts are given by $\frac{E^2}{R}$, the power loss is proportional to the square of the maximum flux density. In the same way, the loss is seen to be proportional to the square of the frequency.

Combining all these factors together, the eddy current loss can be represented by a formula of the type

$$P_e = kt^2B^2f^2v \text{ watts},$$

where k has a value of about $1{\cdot}4 \times 10^{-11}$, all the dimensions being in cm.

The eddy current loss in the case of stalloy is about half what is given by the above formula.

If the iron were not laminated the eddy currents would be so powerful that they would react to an enormous extent upon the field which produces them, diverting the flux from the interior of the metal. Thus a mass of iron, or indeed of any metal, exerts a shielding effect on that which lies behind it, in so far as it keeps back the magnetic flux and forces it into other paths. In the case of armoured cables having a continuous metal sheath the conductors are arranged so that their magnetic effects neutralise one another, thus avoiding large eddy current losses in the sheath.

Flux Measurement.—Measurements of the flux density in an iron specimen can be carried out by a number of methods, but the following is one in which an alternating current is employed. The specimen is made up in the form of a choking coil having a winding which consists of a known number of turns, T. When specimens are specially prepared the cores are usually built up in the form of a rectangle or a circle, a number of laminations being bunched together. The winding is supplied with an alternating pressure which will be, for the present, assumed to be sinusoidal. Further, assuming that the IR drop in the winding is negligible, the whole of this voltage is used up in overcoming the back E.M.F. which is due to the inductance. This back voltage is proportional to minus the rate of change of the magnetic flux and will lag behind it by 90° (see p. 16), and hence the applied pressure will lead the flux by 90°. Further, since the rate of change curve of a sine function also obeys a sine law, it follows that a sinusoidal flux is produced by a sinusoidal E.M.F.

During a complete cycle of magnetic flux each turn is cut by the total maximum flux set up four times, viz., once each time it rises and once each time it falls. If the maximum flux be denoted by Φ, the total lines cut per turn per cycle are 4Φ, and the total lines cut per turn per second are $4\Phi f$, where f is the frequency. The average E.M.F. induced per turn is therefore $4\Phi f \times 10^{-8}$ volts, and since the E.M.F. is sinusoidal, this corresponds to an

R.M.S. voltage of $1{\cdot}11 \times 4\Phi f \times 10^{-8}$ volts, for the form factor of a sine wave is 1·11. The total R.M.S voltage induced in the whole winding is

$$E = 4{\cdot}44 \times \Phi f T \times 10^{-8} \text{ volts,}$$

and, neglecting the IR drop, this is equal to the applied pressure. If the wave form is not sinusoidal the R.M.S voltage is equal to

$$E = 4k\Phi f T \times 10^{-8} \text{ volts,}$$

where k is the form factor. Knowing the number of turns, frequency and voltage, the flux can be calculated as follows :—

$$\Phi = \frac{E \times 10^8}{4{\cdot}44 f T} \text{ lines.}$$

In cases where the IR drop is not negligible, it must be subtracted vectorially from the applied voltage.

By applying different voltages different fluxes are obtained, and the corresponding densities can be determined by dividing by the cross section of the iron. The values obtained really constitute the tips of successive hysteresis loops. If the B—H curve were a straight line, the magnetising ampere-turns would be proportional to the flux density at any instant and the wave form of the magnetising current would be similar to that of the flux and voltage, but this is not so in the case of iron and steel, and hence the current and voltage wave forms are not similar. The actual determination of the current wave form will be dealt with in Chapter IX.

In order to make accurate determinations of the B—H curve, the maximum values of the current and voltage should be obtained, since these correspond to the maximum value of the flux density, this being rendered necessary by the change of wave form.

Measurement of Iron Loss.—In order to measure the iron loss in a specimen, all that is necessary is to measure the input to such a ring or square as is described above. This is done most conveniently by means of a wattmeter (see p. 43). The magnetising component of the current does not cause any loss of power, since it is in quadrature with the voltage, and the I^2R and instrument losses can be allowed for. Since the power factor will be very low, care should be taken to obtain a wattmeter which will read accurately on a low power factor. It is also desirable that the E.M.F. wave form should be as nearly as possible a sine wave.

The great disadvantage of the ring-shaped specimen is the fact that it must be hand wound, but on the other hand any shape which allows a former wound coil to be slipped into position must contain one or more magnetic joints and these exert a considerable effect upon the magnetic reluctance. If, however, it is the core loss which it is desired to measure, this will not be of much moment,

since, although the magnetising current will be increased, the watts lost will remain the same. For this reason, therefore, straight specimens built up into the form of a square are largely used.

Epstein's Iron Testing Apparatus.—In this apparatus, the sample laminations are built up into four cores, which are arranged in the form of a square, as shown in Fig. 62. The dimensions of the laminations are 500 mm. by 30 mm., whilst the cores are built up to a thickness of about 25 mm., having a mass of 2·5 kilogrammes each. The individual laminations are insulated from each other by thin sheets of paper, whilst at each corner, where a butt joint is formed, a thin layer of paper or press-spahn is placed so as to avoid any additional eddy current loss which might be caused by the short-circuiting of adjacent stampings. The four cores are held in position by means of wooden clamps, one of which is placed at each corner.

Each core carries a winding which consists of 150 turns in series, the total cross section of the wire being 14 mm.^{2}. This is wound upon a press-spahn tube having a bore of 38 mm. and a length of 435 mm. These four coils, connected in series, have a total resistance of about 0·18 ohm.

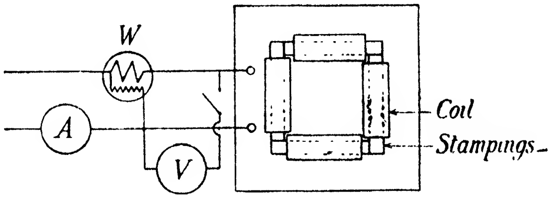


FIG. 62.—Epstein Iron Testing Apparatus.

In making a measurement of the total iron loss, the wattmeter reading is observed, whilst the flux density is calculated after the manner shown on p. 84. Since the total power absorbed is very small, the voltmeter should be disconnected whilst taking the wattmeter reading and the other instrument losses allowed for if necessary, as well as the copper loss of the magnetic square itself.

The loss is usually stated in watts per kilogramme or per lb. at a stated induction density and frequency. A basis of comparison for different grades of iron, adopted in German practice, is the power lost in watts per kilogramme at a flux density of 10,000 lines per cm.^{2} and a frequency of 50. This is termed the *Figure of Loss*. The curves in Fig. 63 show the iron losses in ordinary sheet iron and in the special alloyed iron stampings already referred to. The figure of loss comes out at 3·3 and 1·6 watts per kilogramme respectively.

Sometimes a rectangular core with a removable fourth limb is built up so that different samples of iron can be tested in succession.

The losses in the other three limbs must be measured and allowed for in measurements on different samples.

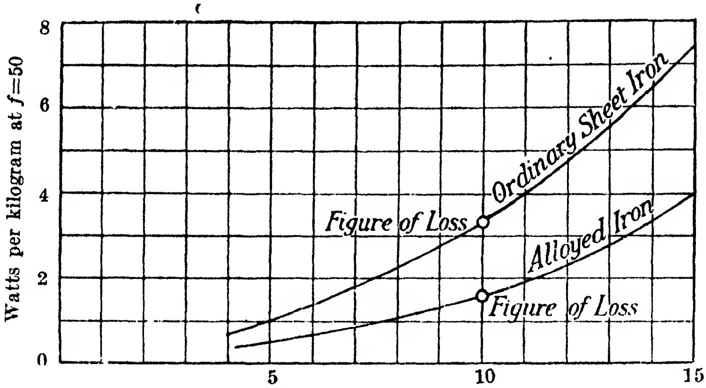


FIG. 63. Total Iron Losses.

Separation of Hysteresis and Eddy Current Loss.—In order to separate the total iron loss into the hysteresis and eddy current components, a series of observations is made at constant flux density, but with varying frequency. To maintain constant flux density, all that is necessary is to keep the ratio $\frac{\text{volts}}{\text{frequency}}$ constant, for

$$\frac{E}{f} = 4{\cdot}44 \times 10^{-8}\Phi T.$$

Since T is constant for a given winding, the flux Φ is solely dependent upon the ratio $\frac{E}{f}$. The voltage must therefore be varied proportionally to the frequency.

In these conditions, the only variable quantity in the expression for the iron loss is the frequency, and the hysteresis and eddy current losses can be represented by the expressions $P_h = af$ and $P_e = bf^2$ respectively, a and b being constants. The total iron loss is therefore equal to

$$P = P_h + P_e = af + bf^2$$

and

$$\frac{P}{f} = a + bf.$$

If the quantity $\frac{P}{f}$ be plotted against frequency, the resulting curve should be a straight line [see Fig. 64 (a)], the value at zero frequency giving the constant a and the slope of the curve determining b. The hysteresis loss at any frequency is given by af, whilst the

remaining power is due to eddy currents. Let a horizontal line be drawn through k [Fig. 64 (a)] and a vertical line lmn at any chosen frequency. The area $olmk = af$ represents the hysteresis loss, whilst the area kmn is proportional to $(km)^2 = f^2$ and represents the eddy current loss.

Another way of arriving at this result is to plot the total watts lost against the frequency, drawing a tangent to the curve at the

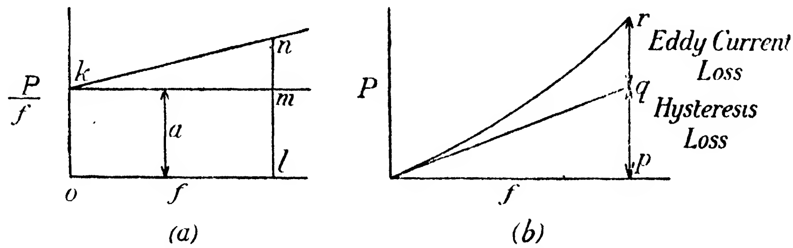


FIG. 64.—Separation of Iron Losses.

origin. At any frequency f [see Fig. 64 (b)] the height pq represents the hysteresis loss, since this height is proportional to the frequency. The remaining height, qr, represents the eddy current loss. The reason why the tangent at the origin is chosen is that at very low frequencies the eddy current loss is negligible, since it is proportional to the square of the frequency. It is important to get some experimental observations at low frequencies in order to determine the position of the tangent with accuracy.

Expression for Total Iron Loss.—From the foregoing it is seen that, for a constant frequency, part of the iron loss is proportional to $B^{1\cdot7}$, whilst the remainder is proportional to B^2, and efforts have been made to obtain an empirical expression of the form kB^n for the total iron loss. The index of B will lie between the limits of 1·7 and 2 in the case of ordinary transformer iron, and between 1·66 and 2 in the case of stalloy, 1·66 being the value of the Steinmetz index for this latter material. The result of experiment shows that for flux densities between 4,000 and 10,000 lines per square centimetre, and for frequencies between 25 and 60, the value of n lies between 1·68 and 1·77 for stalloy (0·5 mm. plates), whilst for ordinary iron (1 mm. plates) it lies between 1·75 and 1·83, the error being within 1·5 per cent. Also for *Lohys*, another type of alloyed iron (0·37 mm. plates), the value of n varies from 1·73 to 1·82 for flux densities between 5,000 and 10,000, and for frequencies between 25 and 60, the error being within 2·5 per cent.[1]

Effect of Iron Loss on Impedance.—In the case of a choking coil having an iron core, the total power losses occurring in it will consist

[1] McLachlan: "Representation of the Magnetisation Losses of Iron," *Journ I.E.E.* vol. 53, p. 350, 1915.

of a copper loss due to its ohmic resistance and an iron loss due to hysteresis and eddy currents. Such a choking coil could be replaced by an equivalent one having no iron loss at all, but an increased copper loss to make the total the same in the two cases. This results in an apparent increase in the ohmic resistance, and the choking coil can be considered to have an equivalent resistance higher than its true ohmic resistance so as to take into account the whole of the losses occurring in the choking coil. Thus iron loss has the effect of increasing the impedance of a choking coil.

Circuit Equivalent to an Actual Choking Coil.—When a voltage is applied to a choking coil, part of it is absorbed in overcoming the ohmic resistance, whilst the remainder, considered vectorially, is used for overcoming the back E.M.F. set up, due to the change of magnetic flux. In other words, the circuit may be considered as a pure resistance connected in series with a resistanceless choking coil. But, due to the iron loss in the latter, the current will not lag behind its volts by exactly 90°, since there must be a power component. The current may therefore be resolved into two components, one in phase and the other in quadrature with the voltage acting on this part of the circuit, which may, consequently, be considered as a pure reactance, L, in parallel with a pure resistance, r. The value of r is such that $\frac{E_L^2}{r}$ is the total iron loss, where E_L is the voltage remaining after the IR drop has been subtracted vectorially. Fig. 65 shows the final equivalent circuit.

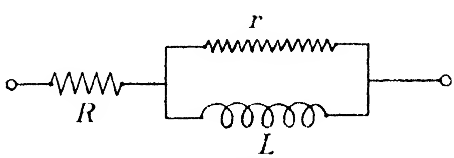


FIG. 65.—Circuit Equivalent to Choking Coil.

Flux Distribution.—In most cases in practice the flux is more or less non-uniformly distributed. This is due to the fact that the flux endeavours to choose the shortest path and hence it tends to crowd together in certain parts of the magnetic circuit. This effect is counterbalanced by the repelling action of the lines on one another resulting in a tendency to cover as large a cross section as possible. In the case of a ring or a rectangular specimen, the former effect tends to make the lines crowd about the inside edge, whilst the latter effect forces some of them towards the outside into longer paths. The flux densities over different paths will be inversely proportional to the relative reluctances of these paths, and hence the density gradually increases as the inner edge is approached.

The actual iron losses as determined by experiment are frequently in excess of the calculated losses, this being accounted for by the

non-uniform distribution of the flux, which always results in an increase in the losses. This can be illustrated by means of an example. Imagine a ring-shaped specimen with an iron cross section of 2 cm.^{2}, the flux density being 10,000. Taking the hysteretic constant as 0·001 and the frequency as 50, the watts lost per c.c. would be

$$0{\cdot}001 \times 50 \times 10000^{1{\cdot}7} \times 10^{-7} = 0{\cdot}0315 \text{ watt.}$$

Now imagine that over one half of the cross section the flux density is 11000, whilst over the other half it is 9000. The watts lost per c.c. will be

$$0{\cdot}001 \times 50 \times 11000^{1{\cdot}7} \times 10^{-7} = 0{\cdot}0371 \text{ watt}$$

and

$$0{\cdot}001 \times 50 \times 9000^{1{\cdot}7} \times 10^{-7} = 0{\cdot}0263 \text{ watt}$$

respectively, giving an average value of 0·0317 watt per c.c., showing a slight increase.

In a number of cases of machines of different types, the flux crosses an air gap and then has to traverse a path which consists of a number of iron teeth in parallel with a number of slots. In a case like this, the major portion of the flux passes down the teeth, but some of the lines must pass down the slots since they have a definite finite reluctance. The actual distribution of the flux is somewhat complicated, but lends itself to mathematical treatment.

Calculation of Magnetising Ampere-turns.—The simplest case is that of a ring specimen without any magnetic joints. Knowing the magnetisation curve of the material, the maximum value of H can be determined for a given maximum value of B, and the total maximum ampere-turns required are given by the formula

$$H_m l = \frac{4\pi}{10} \times I_m T.$$

Knowing the number of turns in the winding, the maximum and R.M.S. values of the current can be obtained.

Let the total maximum flux be $\Phi = B_m a$, where a is the cross sectional area. Then

$$H_m = \frac{\Phi}{a\mu},$$

$$I_m T = \frac{10}{4\pi}\frac{\Phi l}{a\mu},$$

and

$$I_i = \frac{10}{4\pi\sqrt{2}} \times \frac{\Phi l}{a\mu T},$$

assuming sinusoidal wave forms.

This is a purely idle current lagging by 90°, but there is also

a power component due to the iron losses. This can be determined by dividing the watts lost by the voltage. The resultant current is the vectorial sum of the two.

The effect of magnetic joints is quite appreciable, and is equivalent to adding a small air gap, thus resulting in a considerable increase in the magnetising current.

Where composite magnetic circuits are in question, the problem is more complicated and is dealt with by the method of determining the ampere-turns necessary to force the flux through each part of the magnetic circuit in succession. The sum total of these ampere-turns enables the magnetising current to be determined.

CHAPTER IX

WAVE FORM

Non-sinusoidal Wave Form.—All wave forms actually obtained in practice differ more or less from the standard sine wave, and in cases where the difference is considerable it is necessary to take account of it. Whether sinusoidal or not, the wave form is periodic in its character, each cycle being similar to the preceding one. It can be demonstrated mathematically that any periodic curve can be split up into a number of pure sine waves of different frequencies and amplitudes superposed on one another. One of these component curves, called the *fundamental*, will have the same frequency as the resultant complex curve. The other components will have frequencies which are exact multiples of the fundamental frequency. If the frequency of one of these other components bore a fractional

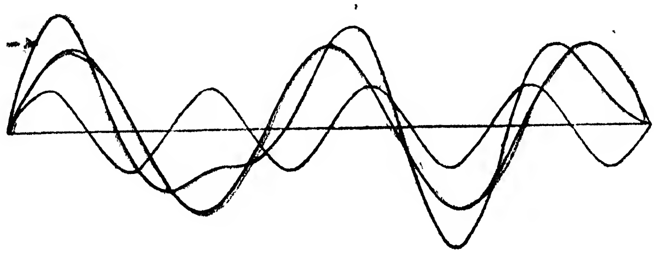


FIG. 66.—Effect of Fractional Harmonic.

ratio to that of the fundamental, it is easy to see that the second cycle would not be a repetition of the first as illustrated in Fig. 66, where the ratio of the frequencies is chosen as 1·6.

These various components are known as *harmonics*, and are distinguished by means of a number. For example, the third harmonic is that component sine wave which has a frequency three times that of the fundamental. The maximum values of these various harmonics may be anything, but usually they get smaller the higher the frequency, although special circumstances may result in a particular one being well developed. The relative phase

of the different harmonics may also be anything ; it is not necessary for them to pass through the zero values at the same instant.

The three things which specify the harmonic are (1) the frequency, (2) the amplitude or maximum value of the component, and (3) the relative phase. Thus the instantaneous value of a complex wave may be expressed as

$$e = E_1 \sin(\theta + \alpha_1) + E_2 \sin(2\theta + \alpha_2) + E_3 \sin(3\theta + \alpha_3) + \ldots,$$

where E_1, E_2, E_3, etc., are the maximum values of the fundamental and the various harmonics respectively, θ is the angle, measured at fundamental frequency, moved through since the commencement, and α_1, α_2, α_3, etc., are angles representing the phase of the various quantities at the instant from which the effects are measured. For example, at the instant of commencement the instantaneous value of the fundamental may have been half the

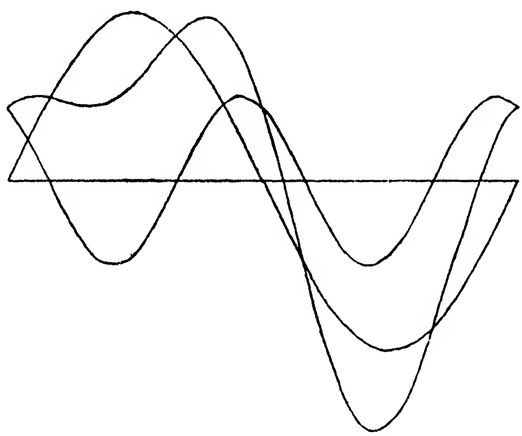


FIG. 67.—Effect of Second Harmonic.

maximum value. Then $\sin(\theta + \alpha_1) = 0{\cdot}5$, and since θ is zero, by hypothesis α_1 must be 30°. Again, at the end of a given time imagine that the fundamental has advanced by 40°. Then the instantaneous value of the fundamental is $E_1 \sin(40° + 30°) = 0{\cdot}94E_1$ or 0·94 times its maximum value.

Even Harmonics.—An even harmonic is one the frequency of which is an even number of times the fundamental frequency. One of the effects of an even harmonic is to make the two halves of the wave dissimilar, and this is not possible with the ordinary types of A.C. generators having a constant speed, for whatever occurs under one pole is repeated under the next which is of opposite polarity. Thus the two halves of the wave must be similar. Fig. 67 represents a wave with a 50 per cent. second harmonic passing through the zero 60° later than the fundamental, and it is seen that not only are the + and − portions dissimilar, but that one occupies a larger proportion of the base line than the other. All the even harmonics,

therefore, are absent in the wave forms obtained from the ordinary types of generators.

Odd Harmonics.—The presence of odd harmonics does not render the two halves of the wave dissimilar, for when the fundamental has advanced through half a period the odd harmonics have advanced through a number of complete periods plus half a period, and hence

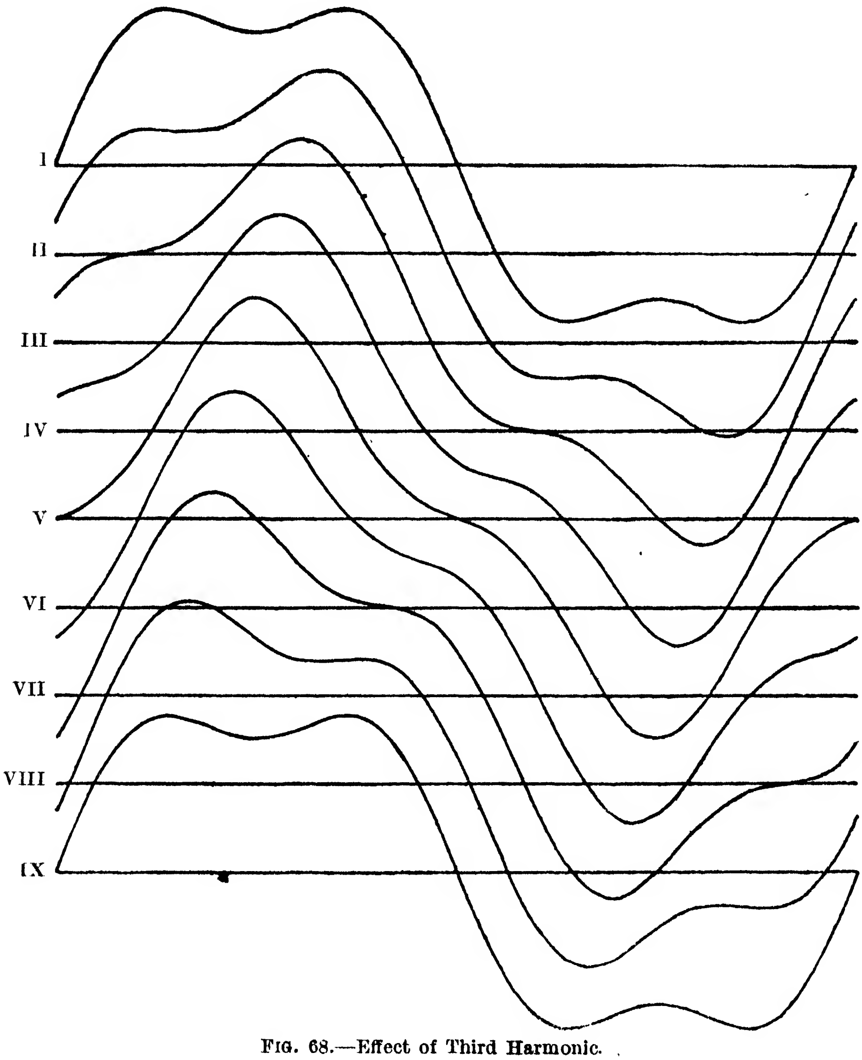


FIG. 68.—Effect of Third Harmonic.

their instantaneous values are in the same direction relative to the instantaneous value of the fundamental. There is therefore no inherent reason why odd harmonics should not be present in E.M.F. and current waves, and they are, indeed, found there.

Phase of Harmonics.—The various harmonics may go through their zero values at any moment relative to the zero of the fundamental,

and this question of the relative phase alters the resulting character of the wave to a very large extent. Fig. 68 shows a series of curves illustrating the effect of a 25 per cent. third harmonic, *i.e.* a third harmonic the maximum value of which is 25 per cent. that of the

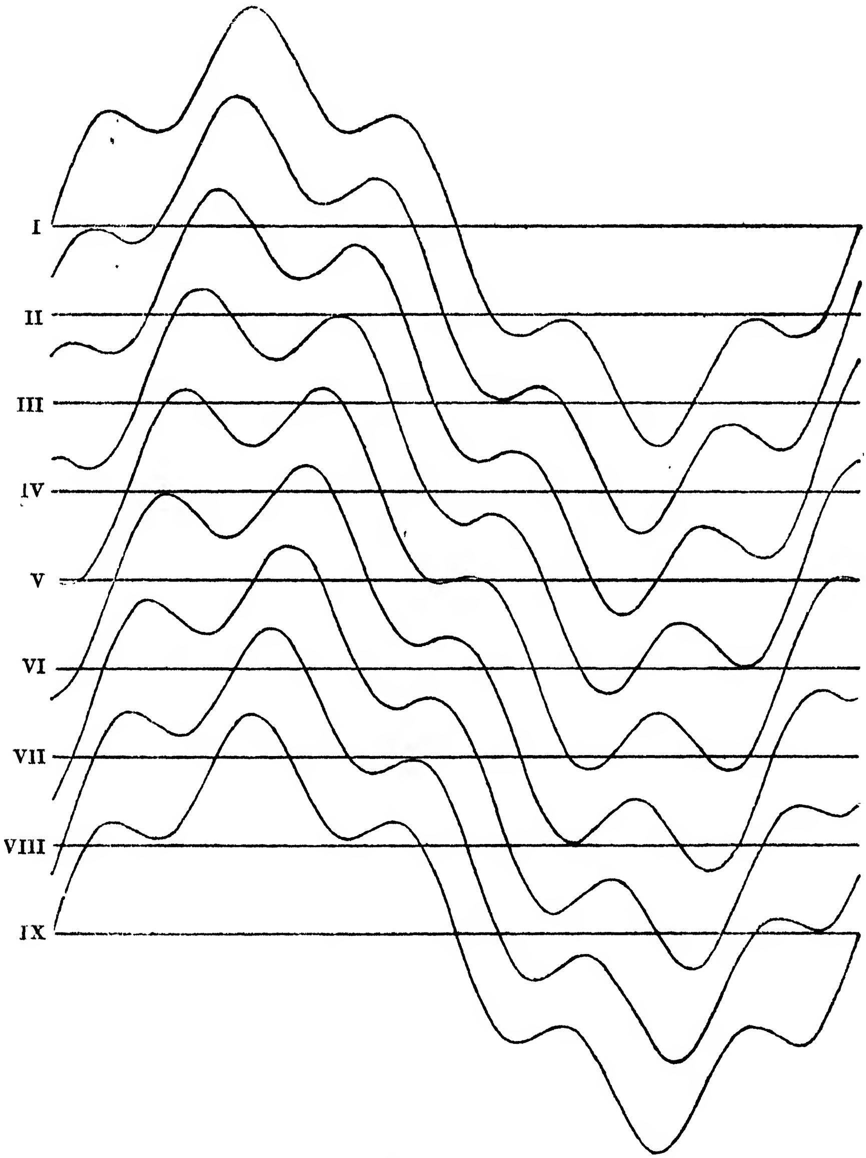


FIG. 69.—Effect of Fifth Harmonic.

maximum value of the fundamental. The different curves result from the application of different values to the angle α in the expression

$$e = E_1 \sin\theta + E_3 \sin(3\theta + \alpha).$$

There is no need to consider an angle in the first term corresponding to α, for this would simply mean drawing the curve from a different starting point. Fig. 69 shows a similar series of curves for a 25 per cent. fifth harmonic.

Curve V in Fig. 68 and curve I in Fig. 69 are termed *peaked* waves, whilst curve I in Fig. 68 and curve V in Fig. 69 are examples of what are termed *dimpled* waves. These two terms are merely indicative of the general character of the wave, there being no rigid demarcation between the two.

R.M.S. Value of a Complex Wave.—Taking an E.M.F. wave as an example, let the instantaneous value be represented by

$$e = E_1 \sin\theta + E_3 \sin(3\theta + \alpha) + E_5 \sin(5\theta + \beta) + \ldots.$$

Then the instantaneous value of e^2 is given by

$$e^2 = E_1^2 \sin^2\theta + E_3^2 \sin^2(3\theta + \alpha) + E_5^2 \sin^2(5\theta + \beta) + \ldots$$

$+$ (a number of terms containing the products of two sines).

It can be shown mathematically that the average product of two sines of different frequencies is zero.[1] Consequently the average value of e^2 is equal to the average value of

$$E_1^2 \sin^2\theta + E_3^2 \sin^2(3\theta + \alpha) + E_5^2 \sin^2(5\theta + \beta) + \ldots.$$

But the average value of $\sin^2\theta$, or $\sin^2(3\theta + \alpha)$, etc., is $\frac{1}{2}$.[2]

[1] Average value of product of two sines

$$= \frac{1}{2\pi}\int_0^{2\pi} \sin a\theta \sin b\theta \, d\theta$$

$$= \frac{1}{4\pi}\int_0^{2\pi} \left\{\cos(a\theta - b\theta) - \cos(a\theta + b\theta)\right\} d\theta$$

$$= \frac{1}{4\pi}\int_0^{2\pi} \left\{\cos(a - b)\theta - \cos(a + b)\theta\right\} d\theta$$

$$= \frac{1}{4\pi}\left[\frac{1}{a - b}\sin(a - b)\theta - \frac{1}{a + b}\sin(a + b)\theta\right]_0^{2\pi}$$

$$= \frac{1}{4\pi} \times 0 = 0.$$

[2] This follows from the following reasoning:—

$$\text{Average value of } \sin^2\theta = \frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{2\pi}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left(\frac{2\pi}{2} - \frac{1}{4}\sin 4\pi - 0 + \frac{1}{4}\sin 0\right)$$

$$= \frac{1}{2}.$$

Therefore the average value of e^2 is equal to

$$\frac{E_1^2}{2} + \frac{E_3^2}{2} + \frac{E_5^2}{2} + \dots$$

and the R.M.S value is

$$E = \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \dots}{2}}.$$

In a similar manner, the R.M.S. value of a complex current wave is found to be

$$I = \sqrt{\frac{I_1^2 + I_3^2 + I_5^2 + \dots}{2}}.$$

Effect of Magnetic Saturation on Current Wave Form.—In the case of a choking coil having an iron core, the current wave form is distorted when magnetic saturation occurs, this being due to the fact that the *B*—*H* curve is not a straight line. The simplest case to take is that of a choking coil having no ohmic resistance at all. The applied volts must therefore be balanced at every instant by the induced back E.M.F. If the wave form of the applied voltage is sinusoidal, it follows that the wave form of the back E.M.F. must also be sinusoidal, and since this latter is proportional to the rate of change of flux, it follows that the flux itself must be sinusoidal as well, although displaced by 90° in phase. The problem resolves itself, therefore, into determining the current wave form necessary to produce a sine wave of flux. If the flux were proportional to the current, no distortion would occur, but this is not so. As the *B*—*H* curve bends over, each additional ampere of magnetising current produces a smaller and smaller amount of flux, and consequently each increment of flux requires a larger and larger increase in the current. The actual wave form can be determined by means of the following graphical construction.

Let the left-hand diagram in Fig. 70 represent the *B*—*H* curve of the iron used, or, preferably, let it represent the relation between the total flux and the magnetising current, the total flux being obtained by multiplying the density by the cross sectional area and the magnetising current from the formula

$$H = \frac{4\pi}{10} \times \text{ampere-turns per cm.}$$

The sinusoidal flux wave is drawn to the right. In order to obtain the current required at any instant to produce a given flux, such as *ab*, a projection is drawn on to the left-hand diagram, the necessary current being given by *oc*. The point *d* is then obtained by making *ad* equal to *oc*, and *d* is then a point on the required current wave. This is repeated until a sufficient number of points is obtained

enabling the current wave to be drawn in, a characteristic example of which is shown in the figure. The outstanding feature of the curve is the fact that it is more peaked than the sine wave and usually contains a prominent third harmonic. It is a purely idle current, since it passes through the zero at the same instant as

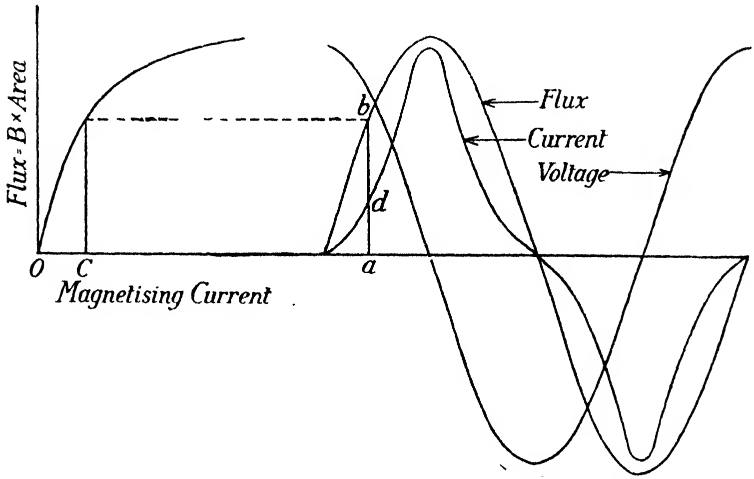


FIG. 70.—Effect of Magnetic Saturation on Wave Form.

the flux which lags behind the applied voltage by 90°, and, moreover, both halves of the wave are equal. The resulting power curve is a double frequency one, but is not sinusoidal in character, although the average value is zero. Thus it is seen that magnetic saturation in the iron causes a distortion in the current wave, but does not result in any loss of power.

Effect of Hysteresis on Current Wave Form.—In the preceding paragraph the same B—H curve was taken for ascending and descending values, with the result that the current wave was symmetrical about its maximum value and was purely idle. When the effect of hysteresis is considered, the value of the magnetising current required to produce a given flux is less when the current is decreasing than when it is increasing, and since the flux dies away at the same rate that it is built up, it follows that the current falls away at a greater rate, resulting, in general, in the downward part of the current wave being steeper than the upward portion. The current wave thus loses its symmetry, with the result that more energy is supplied to the circuit in building up the field than is returned on its destruction. Consequently, a power component has been introduced into the current wave which was to be expected, since hysteresis results in a loss of power.

In order actually to determine the current wave, the graphical construction explained in the last paragraph is again employed,

using a hysteresis loop instead of a simple B—H curve (see Fig. 71). The maximum value of B in the hysteresis loop is made equal to the maximum value of the flux density in the iron. Care must be exercised in order that the correct portion of the hysteresis loop

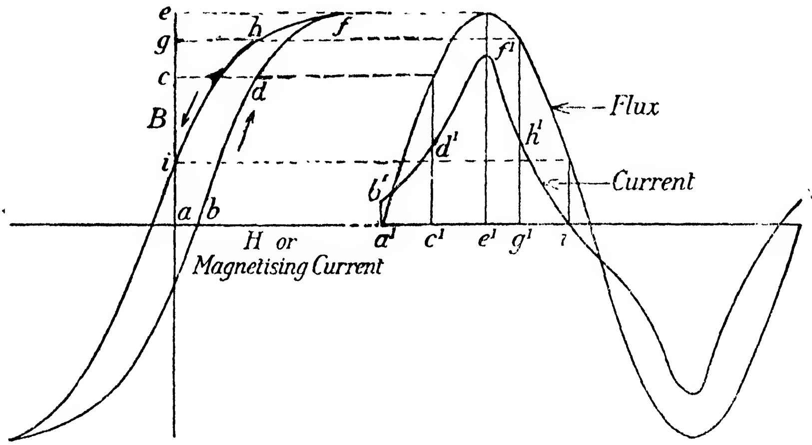


FIG. 71.—Effect of Hysteresis on Wave Form.

shall be used. When the flux is zero and about to take a positive value, the necessary magnetising current is given by ab, which length is transferred to $a'b'$ in the right-hand diagram. When the flux has a value ac in the positive direction, the corresponding magnetising current is cd, which is reproduced at $c'd'$. Similarly,

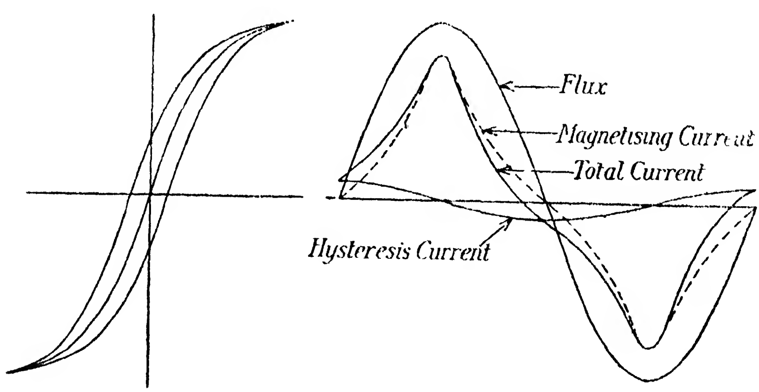


FIG. 72.—Separation of Hysteresis and Magnetising Currents.

ef is reproduced at $e'f'$. After the maximum value has been passed the flux follows the upper curve in the hysteresis loop, and when it has decreased to a value ag the magnetising current is gh, from which $g'h'$ is plotted. When the flux has dropped to ai the magnetising

current has dropped to zero, after which it commences to rise in the opposite direction before the flux has changed sign. It is thus seen that the current wave leads the flux wave to a certain extent and hence is less than 90° behind the voltage wave. On this account, therefore, the average value of the power assumes a positive value, representing the hysteresis loss.

The component of the current which refers to the hysteresis loss can be separated from the component which refers solely to the magnetisation in the following manner. The hysteresis loop is plotted and a mean B—H curve drawn representing the conditions which would obtain if the hysteresis loop were to shrink to a line enclosing no area (see Fig. 72). For a given flux, the magnetising current can be considered as the mean of the two values obtained from the curve. A current wave can be obtained as described above from the mean B—H curve, and another current wave can be obtained by the use of the actual hysteresis loop. The first represents the purely magnetising current, whilst the second represents the sum of the currents necessary for the magnetisation and for supplying the hysteresis loss. By taking the difference of these two curves, a third one may be obtained representing the current supplied solely to make up for the hysteresis loss. This will not be a sine wave, and may, in certain cases, take up a very irregular form; but if a power curve is drawn by multiplying the instantaneous values of this current and the applied voltage, the result will show an average positive value which will represent the hysteresis loss. This component of the current can therefore be considered as a power component.

Effect of Harmonics on Inductive Reactance.—When the wave form of the applied E.M.F. is not sinusoidal, it can be split up into a number of component sinusoidal E.M.F.'s of different frequencies, the fundamental frequency being the same as that of the complex wave. These different E.M.F.'s can be considered as acting independently of each other, each producing its own current. But the reactance of a given choking coil is proportional to the frequency of supply, and if the frequency is raised the reactance increases. Consequently, the reactance, as far as the third harmonic is concerned, is three times what it is to the fundamental, and the amperes produced per volt of the third harmonic will only be one-third of the amperes produced per volt of the fundamental. With the higher harmonics this effect is accentuated still more. In other words, the circuit offers a higher impedance to the harmonics than it does to the fundamental, with the result that the corresponding harmonics in the current wave will be diminished in magnitude and the current wave will be a closer approximation to the ideal than the voltage wave. Thus reactance has the effect of damping out the harmonics in the current wave, and the higher the harmonic the greater is the damping effect.

Consider a circuit having an inductance L and an applied voltage obeying the law

$$e = E_1 \sin \theta + E_3 \sin (3\theta + \alpha) + E_5 \sin (5\theta + \beta) + \ldots.$$

The R.M.S. value of the voltage is

$$E = \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}}$$

and the fundamental E.M.F. produces a maximum current of $\frac{E_1}{2\pi f L}$ amperes, whilst the third and fifth harmonics produce maximum currents of $\frac{E_3}{2\pi \times 3f \times L}$ and $\frac{E_5}{2\pi \times 5f \times L}$ amperes respectively, and so on. The R.M.S. value of all these currents combined is

$$I = \sqrt{\frac{\left(\frac{E_1}{2\pi f L}\right)^2 + \left(\frac{E_3}{6\pi f L}\right)^2 + \left(\frac{E_5}{10\pi f L}\right)^2 + \ldots}{2}}$$

$$= \frac{1}{2\pi f L} \times \sqrt{\frac{E_1^2}{2} + \frac{E_3^2}{18} + \frac{E_5^2}{50} + \ldots}$$

The reactance, considered with respect to the complex wave, is given by

$$X' = \frac{E}{I} = \frac{\sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}}}{\frac{1}{2\pi f L} \times \sqrt{\frac{E_1^2}{2} + \frac{E_3^2}{18} + \frac{E_5^2}{50} + \ldots}}$$

$$= 2\pi f L \times \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{E_1^2 + \frac{E_3^2}{9} + \frac{E_5^2}{25} + \ldots}}.$$

Thus another effect of harmonics in the E.M.F. wave form is to increase the reactance in the ratio

$$\sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{E_1^2 + \frac{E_3^2}{9} + \frac{E_5^2}{25} + \ldots}}.$$

The relative phases of the various harmonics do not affect the value of this ratio, which is only dependent upon the relative magnitudes of the various harmonics.

As an example, consider the case where there is a 20 per cent. third harmonic, a 10 per cent. fifth harmonic and a 5 per cent.

seventh harmonic in the E.M.F. wave form. This voltage is applied to a circuit having an inductance L. The reactance corresponding to this on a sine wave would be $2\pi fL$, but on the complex wave in question it is

$$2\pi fL \times \sqrt{\frac{1^2 + 0{\cdot}2^2 + 0{\cdot}1^2 + 0{\cdot}05^2}{1^2 + \frac{0{\cdot}2^2}{9} + \frac{0{\cdot}1^2}{25} + \frac{0{\cdot}05^2}{49}}}.$$

$$= 2\pi fL \times 1{\cdot}023.$$

Thus the inductance would appear to be 2·3 per cent. higher on this particular wave form than on a pure sine wave. Strictly speaking, the factor 2π is only correct on the sine wave hypothesis, and for other wave forms a slightly different constant should be used.

If resistance is also present in the circuit the conditions are somewhat modified. The current produced by each component of the E.M.F. is calculated from a knowledge of the impedance corresponding to the particular frequency, whilst the angle of lag for each component is calculated from the formula

$$\theta = \tan^{-1}\frac{X}{R}.$$

This angle will be different for the various harmonics on account of the change in X. The sum of these component currents gives the resultant from which the wave may be plotted.

Effect of Harmonics on Capacity Reactance.—In the case of a circuit containing nothing but condensers, the impedance is inversely proportional to the frequency, and treating a complex wave in the same way as in the previous paragraph, it is seen that each harmonic produces a proportionally greater current than the fundamental. Thus the third harmonic produces three times the current per volt that the fundamental does, and so on. Consequently, the circuit appears to possess a reduced impedance when the E.M.F. wave contains harmonics.

If the circuit contains a capacity of C farads the fundamental E.M.F. will produce a maximum current of $2\pi fCE_1$ amperes where E_1 is the maximum value of the fundamental E.M.F. The third harmonic will produce a current of $2\pi \times (3f) \times CE_3$ amperes, and so on. The R.M.S. value of the resultant current will be

$$I = \sqrt{\frac{(2\pi fCE_1)^2 + (6\pi fCE_3)^2 + (10\pi fCE_5)^2 + \ldots}{2}}$$

$$= 2\pi fC \times \sqrt{\frac{E_1{}^2 + 9E_3{}^2 + 25E_5{}^2 + \ldots}{2}},$$

The impedance is therefore given by

$$\frac{E}{I} = \frac{\sqrt{\dfrac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}}}{2\pi fC \times \sqrt{\dfrac{E_1^2 + 9E_3^2 + 25E_5^2 + \ldots}{2}}}$$

$$= \frac{1}{2\pi fC} \times \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{E_1^2 + 9E_3^2 + 25E_5^2 + \ldots}}$$

The impedance is therefore reduced in the ratio

$$\sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{E_1^2 + 9E_3^2 + 25E_5^2 + \ldots}}.$$

Considering the same E.M.F. wave form as in the example in previous paragraph, this ratio works out to be

$$\sqrt{\frac{1^2 + 0{\cdot}2^2 + 0{\cdot}1^2 + 0{\cdot}05^2}{1^2 + 9 \times 0{\cdot}2^2 + 25 \times 0{\cdot}1^2 + 49 \times 0{\cdot}05^2}}$$

$$= 0{\cdot}779.$$

This example serves to show the magnitude of the errors which are liable to creep in when measuring the capacity of a condenser by determinations of the current, voltage and frequency. The higher harmonics are particularly objectionable in such a case. Even a 1 per cent. seventh harmonic results in a decrease in the impedance by 0·25 per cent.

Any variation from the ideal sine wave of E.M.F. results in an increase in the current taken by a condenser, and this shows why it is desirable to obtain in practice as near an approximation as possible to a sinusoidal wave form in order that the capacity current in the circuit, which is undesirable, shall be reduced to a minimum.

If the circuit also contains resistance, the problem is further complicated as in the case of an inductance, the relative phase angles of the various harmonics being considerably modified.

In Fig. 73 are shown the current waves obtained by the application of an E.M.F. given by the expression

$$e = 100 \sin\theta + 25 \sin(3\theta + 180)$$

to (*a*) a pure choking coil and (*b*) a condenser.

Resonance with Harmonics.—There is for every circuit a particular frequency at which the effects of resonance reach a maximum. This critical frequency may be a multiple of the frequency of supply and may coincide more or less with that of one of the harmonics. The effect of resonance is to produce a very large current per volt

and to render the impedance very small over a certain limited range of frequency. Thus the current due to a particular harmonic in the E.M.F. wave may be greatly magnified when it appears in the current wave, resulting in an extraordinary distortion. In fact, in some extreme cases it appears at first sight as if the frequency of the complex wave were that of the resonating harmonic, but this is not so.

The conditions which arise in such a circuit are very well illustrated by means of a concrete example. Consider the case of an E.M.F. wave represented by

$$e = 100 \sin \theta + 10 \sin (3\theta + 180°).$$

The circuit consists of a resistance of 2 ohms, an inductance of 0·02 henry, and a capacity of 55 mfds., all connected in series, the frequency being 50 cycles per second.

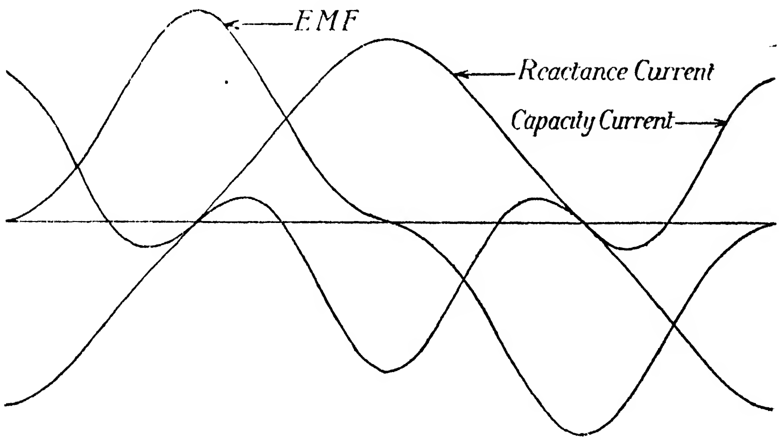


FIG. 73.—Capacity and Reactance Current Wave Forms.

The maximum value of the fundamental current is

$$\frac{100}{\sqrt{2^2 + \left(2\pi \times 50 \times 0{\cdot}02 - \dfrac{10^6}{2\pi \times 50 \times 55}\right)^2}}$$

$$= 1{\cdot}94 \text{ amperes.}$$

The maximum value of the third harmonic current is

$$\frac{10}{\sqrt{2^2 + \left(2\pi \times 150 \times 0{\cdot}02 - \dfrac{10^6}{2\pi \times 150 \times 55}\right)^2}}$$

$$= 4{\cdot}88 \text{ amperes.}$$

The angle of lead of the fundamental current over the fundamental voltage is

$$\tan^{-1}\frac{\left(2\pi \times 50 \times 0{\cdot}02 - \dfrac{10^6}{2\pi \times 50 \times 55}\right)}{2} = 88°.$$

The angle of lead of the harmonic current over the harmonic voltage is

$$\tan^{-1}\frac{\left(2\pi \times 150 \times 0{\cdot}02 - \dfrac{10^6}{2\pi \times 150 \times 55}\right)}{2} = 13°.$$

Strictly speaking, these angles are negative angles of lag.

The expression for the current is therefore

$$1{\cdot}94 \sin(\theta + 88°) + 4{\cdot}88 \sin(3\theta + 193°).$$

Both the voltage and the current waves are represented in Fig. 74, which shows the effect of resonance with the third harmonic.

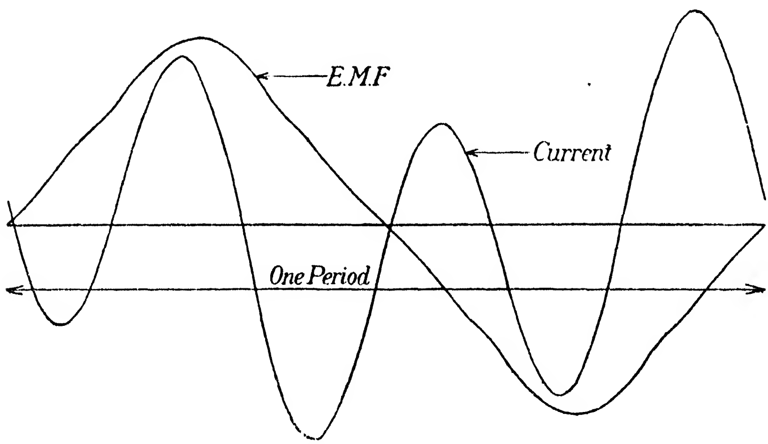


FIG. 74.—Effect of Resonance with Third Harmonic.

When a length of alternating current mains is switched into circuit, a capacity or charging current will flow even on open circuit, since the two mains, being separated by a dielectric, constitute a condenser. The line will also possess a certain amount of inductance, and hence there is a definite frequency at which maximum resonance will occur. If there is a harmonic of this frequency in the E.M.F. wave it will be reproduced to an enormously exaggerated extent in the current wave, resulting in a considerable increase in the R.M.S. value of the charging current. It is very rare that resonance occurs with the fundamental in cases like this; but it is not at all out of the limits of possibility that trouble should be experienced with

harmonics. This forms an additional argument in favour of obtaining as close an approximation to a sine wave as possible.

Power resulting from a Complex Wave.—In determining the power in a circuit due to a complex E.M.F. wave, every component voltage must be considered with respect to every component current. But the average value of the power resulting from a voltage of one frequency and a current of another is zero, and thus the problem is greatly simplified. Each voltage, therefore, need only be considered with respect to the current of the same frequency which it produces. The total power is the sum of the powers resulting from each harmonic separately in addition to that due to the fundamental, the frequency of the power in each case being double that of the current and voltage of which it is the result. The power factor of these various components will be different, the resultant power factor being obtained by dividing the total power by the product of the R.M.S. current and the R.M.S. voltage.

As an example, consider the case of a circuit consisting of a resistance of 10 ohms and an inductance of 0·2 henry. The expression for the applied E.M.F. is

$$e = 100 \sin\theta + 20 \sin(3\theta + 60°) + 10 \sin(5\theta + 150°) + 5 \sin(7\theta + 300°)$$

and the frequency is 50.

The impedance with respect to the various components is

Fundamental	$\sqrt{10^2 + (2\pi \times 50 \times 0{\cdot}02)^2} = 11{\cdot}8$	apparent ohms.
3rd Harmonic	$\sqrt{10^2 + (2\pi \times 150 \times 0{\cdot}02)^2} = 21{\cdot}3$	,, ,,
5th ,,	$\sqrt{10^2 + (2\pi \times 250 \times 0{\cdot}02)^2} = 33{\cdot}0$	,, ,,
7th ,,	$\sqrt{10^2 + (2\pi \times 350 \times 0{\cdot}02)^2} = 45{\cdot}0$	,, ,,

The angles of lag of the currents behind their respective voltages are

Fundamental	$\tan^{-1}\dfrac{2\pi \times 50 \times 0{\cdot}02}{10} = 32{\cdot}1°$
3rd Harmonic	$\tan^{-1}\dfrac{2\pi \times 150 \times 0{\cdot}02}{10} = 62{\cdot}0°$
5th ,,	$\tan^{-1}\dfrac{2\pi \times 250 \times 0{\cdot}02}{10} = 72{\cdot}3°$
7th ,,	$\tan^{-1}\dfrac{2\pi \times 350 \times 0{\cdot}02}{10} = 77{\cdot}2°$

The various power factors are

Fundamental	$\cos 32{\cdot}1° = 0{\cdot}85$
3rd Harmonic	$\cos 62{\cdot}0° = 0{\cdot}47$
5th ,,	$\cos 72{\cdot}3° = 0{\cdot}30$
7th ,,	$\cos 77{\cdot}2° = 0{\cdot}22.$

The expression for the current is

$$i = \frac{100}{11{\cdot}8}\sin(\theta - 32^\circ) + \frac{20}{21{\cdot}3}\sin(3\theta + 60^\circ - 62^\circ)$$

$$+ \frac{10}{33{\cdot}0}\sin(5\theta + 150^\circ - 72^\circ) + \frac{5}{45{\cdot}0}\sin(7\theta + 300^\circ - 77^\circ)$$

$$= 8{\cdot}48\sin(\theta + 328^\circ) + 0{\cdot}94\sin(3\theta + 358^\circ)$$

$$+ 0{\cdot}30\sin(5\theta + 78^\circ) + 0{\cdot}11\sin(7\theta + 223^\circ).$$

The R.M.S. value of the current is

$$\sqrt{\frac{8{\cdot}48^2 + 0{\cdot}94^2 + 0{\cdot}30^2 + 0{\cdot}11^2}{2}}$$

$$= 6{\cdot}02 \text{ amperes.}$$

The R.M.S. value of the voltage is

$$\sqrt{\frac{100^2 + 20^2 + 10^2 + 5^2}{2}}$$

$$= 72{\cdot}5 \text{ volts.}$$

The total power is

$$\frac{100}{\sqrt{2}} \times \frac{8{\cdot}48}{\sqrt{2}} \times 0{\cdot}85 + \frac{20}{\sqrt{2}} \times \frac{0{\cdot}94}{\sqrt{2}} \times 0{\cdot}47$$

$$+ \frac{10}{\sqrt{2}} \times \frac{0{\cdot}30}{\sqrt{2}} \times 0{\cdot}30 + \frac{5}{\sqrt{2}} \times \frac{0{\cdot}11}{\sqrt{2}} \times 0{\cdot}22$$

$$= 360 + 4{\cdot}4 + 0{\cdot}45 + 0{\cdot}06$$

$$= 365 \text{ watts say.}$$

The resultant power factor is given by

$$\frac{\text{Total Average Power}}{\text{R.M.S. Voltage} \times \text{R.M.S. Current}}$$

$$= \frac{365}{72{\cdot}5 \times 6{\cdot}02}$$

$$= 0{\cdot}84.$$

P.D. and Current Waves in the Case of an Arc.—In the case of an arc supplied from an A.C. source, with a steadying resistance in series, a peculiar distortion is brought about in both the current wave and that of the p.d. across the arc terminals. To commence with, the supply voltage must rise to a certain value before the arc is struck, and thus the current wave is flat during this interval. When the arc is struck the current increases rapidly, and this

current, flowing through the series steadying resistance, causes a considerable drop in voltage which has the effect of suddenly lowering the p.d. across the arc itself. This sudden peak is shown in Fig. 75, which represents a typical case. As the current rises the voltage drop in the series resistance increases so that the wave form of the p.d. across the arc shows a broad hollow which continues until the arc suddenly goes out. The current then drops to zero, whilst another smaller peak is obtained in the p.d. wave. The arc is then built up in the opposite direction and the operations are repeated.

This striking distortion of the current wave gives rise to a peculiar state of affairs with regard to the power consumed. The power curve can be constructed in the usual way from the current and voltage waves, but when its average value is measured the result is very much lower than the product of the R.M.S. current and the R.M.S. volts. Therefore, notwithstanding the fact that

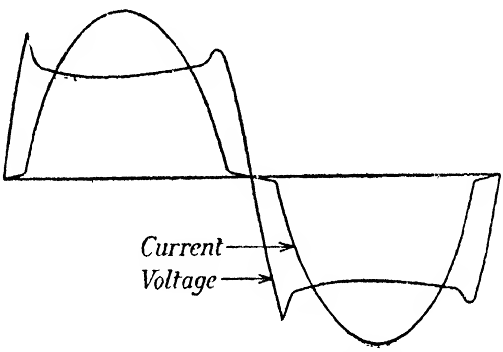


FIG. 75.—Current and Voltage Waves for an Arc.

the current is in phase with the voltage, as far as this can be said of two waves so widely dissimilar, the power factor of the arc is considerably less than unity and may fall as low as 0·6 in certain cases where hard carbons are used.

The measurement of the amperes, volts and watts consumed by an A.C. arc forms an instructive experiment, a power factor of less than unity being obtained in spite of the absence of inductance and capacity.

The horned appearance of the p.d. wave is very much accentuated if the arc is struck in coal gas, the tip of the initial peak representing a voltage many times as great as is obtained throughout the remainder of the time.

Other Causes of Wave Distortion.—Ordinary resistances are often the cause of a slight distortion of the current wave form on account of their variation of temperature throughout the cycle. In the case of an ordinary metal filament glow lamp, the temperature of the filament varies to a considerable extent throughout the cycle,

and since it cannot get rid of its heat instantaneously it follows that the resistance when the voltage is decreasing is on the whole higher than when the voltage is increasing. Consequently, the current in the second quarter of the cycle is slightly less than it is in the first quarter. This results in a small distortion of the wave form, which is thrown slightly forward. The angle of lead obtained in this manner is of the order of 2° or 3° resulting in a drop in the power factor of approximately 0·1 per cent. In all ordinary measurements this is, of course, negligible.

Another prolific source of wave distortion is the variation of the inductance of certain machines in different positions, certain cases of which will be dealt with later.

Equivalent Sine Wave.—In the case of a distorted wave form the power factor is no longer equal to $\cos\phi$, for this is based on the assumption of a sinusoidal wave form. Indeed, ϕ will probably vary throughout the cycle. In some cases it may be simpler to treat problems by substituting an equivalent sine wave for the actual one in question. The R.M.S. value of this equivalent sine wave will be the same as that of the actual wave, and consequently the maximum value of the sine wave will be $\sqrt{2}$ times the R.M.S. value of the actual wave. The phase of the equivalent sine wave can be determined from the power factor. The angle of lag, ϕ', is chosen so that $\cos\phi'$ is equal to the actual power factor. In the case of an A.C. arc, the equivalent current would be taken to be in phase with the voltage, its magnitude being reduced so as to get the right power.

A warning is issued against a too promiscuous use of the equivalent sine wave, as it is only an approximation, and often only a very rough one at that.

Effect of Wave Form on Insulation Testing.—When insulation is subjected to an alternating electrical stress it is the maximum voltage which causes the greatest effect. The application of a peaky wave form is a severer test than if a flat wave form is used, assuming the two R.M.S. values to be equal. On the other hand, the breakdown of the insulation is not solely due to the maximum voltage, but depends to some extent upon the rapidity of its growth in the near neighbourhood of the maximum point.

Harmonic Analysis.—A number of different methods of analysing complex periodic waves have been devised, but it is not expedient to discuss them at this juncture. One method, due to Perry,[1] will be here described, not because it is the best, but because it is one of the simplest to understand.

Perry's Method.—Commencing from any point, the wave may be represented by

$$e = E_1 \sin(\theta + \alpha_1) + E_3 \sin(3\theta + \alpha_3) + E_5 \sin(5\theta + \alpha_5) + \dots$$

[1] *Electrician*, 1898, vol. xxviii. p. 362.

This can be expanded as follows:—

$$\begin{aligned} e = {} & E_1 \sin\theta \cos\alpha_1 + E_1 \cos\theta \sin\alpha_1 \\ & + E_3 \sin 3\theta \cos\alpha_3 + E_3 \cos 3\theta \sin\alpha_3 \\ & + E_5 \sin 5\theta \cos\alpha_5 + E_5 \cos 5\theta \sin\alpha_5 \\ & + \ldots\ldots \end{aligned}$$

Since α_1, α_3, α_5, etc., are constants, this can be written

$$\begin{aligned} e = {} & A_1 \sin\theta + A_3 \sin 3\theta + A_5 \sin 5\theta + \ldots \\ & + B_1 \cos\theta + B_3 \cos 3\theta + B_5 \cos 5\theta + \ldots, \end{aligned}$$

where $$A_1 = E_1 \cos\alpha_1$$

and $$B_1 = E_1 \sin\alpha_1, \text{ etc.}$$

To determine A_1, multiply both sides of the equation by $\sin\theta$ and find the average value over half a cycle. Then

$$\begin{aligned} e \sin\theta = {} & A_1 \sin^2\theta + A_3 \sin 3\theta \sin\theta + A_5 \sin 5\theta \sin\theta + \ldots \\ & + B_1 \cos\theta \sin\theta + B_3 \cos 3\theta \sin\theta + B_5 \cos 5\theta \sin\theta + \ldots \end{aligned}$$

The average value of all the terms on the right-hand side of the equation with the exception of the first is zero, and the average value of $\sin^2\theta$ is $\frac{1}{2}$. Hence A_1 is equal to twice the average value of $e \sin\theta$ taken throughout the half-period.

In a similar manner, by multiplying throughout by $\sin 3\theta$, $\sin 5\theta$ and by $\cos\theta$, $\cos 3\theta$, $\cos 5\theta$, etc., the other co-efficients A_3, A_5, B_1, B_3, B_5, etc., are determined as follows:—

$$\begin{array}{llllll} A_1 = & \text{twice} & \text{the average} & \text{value of} & e \sin\theta, \\ A_3 = & ,, & ,, & ,, & e \sin 3\theta, \\ A_5 = & ,, & ,, & ,, & e \sin 5\theta, \\ B_1 = & ,, & ,, & ,, & e \cos\theta, \\ B_3 = & ,, & ,, & ,, & e \cos 3\theta. \\ B_5 = & ,, & ,, & ,, & e \cos 5\theta, \\ \text{etc.} \end{array}$$

It is now required to cast the terms of the expression in the form $E_1 \sin(\theta + \alpha_1)$ instead of $A_1 \sin\theta$ and $B_1 \cos\theta$.

We have $$E_1 \sin(\theta + \alpha_1) = A_1 \sin\theta + B_1 \cos\theta.$$

Also $$E_1 \sin(\theta + \alpha_1) = E_1 \cos\alpha_1 \sin\theta + B_1 \sin\alpha_1 \cos\theta.$$

Therefore $$A_1 = E_1 \cos\alpha_1$$

and $$B_1 = E_1 \sin\alpha_1.$$

Again $$A_1^2 + B_1^2 = E_1^2 (\cos^2\alpha_1 + \sin^2\alpha_1) = E_1^2$$

and $$E_1 = \sqrt{A_1^2 + B_1^2}.$$

Also $$\frac{B_1}{A_1} = \frac{E_1 \sin\alpha_1}{E_1 \cos\alpha_1} = \tan\alpha_1.$$

Summarising

$$E_1 = \sqrt{A_1^2 + B_1^2}$$
$$E_3 = \sqrt{A_3^2 + B_3^2}$$
$$E_5 = \sqrt{A_5^2 + B_5^2}, \text{ etc.,}$$

and

$$a_1 = \tan^{-1}\frac{B_1}{A_1}$$
$$a_3 = \tan^{-1}\frac{B_3}{A_3}$$
$$a_5 = \tan^{-1}\frac{B_5}{A_5}, \text{ etc.}$$

The results are best drawn up in the form of a table as shown on page 111. The columns in the table are filled in with observations taken from Fig. 76, from which the following results are obtained :—

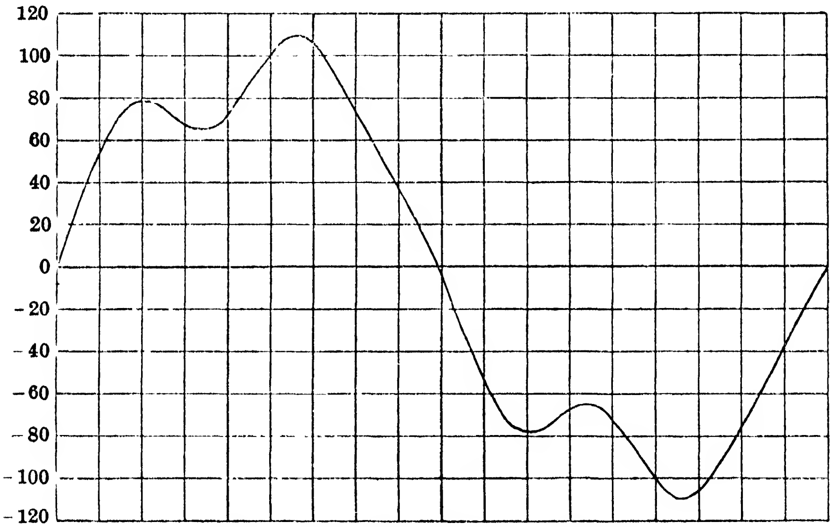


FIG. 76.—Wave Form Analysed on page 111.

$A_1 = 2 \times 50 = 100$ $\quad A_3 = 2 \times 7{\cdot}1 = 14{\cdot}2$ $\quad A_5 = 2 \times 0 = 0$

$B_1 = 2 \times 0 = 0$ $\quad B_3 = 2 \times 7{\cdot}1 = 14{\cdot}2$ $\quad B_5 = 2 \times (-5) = -10$

$E_1 = \sqrt{100^2 + 0^2} = 100$ $\quad E_3 = \sqrt{14{\cdot}2^2 + 14{\cdot}2^2} = 20$ $\quad E_5 = \sqrt{0^2 + (-10)^2} = 10$

$a_1 = \tan^{-1}\frac{0}{100} = 0°$ $\quad a_3 = \tan^{-1}\frac{14{\cdot}2}{14{\cdot}2} = 45°$ $\quad a_5 = \tan^{-1}\frac{-10}{0} = 270°.$

The final expression is

$$e = 100 \sin(\theta + 0°) + 20 \sin(3\theta + 45°) + 10 \sin(5\theta + 270°).$$

Observed.		Calculated.											
Angle	e	sin θ	cos θ	sin 3θ	cos 3θ	sin 5θ	cos 5θ	e sin θ	e cos θ	e sin 3θ	e cos 3θ	e sin 5θ	e cos 5θ
5°	17·0	·0872	·9962	·2588	·9569	·4226	·9063	1·5	16·9	4·4	16·3	7·2	15·4
15°	43·3	·2588	·9659	·7071	·7071	·9659	·2588	11·2	41·9	30·6	30 6	41·9	11·2
25°	65·3	·4226	·9063	·9659	·2588	·8192	– ·5736	27·6	59 2	63·1	16·9	53·4	– 37·4
35°	77·3	·5736	·8192	·9659	– ·2588	·0872	– ·9962	44·3	63·3	74·7	– 20·0	6·7	– 77·0
45°	77·8	·7071	·7071	·7071	– ·7071	– ·7071	– ·7071	55·0	55·0	55·0	– 55·0	– 55·0	– 55·0
55°	71·0	·8192	·5736	·2588	– ·9659	– ·9962	·0872	58·1	40·7	18·4	– 68·6	– 70·7	6·2
65°	65·1	·9063	·4226	– ·2588	– ·9659	– ·5736	·8192	59·0	27·5	– 16·9	– 63·0	– 37·4	53·3
75°	66·9	·9659	·2588	– ·7071	– ·7071	·2588	·9659	64·6	17·3	– 47·3	– 47·3	17·3	64·6
85°	78·1	·9962	·0872	– ·9659	– ·2588	·9063	·4226	77·9	6·8	– 75·5	– 20·2	70·8	33·0
95°	93·8	·9962	– ·0872	– ·9659	·2588	9063	– ·4226	93·5	– 8·2	– 90·6	24·3	85·0	– 39·6
105°	106·2	·9659	– ·2588	– ·7071	·7071	·2588	– ·9659	102·7	– 27·5	– 75·2	75·2	27·5	– 102·7
115°	108·8	·9063	– ·4226	– ·2588	·9659	– ·5736	– ·8192	98·5	– 46·0	– 28 2	105·0	– 62·4	– 89·0
125°	100·1	·8192	– ·5736	·2588	·9659	– ·9962	– ·0872	81·9	– 57·4	25·9	96·7	– 99·7	– 8·7
135°	83·6	·7071	– ·7071	·7071	·7071	– ·7071	·7071	59·1	– 59·1	59·1	59·1	– 59·1	59·1
145°	64·7	·5736	– ·8192	·9659	·2588	·0872	·9962	37·1	– 52·9	62·5	16·7	5·6	64·5
155°	46·5	·4226	– ·9063	·9659	– ·2588	·8192	·5736	19·7	– 42·1	44·9	– 12·0	38·0	26·7
165°	28 5	·2588	– ·9659	·7071	– ·7071	·9659	– ·2588	7·4	– 27·5	20·1	– 20·1	27·5	– 7·4
175°	7·8	·0872	– ·9962	·2588	– ·9659	·4226	– ·9063	0·7	– 7·8	2·0	– 7·5	3·3	– 7·1
		Average Values ...		...		...		50·0	0	7·1	7·1	0	– 5·0

This method involves some very tedious calculations, particularly if the higher harmonics are to be determined, for the above example is only worked out up to the fifth harmonic.

Author's Method.—Most of the methods of analysing periodic wave forms at present in use are rather tedious to carry out and necessitate a good deal of time being spent on the evaluation of the various constants. The method here outlined is an effort to expedite calculations of this kind and to provide a ready method, by means of a simple series of equations, for the analysis of periodic wave forms.

The method consists in taking a series of pairs of points on the wave to be analysed, equidistantly spaced on either side of the point chosen as the zero. The algebraic sum of each pair of readings is equal to a series of terms of the type $2E \sin\alpha \cos\theta$, since they are the sum of two sines. Here, θ is equal to some known angle, determined by the position of the points chosen, and E and α are constants depending upon the composition of the wave. In a similar manner, the algebraic differences of the same pairs of points give rise to a series of terms of the type $2E \cos\alpha \sin\theta$, the symbols having the same meaning as before. When a sufficient number of such equations are obtained, they can be solved in terms of the quantities $E \sin\alpha$ and $E \cos\alpha$.

In solving up to, say, the seventeenth harmonic there are eighteen unknown quantities to be determined, namely, the amplitudes and phase angles of the fundamental and the eight harmonics. Thus eighteen simultaneous equations are required, the necessary data being obtained from eighteen chosen ordinates. In order to minimise errors due to the incorrect drawing of the curve, those ordinates are chosen at equal distances apart, viz., at 5°, 15°, 25°, . . . 165° and 175° after the point which it is desired to regard as zero. The values of these various ordinates will be represented by V_5, V_{15}, V_{25}, . . . V_{165} and V_{175}.

Let the wave be represented by the expression:—

$$V_\theta = E_1 \sin(\theta + \alpha_1) + E_3 \sin(3\theta + \alpha_3) + \ldots + E_{17} \sin(17\theta + \alpha_{17}).$$

$$\text{Then} \quad V_{85} = E_1 \sin(85° + \alpha_1) + E_3 \sin(255° + \alpha_3) + E_5 \sin(425° + \alpha_5) + \ldots + E_{17} \sin(1445° + \alpha_{17}),$$

$$\text{and} \quad V_{-85} = E_1 \sin(-85° + \alpha_1) + E_3 \sin(-255° + \alpha_3) + E_5 \sin(-425° + \alpha_5) + \ldots + E_{17} \sin(-1445° + \alpha_{17}) = -V_{95}.$$

$$V_{85} + V_{95} = V_{85} - V_{-85} = 2E_1 \cos\alpha_1 \sin 85° + 2E_3 \cos\alpha_3 \sin 255° + 2E_5 \cos\alpha_5 \sin 425° + \ldots + 2E_{17} \cos\alpha_{17} \sin 1445°.$$

$$\text{Similarly,} \quad V_{75} + V_{105} = V_{75} - V_{-75} = 2E_1 \cos\alpha_1 \sin 75° + 2E_3 \cos\alpha_3 \sin 225° + 2E_5 \cos\alpha_5 \sin 375° + \ldots + 2E_{17} \cos\alpha_{17} \sin 1275°.$$

Other readings are taken every 10° until V_5 and V_{175} are reached. Then the resulting nine simultaneous equations are solved, thus obtaining $E_1 \cos a_1$, $E_3 \cos a_3$, etc., in terms of $(V_{85} + V_{95})$, $(V_{75} + V_{105})$, . . . $(V_5 + V_{175})$.

A second series of calculations must now be made as follows :—

$$\begin{aligned} V_{85} - V_{95} &= V_{85} + V_{-85} \\ &= 2E_1 \sin a_1 \cos 85° + 2E_3 \sin a_3 \cos 255° \\ &\qquad + 2E_5 \sin a_5 \cos 425° + \ldots + 2E_{17} \sin a_{17} \cos 1445°. \end{aligned}$$

$$\begin{aligned} V_{75} - V_{105} &= V_{75} + V_{-75} \\ &= 2E_1 \sin a_1 \cos 75° + 2E_3 \sin a_3 \cos 225° \\ &\qquad + 2E_5 \sin a_5 \cos 375° + \ldots + 2E_{17} \sin a_{17} \cos 1275°. \end{aligned}$$

In this way another nine simultaneous equations are obtained and $E_1 \sin a_1$, $E_3 \sin a_3$, etc., are evaluated in terms of $(V_{85} - V_{95})$, $(V_{75} - V_{105})$, . . . $(V_5 - V_{175})$.

Then
$$E_1 = \sqrt{(E_1 \sin a_1)^2 + (E_1 \cos a_1)^2},$$
$$a_1 = \tan^{-1}\left[\frac{E_1 \sin a_1}{E_1 \cos a_1}\right],$$

and similarly for the various harmonics.

Care must be taken to observe the signs of $E \sin a$ and $E \cos a$, as these enable the quadrant in which a is situated to be determined.

The initial labour in solving the above equations is considerable, but this having been accomplished once and for all, the problem resolves itself into simply multiplying the various chosen ordinates by certain known constants, and the analysis becomes much less tedious an operation than it is by many of the other methods in common use.

In order further to facilitate the actual calculations, a schedule has been drawn up as shown in Table I. This table is self-explanatory ; in fact it is not even necessary to be familiar with the underlying principles in order to work out an example.

If so desired, the fundamental or any particular harmonic may be determined by itself without carrying out the analysis any further.

A modified set of constants, for use in connection with the approximate analysis of a wave up to the fifth harmonic, may be of value in some cases. For this purpose six readings from the curve are necessary and they may be taken conveniently at 15°, 45°, 75°, 105°, 135° and 165° after the zero.

It will be found convenient for quickness in working to arrange these approximate calculations also in the form of a schedule, an example of which is shown in Table II.

If no harmonics higher than the fifth are present, the constants

enumerated in this table will give correct results, but, of course, the more pronounced the higher harmonics are, the larger the error will be in the determination of the equation to the wave by means of this second set of constants.

Experimental Determination of Wave Form.—A number of different types of instruments have been devised for the purpose of determining wave forms experimentally, and some of these will be described in Chapter XII. One method which does not involve the use of any special type of measuring instrument, except an electrostatic voltmeter, will be described now.

Joubert's Contact Method.—The special piece of apparatus used consists of an arrangement whereby momentary contact is made once per cycle to an electrostatic voltmeter (see p. 138), which consequently gives a deflection which is an indication of the instantaneous value rather than the R.M.S. value. By shifting the point of contact, the instantaneous values can be determined at other points of the wave, thus enabling it to be plotted.

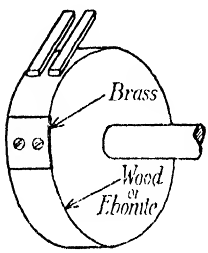


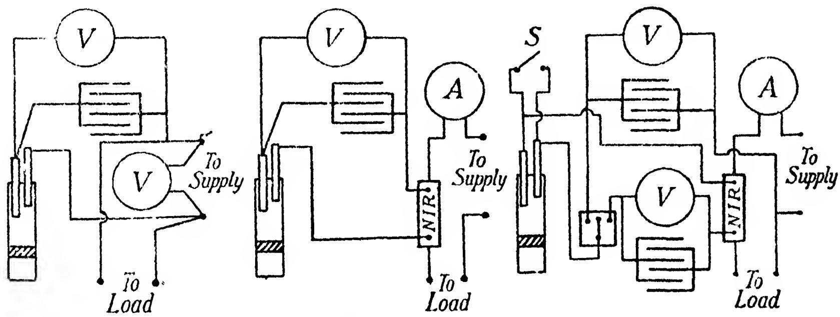
FIG. 77.—Joubert Contact.

One form of the Joubert contact consists of an insulated disc mounted on a metal hub, the whole being directly driven by means of a shaft extension of the machine supplying the power. At one portion of the rim a little strip of brass is let in so that two copper brushes placed side by side and insulated from each other will be connected together every time the strip of brass comes under them. One of these brushes is capable of being given a certain amount of lead by means of an adjustment, this regulating the duration of the interval of time over which contact is made. This affects the steadiness and magnitude of the voltmeter reading and the best position is obtained by trial.

The two brushes are supported from a movable arm the position of which is indicated by means of a pointer moving over a scale. Assuming that the machine supplying the circuit has four poles, a displacement of 10° of the brushes corresponds to a phase displacement of 20°. If the brushes are moved round in the direction of rotation the instant of contact is deferred and the point obtained comes later in the wave, whilst if the brushes are moved in the opposite direction the reverse is the case.

In making a determination of the voltage wave the circuit is connected according to Fig. 78 (a), the electrostatic voltmeter measuring directly the instantaneous pressure across the mains, whilst the connections shown in Fig. 78 (b) are used when it is desired to make a determination of the current wave. Here the instantaneous voltage drop across a known non-inductive resistance is obtained, enabling the instantaneous current to be calculated. The other instruments indicated in the diagram serve to measure

the R.M.S. values of the quantities concerned. An alternative method of doing this is to connect a short-circuiting switch across the two brushes and use one voltmeter throughout. A condenser is also placed in parallel with the voltmeter in order to maintain the deflection during the period when no electrical connection is made at the contact. The magnitude of the condenser should be determined by trial, but the best value will in most cases be found to be somewhere under a microfarad. By means of a change-over switch it can be arranged to take readings on the voltage and current waves together as indicated in Fig. 78 (*c*).



(*a*) For Volt Wave. (*b*) For Current Wave. (*c*) For Volt and Current Waves.

FIG. 78.—Connections for Joubert Contact.

For the purpose of current wave form determinations, it is better to have a low reading electrostatic voltmeter such as the Ayrton and Mather type, or else to set up the voltage by the addition of a constant voltage. This latter must be very constant, as otherwise the accuracy of the measurement is vitiated. A convenient piece of apparatus for this purpose is a battery of 100 cadmium cells arranged so that they can be switched in ten at a time.

CHAPTER X

POLYPHASE CURRENTS

Production of Two Phase Currents.—The simple elementary alternating current dynamo consists of a single turn rotating with uniform speed in a bipolar magnetic field. Such a turn has a sinusoidal E.M.F. induced in it if certain conditions are fulfilled, and a *single*

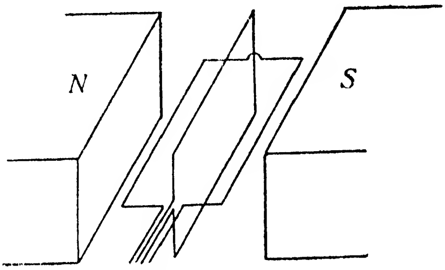


FIG. 79.—Simple Two Phase A C. Dynamo.

phase current is the result if the circuit is closed. Now imagine a second turn rigidly fixed to the first, the planes of the two coils being at right angles (see Fig. 79). This second turn will also produce a sinusoidal E.M.F. of the same magnitude as the first, the only difference being that when one coil has its maximum E.M.F. induced, the E.M.F. in the second is zero. In other words, there is a phase difference of 90° between the two E.M.F.'s induced in the two coils. The two E.M.F.'s are represented graphically in Fig. 80. Each coil may be connected across a non-inductive resistance producing a current, and these two currents will also have a phase difference of 90°, since they are in phase with their respective E.M.F.'s. Such a circuit is termed a *two phase* circuit. If reactance is included in equal amounts in each circuit so that the magnitudes and phases of the two currents are equal, the two currents will still be 90° out of phase with each other. Such a combined circuit is

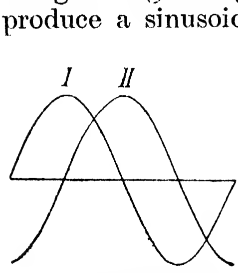


FIG. 80.—Two Phase E.M.F.'s.

termed *balanced*, which term implies quadrature of phase and equal magnitudes of the two currents. Fig. 81 shows a vector diagram of a two phase circuit where each current lags behind its E.M.F. by an angle ϕ. This circuit requires four wires to transmit the power, the two circuits being separated from each other and completely insulated.

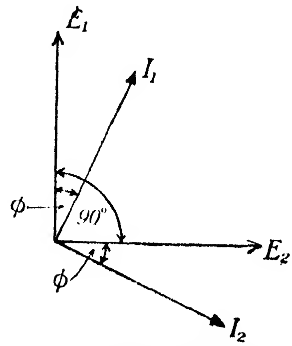


FIG. 81.—Vector Diagram of Two Phase Circuit.

In actual practice it is usual to connect the two phases together at one point so as to avoid the possibility of a great potential difference between the two coils. The two phases are then said to be *linked* together. This linking is very often accomplished by connecting the centre points of each phase together as illustrated diagrammatically in Fig. 82. Such a system is sometimes called by the alternative name of *quarter phase*.

Instead of linking the two phases together at their centres, they may be connected together at the extreme end of the coils,

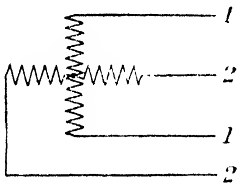


FIG. 82.—Two Phase Linked Circuit.

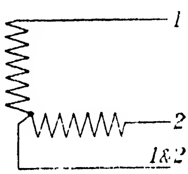


FIG. 83.—Two Phase Three Wire Circuit.

in which case they would be represented diagrammatically as in Fig. 83. This system of connections results in the reduction of the wires necessary for transmission from four to three, but labours under certain disadvantages from which the four wire transmission is free. The third wire serves as the return for both phases and hence carries a current which is the vector sum of the two phase currents. Since these two currents are in quadrature, their vector sum is $\sqrt{2}$ times the value of either of them, and hence the third wire must be made larger in cross-section to allow for this.

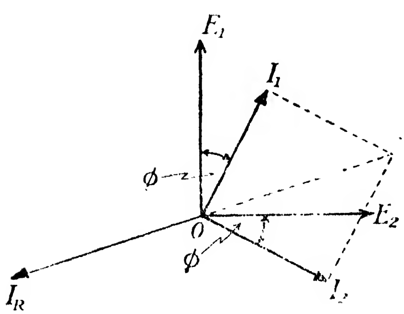


FIG. 84.—Vector Diagram for Two Phase Three Wire System.

The vector diagram for a two phase three wire system is illustrated in Fig. 84. The current flowing *inwards* in the third wire

is the vector sum of the currents in the other two wires ; therefore the current flowing *outwards* is represented by the vector OI_R, which is minus the current flowing inwards, since a reversal of the current is equivalent to changing its phase by 180°. The phase difference between the current in the third wire and either of the other two is 135°. In one case it is a lead and in the other a lag.

Power in a Two Phase Circuit.—The power developed in a two phase circuit is the sum of the powers developed in each phase separately

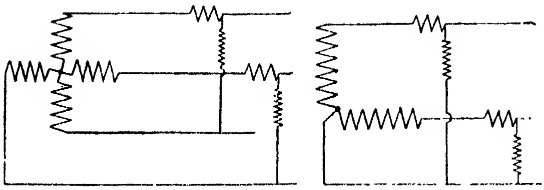


FIG. 85.—Measurement of Power in a Two Phase Circuit.

and can be determined by two wattmeters connected so that each measures the power in one phase, as shown in Fig. 85. Sometimes these two wattmeters are combined in one instrument, both tending to deflect the pointer in the same direction, so that the resultant deflection is proportional to the total power. In measuring the power in this manner, it is not necessary to have the circuits balanced.

If the circuits are balanced, the power factor of each phase will be the same, but if the circuits are unbalanced the resultant power factor will be given by

$$\frac{\text{Total Watts}}{\text{Volt-amperes of phase I} + \text{Volt-amperes of phase II.}}$$

Production of Three Phase Currents.—If, instead of placing two coils at right angles on the elementary alternating current dynamo,

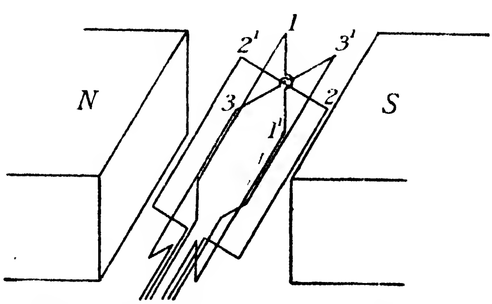


FIG. 86.—Simple Three Phase A.C. Dynamo.

three coils had been placed on it mutually inclined at 120° to each other, as in Fig. 86, then the R.M.S. voltage induced in all three would be the same, but there would be a phase difference of 120°

between each pair of coils. Such a combination is termed a *three phase* system, and the three E.M.F.'s are graphically represented in Fig. 87 both in curve form and in a vector diagram.

A three phase circuit may be represented diagrammatically in

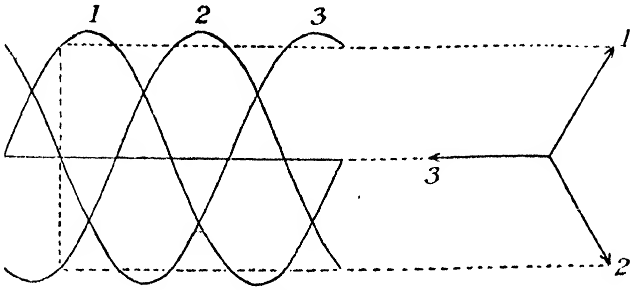


FIG. 87.—Three Phase E.M.F.'s.

the way shown in Fig. 88, where the three coils of the simple A.C. dynamo are independently connected to three equal resistances. According to this method of connection, six wires are required for the transmission of the power, but all the return leads can be combined in one without upsetting the electrical conditions, since they are only joined on one pole. Thus the six wires can be reduced

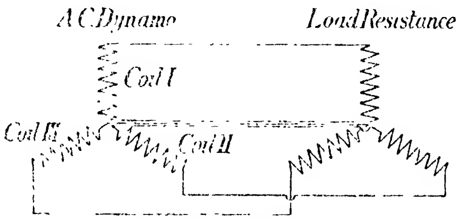


FIG. 88.—Three Phase Circuits.

to four. The current returning through this fourth wire is the vector sum of all the currents flowing *outwards* in the other three wires. Assuming that the system is balanced, these three currents will be equal and will have a phase displacement of 120° with each other. Thus the current returning by the fourth wire can be represented by the expression

$$i = I \sin\theta + I \sin(\theta - 120^\circ) + I \sin(\theta - 240^\circ).$$

This can be expanded with the following result:—

$$i = I[\sin\theta + \sin\theta\cos 120^\circ - \cos\theta\sin 120^\circ + \sin\theta\cos 240^\circ - \cos\theta\sin 240^\circ]$$

$$= I\left[\sin\theta - \frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right]$$

$$= 0.$$

Thus at every instant the current returning by way of the fourth wire is zero, and consequently this wire can be dispensed with. In this way, the important result is arrived at that only three wires are necessary to transmit power by means of a three phase system. In certain cases in practice, however, where unbalanced circuits are dealt with, four conductors are used, the earth being commonly used for the fourth conductor.

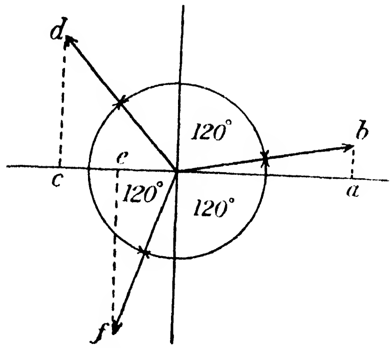


FIG. 89.—Vector Diagram of Three Phase E.M.F.'s.

The three line wires may also be regarded from the point of view that each in turn serves as the return wire for the other two, for it is seen that the instantaneous current flowing outwards in any one is always equal and opposite to the vector sum of the instantaneous currents flowing outwards in the other two. This can also be seen from the vector diagram shown in Fig. 89, where, considering the vertical components, $ab + cd$ is equal to ef and, considering the horizontal components, oa is equal to $oc + oe$.

Star System of Connection.—In developing the circuits shown in Fig. 88, the first operation was to join all the inner ends together

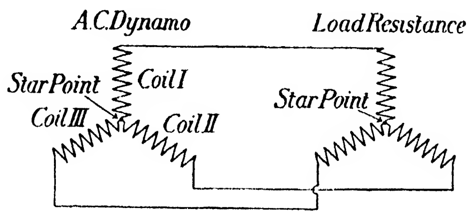


FIG. 90.—Three Phase Star Connections.

into what is known as a *star point* and then to omit the fourth wire. The resulting system of connections, shown in Fig. 90, is known as a *star connected* circuit, or sometimes as a Y-connected circuit.

An important relationship to be established is that existing between the volts per coil, or the phase volts, and the volts between any two line wires, or the line volts. Referring to the vector diagram (see Fig. 91), it is seen that the voltage between lines is equal to the vector *difference* of the two phase voltages concerned, for the vectors represent the E.M.F.'s acting away from the star point. Reversing E_2, therefore, and adding the voltage thus obtained to E_1 the vector E_L is obtained, and it is seen that there is a phase difference of 30° between E_1 and E_L. By dropping a

perpendicular from E_1 on to OE_L it is seen that $\frac{E_L}{2}$ is equal to $E_1 \cos 30° = \frac{\sqrt{3}}{2}E_1$, and therefore E_L is equal to $\sqrt{3}E_1$. The important fact is therefore established that the line voltage is $\sqrt{3}$ times the volts per coil or per phase, and that there is a difference of phase of 30° in each case.

If the load at the receiving end of the line consists of three non-inductive resistances arranged in star, then the current through each resistance will be in phase with the voltage across it, and the

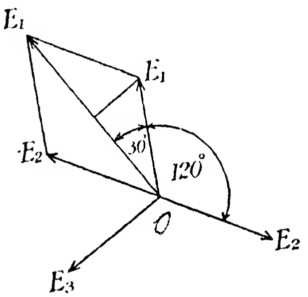


FIG. 91.—Line and Phase Volts for Star Connection.

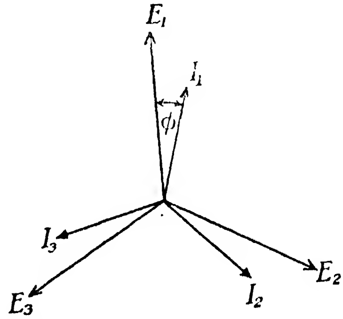


FIG. 92.—Vector Diagram showing Currents in Star Connected System.

same will hold good at the generator end of the line. But since there is a phase difference of 30° between the coil volts and the line volts, it follows that in a non-inductive load there is a phase difference of 30° between the line current and the line voltage. The voltage across lines I and II will lead the current in I by 30°, whilst the voltage across lines I and III will lag behind the current in I by 30°, as can be seen from Fig. 91. If the load circuit contains reactance as well as resistance, the current will lag behind the coil voltage by an angle ϕ, as shown in the vector diagram in Fig. 92. The angle of phase difference between current and line voltages will be $30° + \phi$ and $30° - \phi$.

Mesh System of Connection.—An alternative method of connecting the three coils of the simple alternating current dynamo is to connect the rear end of I to the front end of II, the rear end of II to the front end of III, and the rear end of III to the front end of I, thus making a closed local circuit. The three line wires are connected to the three joining points (see Fig. 93). Such a system is called a *mesh connected* or *delta* (from the Greek letter Δ) *connected* system. At first sight it appears as if there is a short circuit formed by the three coils, since they are all connected in series with one another and the circuit is closed. But if the three E.M.F.'s are added together at any instant it will be found that they always add up to

zero. In other words, the sum of the E.M.F.'s of any two coils is always equal and opposite to the E.M.F. of the third. Here the line voltage is obviously equal to the phase voltage, but the current in each line wire is the sum of the currents in two coils. Again, it

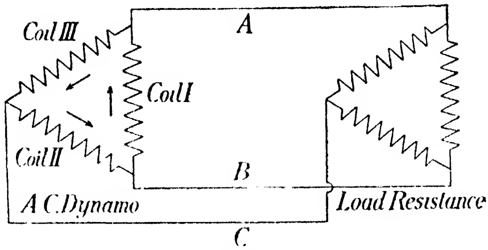


FIG. 93.—Three Phase Mesh or Delta Connections.

will be seen that the current flowing out of line A is the vector difference of the currents in coils I and III. In order to determine what this is, reverse I_3 and add it to I_1, when the resulting line current will be found to be equal to $\sqrt{3}I_1$ in the same way that the line voltage was found to be equal to $\sqrt{3}$ times the coil voltage for a star connected system. If three non-inductive resistances are connected up in the same manner at the receiving end, they will each take a current which is in phase with the voltage across their respective terminals. If reactance is present in equal amounts in the three circuits the current will lag behind the voltage by an angle ϕ, and combining each pair of phase currents the three line currents are obtained, the resulting vector diagram being shown in Fig. 94.

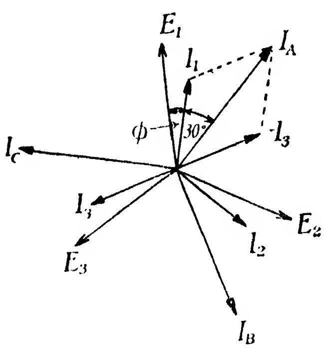


FIG. 94.—Vector Diagram of Three Phase Mesh or Delta Connected System.

The two systems of connection described above are interchangeable and it is possible to have a star-connected generator with a mesh-connected load and *vice versa*.

Power in a Star-connected Circuit.—The power given out by the simple alternating current dynamo when star-connected is the sum of the powers given out by the three coils. Assuming that both resistance and reactance are present so as to make the current lag by an angle ϕ behind the coil voltage, it is seen that the power output of each coil is $E_P I_P \cos\phi$, where E_P is the phase voltage and I_P is the phase current, which is also equal to I_L, the line current. But $E_P = \frac{1}{\sqrt{3}} E_L$ where E_L is the line voltage, and therefore

$$P = 3 \times \frac{1}{\sqrt{3}} E_L I_L \cos\phi$$
$$= \sqrt{3} E_L I_L \cos\phi.$$

It is to be remembered that the power factor is the cosine of the angle of phase difference between the coil voltage and the current, not the line voltage and the current.

Power in a Mesh-connected System.—Here again the total power is the sum of the powers in the three separate phases and is therefore equal to

$$3E_P I_P \cos\phi.$$

But the line current is equal to $\sqrt{3} I_P$ and consequently the total power can be rewritten

$$3E_L \frac{I_L}{\sqrt{3}} \cos\phi$$
$$= \sqrt{3} E_L I_L \cos\phi,$$

the same as before.

Thus the power in a three phase system is the same for both star and mesh if the line voltage, line current, and power factor are the same. In the one case the line voltage is $\sqrt{3}$ times the phase voltage, whilst in the other the line current is $\sqrt{3}$ times the phase current.

Measurement of Power in a Three Phase System.—The first obvious method of measuring the power in a three phase system is to use three wattmeters so that each measures the power developed or absorbed by one phase. The connections for doing this in the case of a star system are shown in Fig. 95, and at first sight it appears as if it is necessary to bring out a fourth wire from the star point. But the three ends of the volt coils of the wattmeters form a star point in themselves, and, considering them as a very small load, it is seen that no current flows through the fourth wire, so that it may be omitted. In other words, the potential of the star point formed by the wattmeter volt coils is the same as that at the generator or load ends.

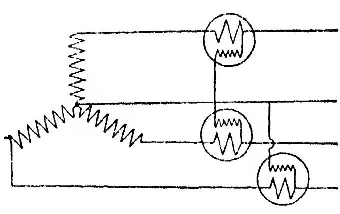


FIG. 95.—Power Measurement in Star System.

If the system were balanced one wattmeter would be sufficient if the star point were available, the reading being multiplied by 3. If a lead is brought out from the generator star point the connections would be those shown in Fig. 96 (*a*). But forming an auxiliary star point by means of three high resistances the connections shown in Fig. 96 (*b*) could be adopted. This obviates the necessity of

bringing a fourth wire out from the generator. But since the resistance of the volt coil will be comparable with each of these fine wire resistances, the value of the resistance in parallel with the volt coil must be chosen higher than the other two, so that when placed in parallel with the volt coil the joint resistance is equal to each of the other two. A further simplification can be made as shown in Fig. 96 (*c*). Here the volt coil is itself used as one of the three resistances, the values of the other two each being made equal to it.

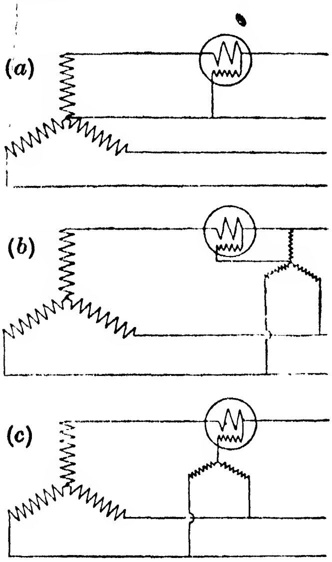


Fig. 96.—Wattmeter Connections for Balanced Three Phase System.

Wattmeters are sometimes provided with these two extra resistances so that they can measure directly the power in a balanced three phase circuit.

In the case of a mesh-connected system three wattmeters could be used as in the previous case, but this would necessitate opening the three branches of the mesh for the purpose of introducing the three current coils as shown in Fig. 97, and this would be very inconvenient, apart from the fact that it could only be done at either the generator or the load end of the line. But since the power in a mesh system is the same as that in a star system, there is no need to resort to this arrangement, and the other more convenient methods can be adopted.

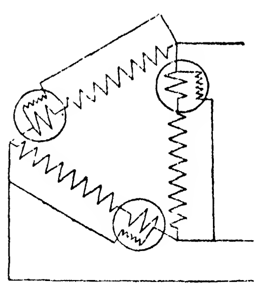


Fig. 97.—Power Measurement in a Mesh System.

Two Wattmeter Method of Measuring Power.—By far the most commonly used method of measuring the power in a three phase system is that known as the *two wattmeter method.* Each wattmeter has its current coil in a different lead, whilst the two volt coils are connected at one end to their respective current coils, the other ends being connected to the third lead, which has no current coil in it, as shown in Fig. 98. It will now be shown that the sum of the two wattmeter readings gives the total power, and that this measurement is independent of balance and wave form if the wattmeters are themselves accurate.

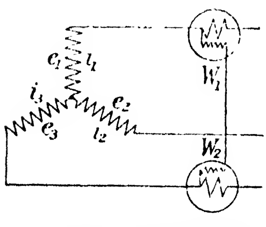


Fig. 98.—Two Wattmeter Method of Measuring Power.

Let the instantaneous volts measured from the star point to

the three line wires be e_1, e_2 and e_3 respectively and let the currents flowing outwards in the three arms be i_1, i_2 and i_3 respectively. Then wattmeter W_1 measures $(e_1 - e_2)\, i_1$ and wattmeter W_2 measures $(e_3 - e_2)\, i_3$. The sum of the two readings is therefore

$$(e_1 - e_2)\, i_1 + (e_3 - e_2)\, i_3$$
$$= e_1 i_1 - e_2 (i_1 + i_3) + e_3 i_3.$$

But $$i_2 = -(i_1 + i_3).$$

Therefore the sum of the two readings is

$$e_1 i_1 + e_2 i_2 + e_3 i_3,$$

which is the total power at that instant, and the wattmeters will indicate the average value of this quantity, in which no assumption as to balance or wave form is made

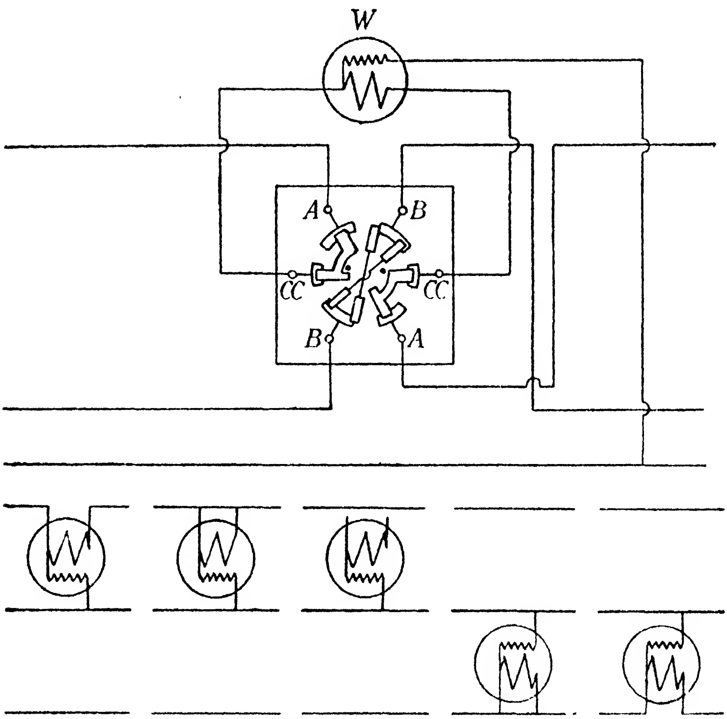


FIG. 99.—Wattmeter Change-over Switch.

The same reasoning holds good for a mesh connection, for it does not matter, as far as the wattmeters are concerned, whether the power is generated in a mesh or star system.

By the use of a specially designed change-over switch one wattmeter can be used for making the necessary measurements. The function of this switch is to change over the current coil from

one lead to the other without opening the circuit even momentarily. One end of the volt coil is attached to the current coil and is changed over with it, whilst the other end of the volt coil remains untouched. Fig. 99 shows a diagrammatical sketch of the switch and its connections, together with the five successive stages in the operation, all of which are performed in the one movement of the switch.

Vector Diagram for Two Wattmeter Method.—It is interesting to observe what each wattmeter is really doing in this measurement, and this can be seen by a reference to the vector diagram in Fig. 100.

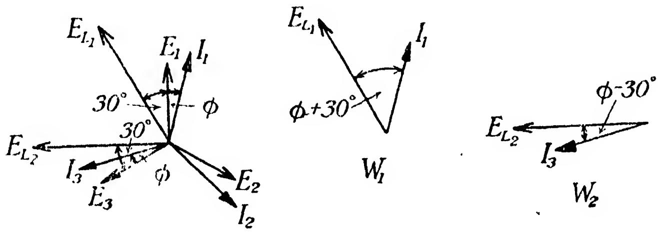


FIG. 100.—Vector Diagram for Two Wattmeter Method.

Assuming again a balanced circuit and sinusoidal wave forms, it is seen that wattmeter W_1 measures the product of I_1, E_{L_1} and the cosine of the angle of phase difference between them. If E_{L_1} had been drawn in exactly the opposite direction, it would have been more than 90° out of phase with I_1 and would correspond to a negative power and a backward reading of the wattmeter. Similarly wattmeter W_2 measures the product of I_3, E_{L_2} and the cosine of the angle of phase difference between them. The quantities which the wattmeters record are shown separately in Fig. 100, where it is seen that

$$W_1 = EI \cos(\phi + 30°)$$

and

$$W_2 = EI \cos(\phi - 30°).$$

Therefore

$$\begin{aligned} W_1 + W_2 &= EI\{\cos(\phi + 30°) + \cos(\phi - 30°)\} \\ &= EI \times 2\cos\phi\cos 30° \\ &= \sqrt{3}EI\cos\phi, \end{aligned}$$

which is again equal to the total power.

When ϕ attains a value of over 60°, $\cos(\phi + 30°)$ becomes negative and wattmeter W_1 commences to read backwards. In order to make the measurement, therefore, either the current coil or the volt coil must be reversed, and the forward reading thus obtained must be *subtracted* from that of the other wattmeter in order to obtain the total power. In the same way, when the

angle of lead becomes greater than 60° the wattmeter W_2 commences to read backwards and the same procedure must be adopted.

Measurement of Power Factor from Wattmeter Readings.—On a sine wave hypothesis the measurement of the power factor devolves into a determination of $\cos\phi$, and this can be obtained from the two wattmeter readings mentioned above, for

$$\begin{aligned} W_1 - W_2 &= EI\{\cos(\phi + 30^\circ) - \cos(\phi - 30^\circ)\} \\ &= -EI \times 2 \sin\phi \sin 30^\circ \\ &= -EI \sin\phi. \end{aligned}$$

Therefore $$\frac{W_1 - W_2}{W_1 + W_2} = \frac{-EI \sin\phi}{\sqrt{3}\,EI\cos\phi}$$

$$= \frac{-\sin\phi}{\sqrt{3}\cos\phi},$$

$$\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = -\tan\phi,$$

$$3\left(\frac{W_1 - W_2}{W_1 + W_2}\right)^2 = \tan^2\phi,$$

$$1 + 3\left(\frac{W_1 - W_2}{W_1 + W_2}\right)^2 = 1 + \tan^2\phi = \sec^2\phi,$$

$$\frac{1}{1 + 3\left(\frac{W_1 - W_2}{W_1 + W_2}\right)^2} = \frac{1}{\sec^2\phi} = \cos^2\phi,$$

and $$\boxed{\cos\phi = \frac{1}{\sqrt{1 + 3\left(\frac{W_1 - W_2}{W_1 + W_2}\right)^2}}.}$$

Letting r equal $\frac{W_1}{W_2}$ and dividing top and bottom of the part in the round brackets by W_2 we get

$$\boxed{\cos\phi = \frac{1}{\sqrt{1 + 3\left(\frac{r-1}{r+1}\right)^2}}}.$$

It is convenient to divide by the larger reading so that $\frac{W_1}{W_2}$ is always less than unity.

Another way of stating this relationship is obtained by multiplying top and bottom of the above equation by $(r + 1)$. Then

$$\cos\phi = \frac{r+1}{\sqrt{\left\{1+3\left(\dfrac{r-1}{r+1}\right)^2\right\} \times (r+1)^2}}$$

$$= \frac{r+1}{\sqrt{(r+1)^2+3(r-1)^2}}$$

$$= \frac{r+1}{2\sqrt{r^2-r+1}}.$$

When the angle of lag is 60° the wattmeter W_1 is measuring

$$EI\cos(60° + 30°) = 0,$$

and thus, when the power factor falls to $0{\cdot}5 = \cos 60°$, the indication of the first wattmeter is zero. This result is also obtained by putting r equal to zero in the last equation. If the power factor is less than 0·5 it follows, from the same equation, that r must have a negative value, indicating that one wattmeter is reading backwards.

Again, when the angle of lead is 60° the wattmeter W_2 measures

$$EI\cos(-60° - 30°) = 0,$$

and for power factors of less than 0·5 with the current leading the second wattmeter gives a negative indication.

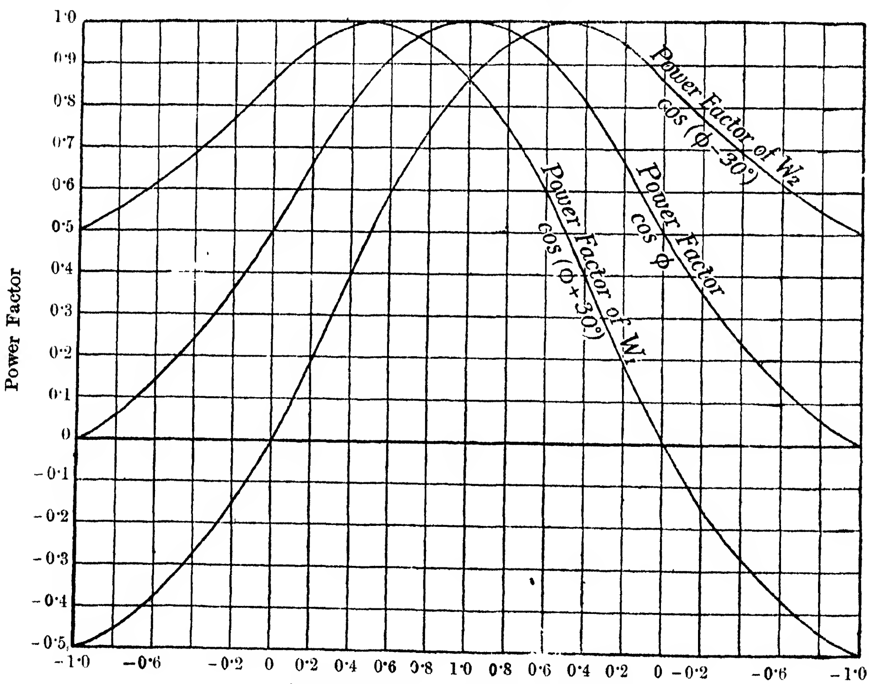


FIG. 101.—Power Factor from Wattmeter Readings.

Fig. 101 shows in curve form the relation between the power factor and the ratio of the two wattmeter readings, this ratio always having a fractional value. The power factors under which the two wattmeters themselves are working are also shown, it being seen that each of them reverses in sign over a portion of the range.

Three Phase Load.—The conditions necessary for a three phase load to be balanced are that the resistances of the three arms must be all equal, the inductive reactances must be all equal and the capacity reactances must be all equal. It is not sufficient that the impedances should be equal, as this result might be attained with different proportions of resistance and reactance.

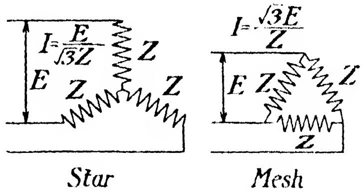


FIG. 102.—Impedances in Star and Mesh.

The same impedances arranged in mesh produce a larger current for a given E.M.F. than when arranged in star, for consider the general case where the impedance of each branch is Z and the line volts are E (see Fig. 102).

When arranged in star, the voltage across each branch is $\frac{E}{\sqrt{3}}$ and the current is $\frac{E}{\sqrt{3}Z}$, but when they are arranged in mesh the current per branch is $\frac{E}{Z}$, so that the line current is $\sqrt{3}\frac{E}{Z}$, or three times the previous value. Thus the equivalent impedance of the system is reduced to one-third of its original value by changing over from star to mesh.

Unbalanced Three Phase Circuit.—An unbalanced system is produced when the impedances in the different branches are unequal. The three currents may be of different magnitude and may also lag behind their respective E.M.F.'s by different amounts, but the vector sum of all the currents flowing outwards must add up to zero if there are only three wires. In order that this condition may be brought about, the voltage is usually distributed in an unsymmetrical manner over the three branches, the general effect being to reduce the voltage on the heavily loaded side. One result of this is that the potential of the unsymmetrical star point is different from what it would be if the system were balanced.

The average power factor is a term which has rather a dubious meaning, but it may be defined as the

$$\frac{\text{Total power}}{\text{Sum of the volt-amperes for all phases}}.$$

This will not necessarily be the same as the average of the three individual power factors.

Effect of Third Harmonic.—All the harmonics which are divisible by 3 are absent in a balanced three phase system, providing no fourth conductor is used. The voltage between any two lines is the vector difference of two phase voltages acting away from the star point, and these voltages differ in phase by a third of a period. But if a third harmonic is present in the E.M.F. wave, there will be a phase difference of one complete period between these harmonic voltages in the two phases, and hence they will always be equal and opposite. This can be graphically demonstrated by subtracting two equal voltages containing third harmonics, as in Fig. 103, the phase difference being 120°. The resulting curve shows no third harmonic.

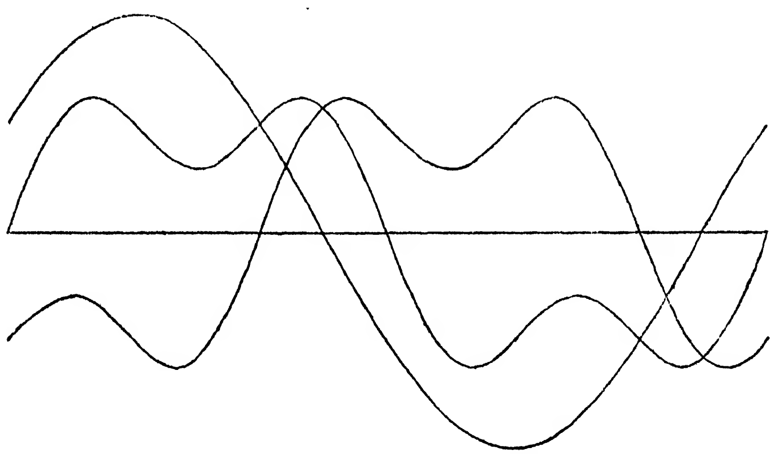


FIG. 103.—Disappearance of Third Harmonic in Three Phase System.

It can also be seen by adding together the voltages as follows :—

$$
\begin{aligned}
e_1 &= E_1 \sin\theta + E_3 \sin(3\theta + a)\\
e_2 &= E_1 \sin(\theta - 120°) + E_3 \sin\{3(\theta - 120°) + a\}\\
&= E_1 \sin(\theta - 120°) + E_3 \sin(3\theta + a - 360°)\\
&= E_1 \sin(\theta - 120°) + E_3 \sin(3\theta + a)\\
e_1 - e_2 &= E_1\{\sin\theta - \sin(\theta - 120°)\} + E_3\{\sin(3\theta + a) - \sin(3\theta + a)\}\\
&= E_1\{\sin\theta - \sin(\theta - 120°)\}\\
&= 2E_1 \cos(\theta - 60°) \sin 60°\\
&= \sqrt{3}E_1 \cos(\theta - 60°)\\
&= \sqrt{3}E_1 \sin(\theta + 30°).
\end{aligned}
$$

The third harmonic has obviously disappeared.

In a similar manner, the ninth, fifteenth, etc., harmonics disappear, so that the only ones possible in a three phase system are the fifth, seventh, eleventh, thirteenth, etc.

This is not necessarily the case when a fourth wire is employed

to join the star points, for although no third harmonic can exist in the voltage between lines, yet it can exist between any line wire and the fourth conductor, and its presence may produce a harmonic current flowing round the circuit consisting of the line wire and the fourth conductor together with the apparatus joining them.

Six Phase Currents.—Six phase currents can be obtained in the same way as two and three phase currents by having six coils spaced 60° apart on the simple alternating current dynamo. But coil No. 4 is exactly the same as coil No. 1, with the exception that it is reversed, and thus six separate phases can be obtained from a three phase source provided they are kept insulated. By means of transformers (see Chapters XIII–XV) it is possible to produce six connected phases from a three phase supply, and in most cases in practice where six phase currents are employed they are obtained from a three phase source. They are used in connection with certain types of apparatus (*e.g.* rotary converters, see Chapter XXIII) the operation of which is improved by the use of a large number of phases.

Nine and twelve phase currents are also occasionally met with in practice, but here again their application is limited to certain particular types of apparatus.

CHAPTER XI

ALTERNATING CURRENT INSTRUMENTS

Moving Iron Ammeters and Voltmeters.—This large class of ammeters is divided into two main groups, (1) where there is only one iron which is attracted into a solenoid carrying the current to be measured, and (2) where there are two irons, one fixed and the other movable, both of them being magnetised by the current to be measured.

The general action can be understood by considering a simple case of the second type. The two irons lie parallel to the axis of the coil, and, assuming their permeabilities to be constant, the pole strengths will be proportional to the current. The force of repulsion will therefore be proportional to the instantaneous square of the current and will always be in the same direction. The square root of the deflecting torque will be proportional to the R.M.S. value, so that these instruments will work all right on A.C. circuits. Unfortunately, the assumption of the constancy of the permeability cannot by any means be justified in practice, and this introduces modifications. When the iron gets highly saturated the deflecting torque falls away from the square law and becomes more nearly proportional to the first power of the current. The result of this is that the indications of the instruments are considerably affected by wave form and they should be calibrated on a wave form similar to that on which they are to be used. A badly designed instrument may show a difference of 10 per cent. in extreme cases, whereas a well designed instrument should not show a greater difference than 1 per cent.

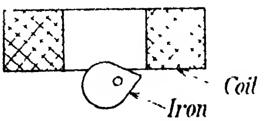


FIG. 104.—Single Iron Type of Moving Iron Instrument.

An example of the single iron type is shown in diagrammatic form in Fig. 104. It consists, in brief, of an iron disc pivoted eccentrically and fixed at one end of the magnetising coil. When a current flows the iron is attracted into the coil, causing a rotation of the spindle.

The second class is indicated in diagrammatic form in Fig. 105. A fixed rod of soft iron repels another rod attached to the moving system. When the needle is deflected the distance between the

irons is increased and the square law no longer holds. This has the effect of improving the scale, the contracted nature of which in the lower parts is one of the disadvantages of the moving iron instrument.

The irons are often shaped in a particular fashion so as to improve

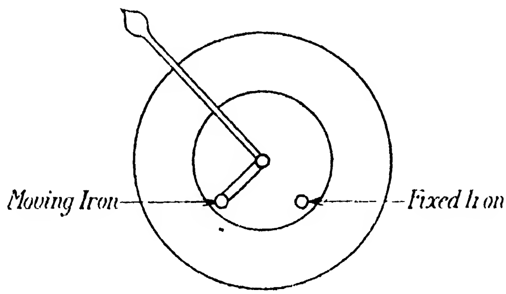


FIG. 105. Double Iron Type of Moving Iron Instrument.

the shape of the scale, and in a good modern instrument this should be moderately uniform from about one-fifth to full scale deflection.

These instruments operate equally well either as ammeters or voltmeters. It is only a case of varying the number of turns and the size of the wire in the magnetising coil, the ampere-turns being kept the same. They can also be used for both continuous and alternating currents. The contraction of the lower part of the scale is not so important in the case of voltmeters, since only the voltage in the region of the normal is of great importance in the majority of cases, and the instrument can be designed so that this comes in the open part of the scale.

In the majority of moving iron instruments a gravity control is adopted, the moving system being balanced on the zero by means of two counterpoise weights. These weights consist of small nuts which travel on two screwed rods at right angles, as shown in Fig. 106. When the pointer is on the zero the nut B lies vertically underneath the centre and its movement does not affect the position of the pointer. The zero adjustment, therefore, is obtained by means of A. In order to vary the magnitude of the reading, an adjustment of B is made, since this will have an effect when the pointer is deflected. A must not be touched now, as this would alter the zero. If B is moved nearer to the centre, the deflection for a given current will be increased. The control is not uniform, since the centre of gravity is raised by

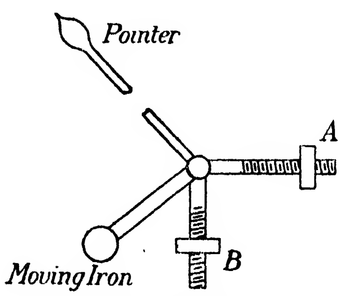


FIG. 106.—Gravity Control.

different amounts for equal deflections at different parts of the scale. The controlling torque is, in fact, proportional to the sine of the angle of deflection.

Moving iron ammeters and voltmeters suffer from the disadvantage of a comparatively large power consumption, and unless shielded they are affected by stray fields. The temperature error in low range voltmeters is also considerable.

Air damping is usually provided either by a piston working in a cylinder or simply by means of a vane attached to the moving system.

Dynamometer Ammeters and Voltmeters.—In these instruments there are two coils connected in series carrying the whole or a proportional part of the current. These two coils are set with their axes inclined to one another, one being fixed and the other movable. The instantaneous torque is proportional to the square of the instantaneous current and the average torque is therefore proportional to the mean square of the current. Owing to the variation of the distance between the two coils, however, the scale does not

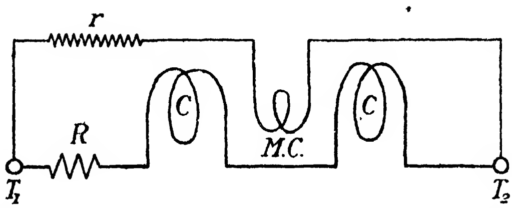


FIG. 107.—Connections for Dynamometer Type Ammeter.

obey a strict square law, but is usually a little contracted at the top end as well as being very contracted at the beginning.

In the case of ammeters, the whole current cannot be led into and out of the moving system by means of the controlling springs, and to get over this difficulty the connections shown in Fig. 107 are sometimes used. The two fixed coils, C, C, are placed in series with one another and a resistance, R, across the terminals, T_1, T_2. The moving coil, $M.C.$, is connected in series with the resistance, r, also across the terminals, T_1, T_2. The ratio of resistance to reactance in each branch circuit must be the same, so that the phase relationships are not disturbed.

They are affected by stray magnetic fields, but to counteract this they can be effectively screened and, if necessary, they can be wound astatically.

Most instruments of this type are provided with a spring control and an air damping device.

The spring control produces a restoring torque proportional to the angle of deflection, and is similar to that used on moving coil instruments, whilst the various air damping devices adopted are similar to those used on moving iron instruments.

The Kelvin Balance and the Siemens Dynamometer are examples of instruments of this type which are suitable for use on continuous as well as alternating currents.

High-class dynamometer instruments are only affected to a very small degree by changes of wave form and frequency and sometimes not at all.

Induction Ammeters and Voltmeters.—The main class of induction instruments are designed upon the *shielded pole* principle and were originally due to Ferraris.

A specially shaped aluminium disc, D (see Fig. 108), is arranged to rotate between the poles of an electromagnet, M, energised by the current to be measured. Two copper discs, C, C, partially shield the poles, so that part of the flux goes straight across the aluminium disc, and part goes through the copper discs on the way. Due to the fact that the flux is alternating, eddy currents are set up in both copper and aluminium discs, and since these currents are flowing in the same direction at any given instant of time, they will attract each other. A clockwise rotation of the movable disc is therefore set up, this being opposed by a spiral spring. By suitably shaping the aluminium disc, a deflection of 300° can be obtained, the long scale being one of the distinctive features of the instrument. The deflecting torque at any instant is proportional to the square of the instantaneous current, and consequently the instruments will indicate R.M.S. values, but owing to the action of the eddy currents they are very sensitive to changes of frequency unless this is compensated for. The indications are practically independent of wave form, but temperature errors are considerable unless these are also compensated for.

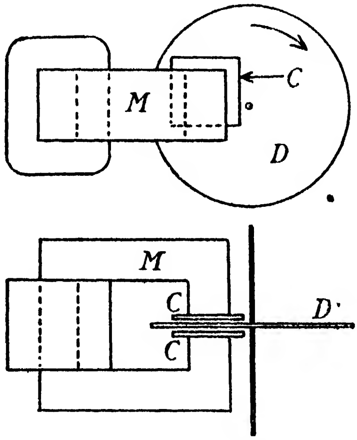


FIG. 108.—Principle of Induction Instrument.

Another type of induction ammeter or voltmeter is the *split phase* instrument. This splitting is done by making one phase non-inductive and the other highly inductive, the instrument acting after the manner of the induction wattmeter.

Iron Cored Ammeters and Voltmeters.—This class of instrument, in which shunt electromagnets are used, has been developed by Dr. Sumpner. The voltmeter consists of an electromagnet, M, with a laminated iron core shaped as shown in Fig. 109. This is excited by means of the E.M.F. to be measured, whilst the moving coil, $M.C.$, is connected in series with a condenser, C, across the same two points as the shunt coil. The instantaneous value of the deflecting

torque is proportional to the product of the instantaneous values of the flux and the current in the moving coil. On the assumption that the resistance drops in the two coils are negligible, the flux lags behind the impressed E.M.F. by 90°, and the current in the moving coil leads the E.M.F. by 90° on account of the condenser. These two will therefore have a phase difference of 180°, or by reversing one of them they can be considered as being in phase. Moreover, both the current in the moving coil and the flux are proportional to the p.d., so that their product is proportional to the instantaneous square of the voltage, and the instrument can be calibrated to read the R.M.S. value without being affected by wave form or frequency.

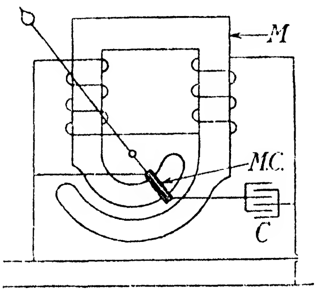


FIG. 109.—Iron Cored Voltmeter.

Hot Wire Ammeters and Voltmeters.—These instruments depend upon the elongation of a wire the temperature of which is raised, due to the passage of the current to be measured. The wire rises in temperature until the heat radiated per second is equal to the heat produced per second, the resulting sag of the hot wire which is stretched between two fixed points being used to actuate the pointer. Since the rate of production of heat is proportional to the square of the current, the instrument tends to obey, with slight modifications, the square law common to all alternating ammeters and voltmeters, and consequently indicates R.M.S. values. In fact the A.C. ampere is defined as being that alternating current which produces the same heating effect as one ampere of continuous current. Obviously, wave form, frequency and stray magnetic fields have absolutely no effect on the indications, but the instruments suffer from other inherent disadvantages which render them not so accurate as some of the other types described.

The construction of ammeters and voltmeters is practically the same, the voltmeters having a large non-inductive resistance placed in series with the hot wire, whilst the ammeters are shunted for currents more than a few amperes.

The general arrangement is shown in Fig. 110, the hot wire being made of platinum-iridium or platinum-silver, and in some cases even of eureka. This is maintained in a state of tension between the two supports, T_1, T_2, one of which is made adjustable in order to give a zero adjustment which is performed by means of the screw, Z. Attached to the point A is a second wire of phosphor-bronze held rigidly at the other end, B. This second wire is kept in tension by means of a fine silk fibre one end of which is attached to the phosphor-bronze wire at C, whilst the other end is carried round a small bone pulley, P, and is attached to the end of a flat

steel spring, S, which has the effect of keeping the whole system in tension. When the hot wire expands a certain amount of sag is produced at A, which causes a sag of greater amount in the phosphor-bronze wire. This action is repeated by the silk fibre, which, by its movement, causes the pulley to rotate, producing a deflection of the pointer, to which it is rigidly attached. Since the amount of sag which is produced is very large compared with the actual increase in length of the hot wire, a double magnification is obtained, the displacement of the silk fibre being some 500 times the actual elongation of the hot wire.

Attached to the moving system is an aluminium vane moving between the jaws of a permanent magnet. When a deflection takes place eddy currents are produced in the vane and a damping action ensues.

In order to prevent a reading being obtained due to variations in the atmospheric temperature, the base plate is made of two

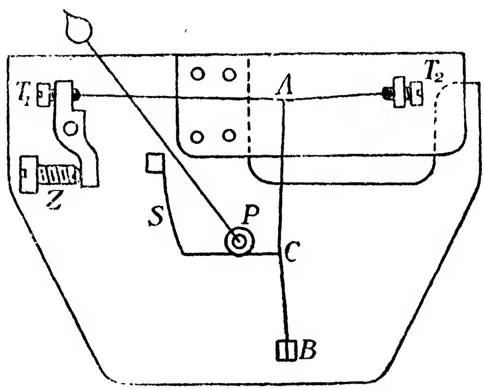


FIG. 110.—Hot Wire Instrument.

materials so proportioned that its coefficient of expansion is equal to that of the wire itself. This can be done by making the main part of copper with an iron extension as shown, or the base might be made of iron and the auxiliary plate of nickel steel. When the instrument is switched off after it has been in circuit for some time, the wire cools down almost instantly, whilst the base plate has a considerable time lag. For this reason, the pointer seldom returns accurately to zero and the instrument has a tendency to read low. Those instruments which have a platinum-silver hot wire are more uncertain with regard to the zero than those having a hot wire of platinum-iridium. The temperature rise involved with the latter is so high that no serious errors occur due to the room temperature. Platinum-iridium has another advantage over platinum-silver in its greater tensile strength. This enables a thinner wire to be used, rendering the instrument less sluggish in its action.

A great disadvantage possessed by hot wire instruments is their

inability to withstand even comparatively small overloads. This is due to the wires being worked at as high a temperature as possible in the first case, in addition to the fact that the production of heat is proportional to the square of the current. Fuses are sometimes employed to protect the instruments, but they are not very satisfactory, as very often the hot wire fuses first, due to the rapidity with which its temperature rises.

Thermo-junction Ammeters and Voltmeters.—Another type of thermal instrument is the moving coil millivoltmeter supplied from a thermo-junction heated by the alternating current to be measured. In the Paul instrument of this type, the thermo-junction and heater are contained in a glass bulb about 25 mm. diameter exhausted to a high degree of vacuum. Both the heater and the thermo-junction, which consists of an iron-eureka couple, are supported on platinum leading in wires and are lightly soldered together. Only a proportional part of the current is used to heat the thermo-junction, J (see Fig. 111), the remainder being carried by the shunt, S. A small resistance, R, is used for purposes of adjustment. These "thermal converters," as they are called, can be used in conjunction with shunts on ammeters as well as on voltmeters.

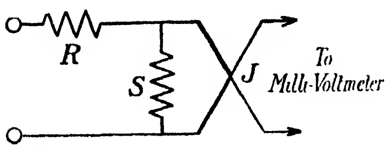


FIG. 111.—Connections of Thermal Converter.

In the Duddell thermo-ammeter the heater consists of a sheet of platinised mica, the platinum being scraped away in places so as to form a kind of grid. In this way high resistances are readily obtained in a very small space. The thermo-junction lies just above the heater, and is part of the moving system of the instrument, so that no current passes through the controlling spring.

These instruments are particularly suitable for high frequency work, since they are absolutely independent of frequency. They are also suitable for the measurement of small alternating currents.

Electrostatic Voltmeters.—These instruments can only be used as voltmeters, since they act like condensers and only take a small capacity current. They can be divided into two main types: (1) where a pair of plates or quadrants are charged to different potentials, whilst a movable vane to which a pointer is attached is connected to one or other of the fixed vanes or is charged to some intermediate potential; and (2) where the moving vanes are connected to one terminal and the fixed vanes are connected to the other, the moving system being attracted bodily to the fixed system, causing a rotation round its axis.

One of the best known electrostatic instruments is the Kelvin multicellular voltmeter, so named because a number of cells act together on a common spindle. The working parts are represented diagrammatically in Fig. 112, where a number of movable vanes,

M.V., are threaded on to a spindle carrying the pointer, *P*, and are interleaved between a number of fixed vanes, *F.V.*

The moving system is suspended by means of a fine wire, *S*, from the underneath side of a coach spring, *C*, to prevent injury due to accidental vibration. The zero is adjusted by means of a torsion head, *T.H.*, and tangent screw, *T.S.*, attached to the suspension. The underneath side of the moving system ends in a vertical perforated paddle, *P*, moving in a glass vessel containing oil, this serving as the damping device. To avoid injury during transit, the moving system is clamped against a collar.

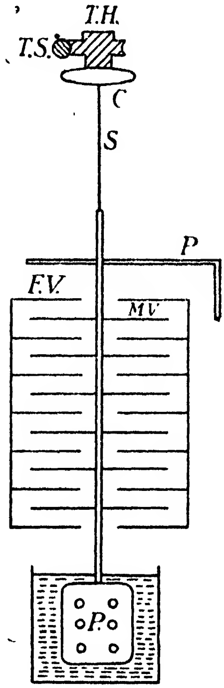


FIG. 112.—Multicellular Electrostatic Voltmeter.

These instruments are only used on low tension circuits; for high tension, a different pattern is used. In this case the vanes are vertical and the moving system is supported on knife edges.

The Ayrton-Mather voltmeter consists of a number of vanes which are portions of concentric cylinders of different radii (see Fig. 113). The movable vanes are attracted into the space between the fixed ones when a difference of potential is set up, thus causing a deflection. In low voltage instruments a spring control is used, but for the medium voltages and high tension a gravity control is adopted. Eddy current damping is provided by means of a permanent magnet.

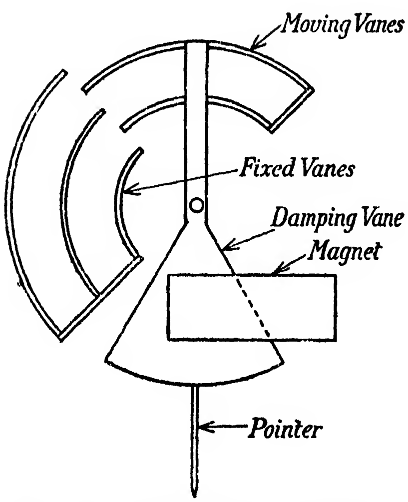


FIG. 113.—Ayrton-Mather Electrostatic Voltmeter.

For very high voltage work, the Kelvin *Volt Balance* may be used. This consists of a metal scale pan balanced over a fixed plate, which is covered with a layer of mica insulation to prevent any accidental short-circuiting (see Fig. 114). The scale pan and pointer are counterpoised so that the whole system is balanced with the pointer at zero. When a p.d. is set up, attraction between the two plates ensues and a deflection is obtained. Such a voltmeter is very bulky, the outer case of a 100,000 volt instrument standing some 5 feet high with a diameter of about 3 feet.

Electrostatic voltmeters have been constructed so as to read up to 200,000 volts, but there is great difficulty in accurately calibrating such instruments.

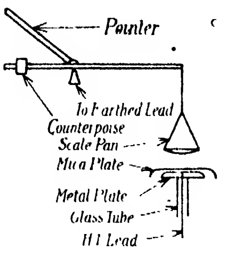


FIG. 114.—Kelvin Volt Balance.

Where oil damping is used, these instruments are sluggish in their action, due to the moving mass being necessarily large, whilst the working force is small.

The scales are usually short and often somewhat unevenly divided, the range being extended in some cases by means of subdivided resistances and in others by means of subdivided condensers.

Wattmeters.—Wattmeters consist of two essential elements, viz., a pressure and a current coil, these two parts being connected in the circuit as a voltmeter and an ammeter respectively. The simplest type of wattmeter to understand is the dynamometer type, where the current passes through the fixed current coils and the voltage is applied to the moving volt coil, producing a current proportional to the voltage.

The torque which is produced at any moment is proportional to the product of the instantaneous values of voltage and current, and, assuming that the current lags behind the voltage by an angle ϕ, the average value of this product is $EI \cos \phi$. In the ideal wattmeter, the volt coil must have negligible inductance and capacity compared with its resistance, so that the current flowing through it shall be in phase with the voltage across it. In actual instruments, however, a certain amount of inductance is bound to be present, although it can be reduced to very small amounts, and the result is that the current lags in the shunt circuit by some small angle α where $\tan \alpha = \frac{x}{r}$, r being the resistance and x the reactance of the shunt coil. The presence of the reactance x causes two effects, (1) the shunt current is reduced in the ratio $\frac{r}{\sqrt{r^2 + x^2}}$, and (2) the shunt current is made to lag by an angle α. The reduction of the shunt current is constant in magnitude and can be allowed for in the design of the instrument. The alteration in the phase of the shunt current, however, causes different variations at different power factors and needs to be examined further.

Instead of measuring at any instant

$$E_m I_m \cos \theta \cos (\theta - \phi)$$

the wattmeter will now measure

$$E_m I_m \cos (\theta - \alpha) \cos (\theta - \phi) \times \frac{r}{\sqrt{r^2 + x^2}}$$

$$= E_m I_m (\cos\theta\cos\alpha + \sin\theta\sin\alpha)(\cos\theta\cos\phi + \sin\theta\sin\phi)\cos\alpha$$

$$= E_m I_m (\cos^2\theta\cos\alpha\cos\phi + \tfrac{1}{2}\sin 2\theta\cos\alpha\sin\phi$$
$$+ \tfrac{1}{2}\sin 2\theta\sin\alpha\cos\phi + \sin^2\theta\sin\alpha\sin\phi)\cos\alpha.$$

Putting the average values of $\cos^2\theta$, $\sin^2\theta$ and $\sin 2\theta$ as $\frac{1}{2}$, $\frac{1}{2}$ and 0 respectively, the wattmeter indication will be proportional to

$$E_m I_m (\tfrac{1}{2}\cos\phi\cos\alpha + \tfrac{1}{2}\sin\phi\sin\alpha)\cos\alpha$$
$$= EI\cos(\phi - \alpha)\cos\alpha.$$

The effect is the equivalent of advancing the current in phase by an angle α.

Let the true power, P, be equal to $k \times W$, where W is the wattmeter reading and k the correction factor.

Then $$P = k \times W,$$
$$EI\cos\phi = kEI\cos(\phi - \alpha)\cos\alpha$$

and $$k = \frac{\cos\phi}{\cos(\phi - \alpha)\cos\alpha}.$$

$$= \frac{\cos\phi}{(\cos\phi\cos\alpha + \sin\phi\sin\alpha)\cos\alpha}$$

$$= \frac{1}{(\cos\alpha + \tan\phi\sin\alpha)\cos\alpha}$$

$$= \frac{1}{(1 + \tan\phi\tan\alpha)\cos^2\alpha}$$

$$= \frac{1 + \tan^2\alpha}{1 + \tan\phi\tan\alpha}.$$

The value of this correction factor is plotted in Fig. 115 for various values of α, the angle of lag in the shunt circuit. A simple

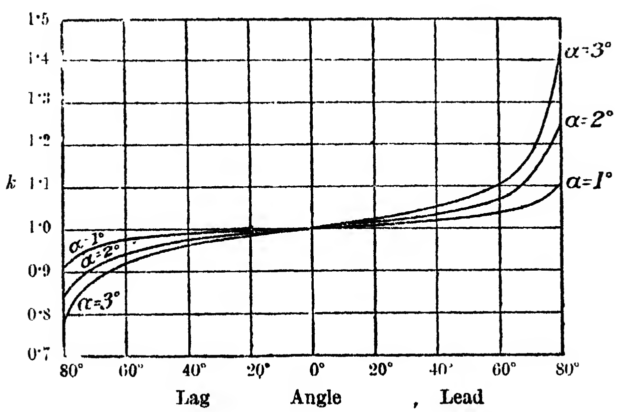


FIG. 115.—Wattmeter Correction Factors.

approximation to the correction factor is obtained by neglecting $\tan^2 \alpha$, since α is small. Then

$$k = \frac{1}{1 + \tan\phi \tan\alpha}$$
$$= 1 - \tan\phi \tan\alpha.$$

This constant k is, however, dependent upon the frequency and wave form to a certain extent, since these affect the value of α.

As the power factor decreases the error increases at a greater and greater rate, so that an instrument which is only subject to a 0·5 per cent. error on a power factor of 0·8 would be liable to a 7 per cent. error on a power factor of 0·1. This source of error is very prominent when low power factors are dealt with, and has led to the development of a number of wattmeters having no iron in their construction, the object being to reduce the inductance of the pressure circuit. High voltage wattmeters are easier to deal with in this respect, since a large non-inductive resistance can be placed in series with the moving coil. These series resistances are wound back on themselves, as in Fig. 116, so as to make them

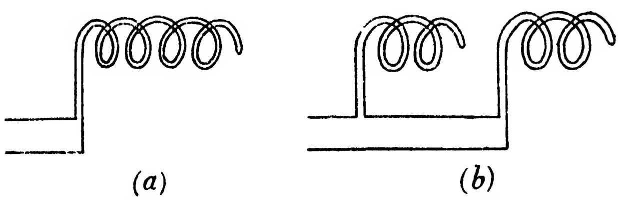


FIG 116.—Non-inductive Winding in Sections.

non-inductive; but this introduces a capacity effect, since there are conductors at different potentials lying close to one another and separated by a dielectric.

To reduce this capacity effect and still have a non-inductive winding the coil is divided into sections as in Fig. 116 (*b*). This reduces the average p.d. between adjacent conductors, the total capacity being inversely proportional to the number of sections.

This capacity, if not too great, tends to neutralise the effect of inductance, and therefore is beneficial up to a certain amount. Eddy currents also tend to produce errors, and to minimise these the case and constructional details are made of some insulating material rather than metal, whilst the current coil is carefully stranded when large currents are dealt with.

An inherent error in wattmeter measurements lies in the fact that the instrument always includes in its reading the power absorbed by either the current coil or the volt coil. In Fig. 117 (*a*) the power lost due to the voltage drop across the current coil is included in the power measured, whilst in Fig. 117 (*b*) the power lost due to the current in the volt coil is included. On a

constant voltage circuit the latter will provide a constant error, whilst in the former case the error will be different for each value of the current. It does not always follow, however, that Fig. 117 (*b*) is the better method of connection, for when very small currents are being measured the watts lost in the current coil will be smaller than those lost in the volt coil, and in such cases the connections shown in Fig. 117 (*a*) are preferable.

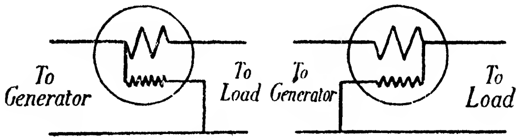


FIG. 117.—Wattmeter Connections.

In some wattmeters a fine wire compensating coil is placed inside the main current coil and in opposition to it, so that a negative torque is produced reducing the deflection by the amount of power lost in the volt coil. The turns on this compensating winding are made equal to those on the main current coil. This unfortunately increases the inductance of the pressure circuit, which is so undesirable.

Dynamometer Type Wattmeters.—Dynamometer type wattmeters form one of the commonest classes met with in practice, the moving coil being used as the pressure element and the fixed coil or coils as the current element. The moving coil, *V.C.*, is attached to a spindle, *S*, in the plane of the fixed current coil, *C.C.*, as shown in Fig. 118. When the current is sufficiently large, the fixed coil is wound with copper strip, which is forced apart at the points where the spindle comes through. Attached to the moving system is an air-damping piston, *D*, to make the instrument aperiodic. When the pointer is on zero the two coils are inclined a little as shown, the winding being in such a direction that the coil tends to turn in a clockwise direction. Variations of wave form and frequency have very little effect in good dynamometer wattmeters.

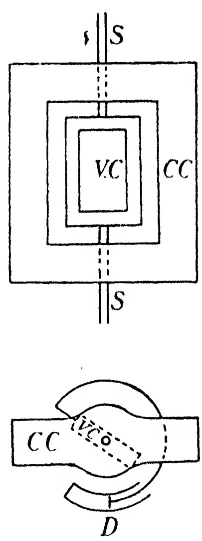


FIG. 118.—Dynamometer Type Wattmeter.

Induction Wattmeters.—Induction wattmeters on the Ferraris principle are often used for switchboard work, one of their advantages being the long scale, which often extends over about 300°. The accuracy is not as a general rule as high as in dynamometer wattmeters, but is sufficient for switchboard purposes.

The wattmeter contains a volt coil, *V.C.*, and a current coil, *C.C.*,

as before, in addition to a copper or aluminium disc, *D* (see Fig. 119), whilst damping is obtained by means of a permanent magnet. The coils are wound on to soft iron cores between the pole pieces of which the disc rotates. The pressure circuit is made very highly inductive, so that the current and flux produced by it lag behind the applied voltage by practically 90°. The eddy currents induced in the disc are further proportional to the rate of change of the flux

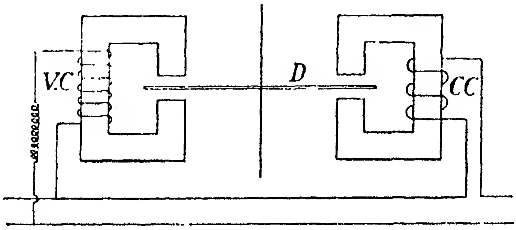


FIG. 119.—Induction Wattmeter.

and are again 90° out of phase. The induced current in the disc is therefore in phase with the line voltage, whilst the flux produced by the current coil is in phase with the main current, since the coil is wound in series with the line. The torque which is developed, therefore, between the disc and the current coil is proportional to their instantaneous product and the deflection is proportional to the true power.

In practice, however, the flux produced by the volt coil does not lag by exactly 90° behind the voltage on account of the resistance, nor is the flux produced by the current coil exactly in phase with the current, since there are power losses produced by hysteresis and eddy currents. The effects of the error produced by this phase displacement have already been discussed (see p. 140).

Various compensating devices are used to get over this difficulty, and usually consist of auxiliary windings arranged so that, when in conjunction with the main coil, a flux of the desired phase is produced.

Frequency has a very marked effect on the accuracy of the instrument, the wattmeter reading increasing as the frequency is reduced. Change of voltage and wave form also affect the accuracy.

Sumpner Wattmeter.—The principle on which this instrument works is the same as that in the case of the iron cored voltmeter, except that the moving coil, instead of having a condenser connected in series with it and being supplied with the line voltage, is connected to a piece of apparatus known as a *quadrature transformer*, *Q.T.*, with a non-inductive resistance, *R*, in series, as shown in Fig. 120. This quadrature transformer (see p. 192) produces a current proportional to the line current and 90° out of phase, so that it serves the same function as the condenser did in the case

of the voltmeter. Since the flux is 90° out of phase with the line voltage and the current in the moving coil is 90° out of phase with the current, the instrument will act as a true wattmeter.

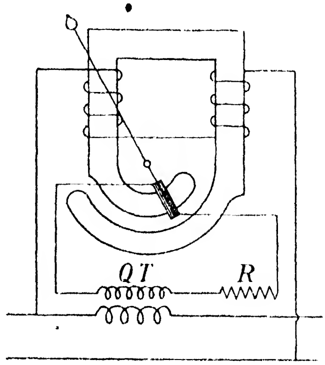


FIG. 120.—Sumpner Wattmeter.

Electrostatic Wattmeters.—These instruments have a moving vane and a system of fixed vanes as in the case of electrostatic voltmeters, but instead of the fixed vanes being connected to points at the same potential, opposite pairs are connected to the two ends of a non-inductive shunt, the potential difference between them being proportional to the current (see Fig. 121). The moving vane is connected to the other line wire, so that the full voltage exists between the moving and fixed vanes, and it can be shown that the deflection is proportional to the true watts.

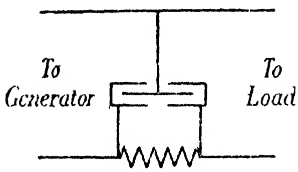


FIG. 121.—Electrostatic Wattmeter Connections.

The sphere of usefulness of these instruments lies, at present, in measurements involving very high pressures and low currents.

Two Phase Wattmeters.—In the case of a two-phase circuit the total power is the sum of the powers in the two phases, and if two wattmeters have their moving systems attached to the same pointer the resulting deflection will depend upon the total power in the circuit. Instruments are made on this principle so as to avoid having to use two separate wattmeters, there being eight terminals in such a case, as shown in Fig. 122. Such wattmeters may be used for balanced or unbalanced circuits. Occasionally in connecting up such instruments the two parts are put in opposition, and the wattmeter will then give a small deflection which is due to the difference of the powers supplied by the two phases.

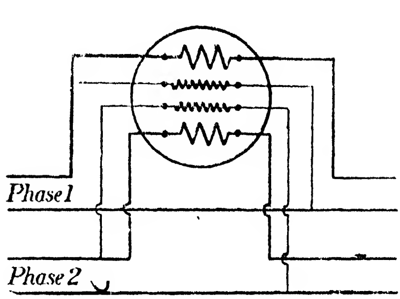


FIG. 122.—Two Phase Wattmeter Connections.

Three Phase Wattmeters.—In the case of three-phase measurements it has been shown that only two wattmeters are necessary, and these can be mounted on the same spindle in the same way as in the two-phase case. Thus the total power can be obtained from a single observation on one instrument which is supplied

with seven terminals and connected as shown in Fig. 123. The number of terminals in some instruments is reduced to five by suitable combinations. When the power factor falls below 0·5 one part of the instrument acts in opposition to the other, the result being that the deflection is due to the arithmetical difference of the powers measured by the two component parts of the wattmeter. By reversing one half of the instrument a reading is obtained giving the value of $(W_1 - W_2)$ (see p. 127), and this, in conjunction with the total power $(W_1 + W_2)$ enables the power factor to be determined from the formula

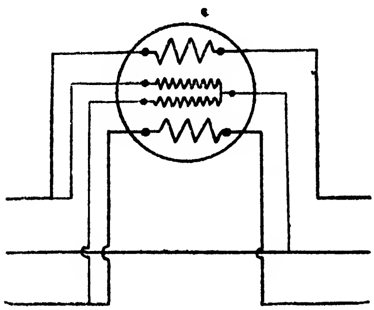


FIG. 123.—Three Phase Wattmeter Connections.

$$\text{Power factor} = \frac{1}{\sqrt{1 + 3\left(\frac{W_1 - W_2}{W_1 + W_2}\right)^2}}.$$

Care must be observed, for when the power factor is less than 0·5 one part of the wattmeter normally gives a backward deflection and the total power $(W_1 + W_2)$ is the low reading, whilst the difference $(W_1 - W_2)$ is the larger reading. The question of whether the power factor is above or below 0·5 can generally be determined from a knowledge of the circuit and the apparatus used.

Drysdale Double Standard Wattmeter.—This is a type of polyphase wattmeter suitable for standardising or accurate testing work. It consists of two separate dynamometer wattmeters connected as shown in the previous paragraph, and can be used for either two or three phase work, and, by connecting the two volt coils in series and the two current coils in series, it can also be used as a single-phase wattmeter.

The two moving systems are each mounted on a mica former arranged so that they are at right angles to each other in order to avoid magnetic interference. These are suspended by a silk fibre and controlled by a helical spring of German silver as shown in Fig. 124. It can be seen from this diagram that the coils are wound astatically, the current being led into and out of the moving system by means of flexible strips of phosphor bronze. A pointer is attached, but since the instrument is always read with the pointer on the zero, the scale need only be sufficiently long to indicate a deflection. The current coils are arranged to surround the pressure coils as shown in the diagram, these being themselves encased in ivoride boxes. The current coils are built up of ten separately insulated strands which can be connected 1, 2, 5 or 10 in parallel

by means of a commutator, one of which is provided for each phase. This stranding of the conductors also tends to eliminate eddy

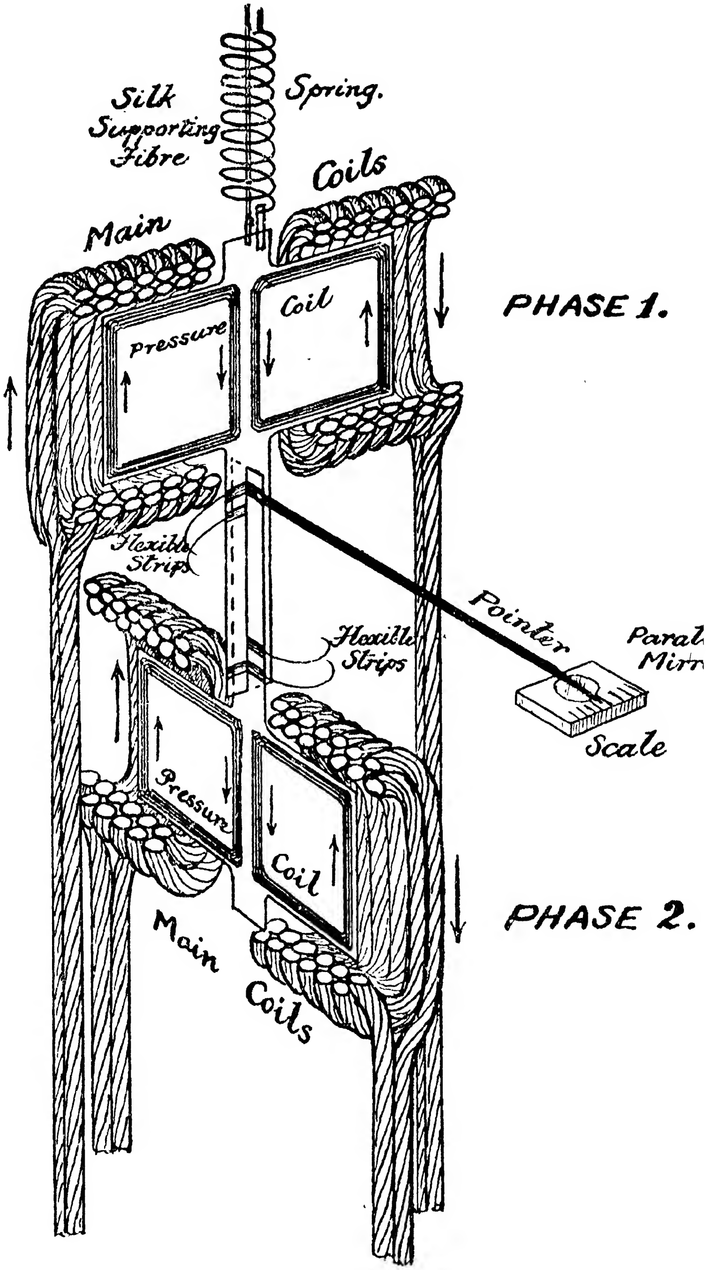


FIG. 124.—Arrangement of Coils in Drysdale Double Standard Wattmeter.

currents, the whole instrument being made of non-metallic materials, as far as possible, for this same purpose. The torque set up by

the interaction of the current and pressure coils when a load is applied causes the swinging coil to deflect. This is brought back to zero by twisting the spring in the opposite direction by means

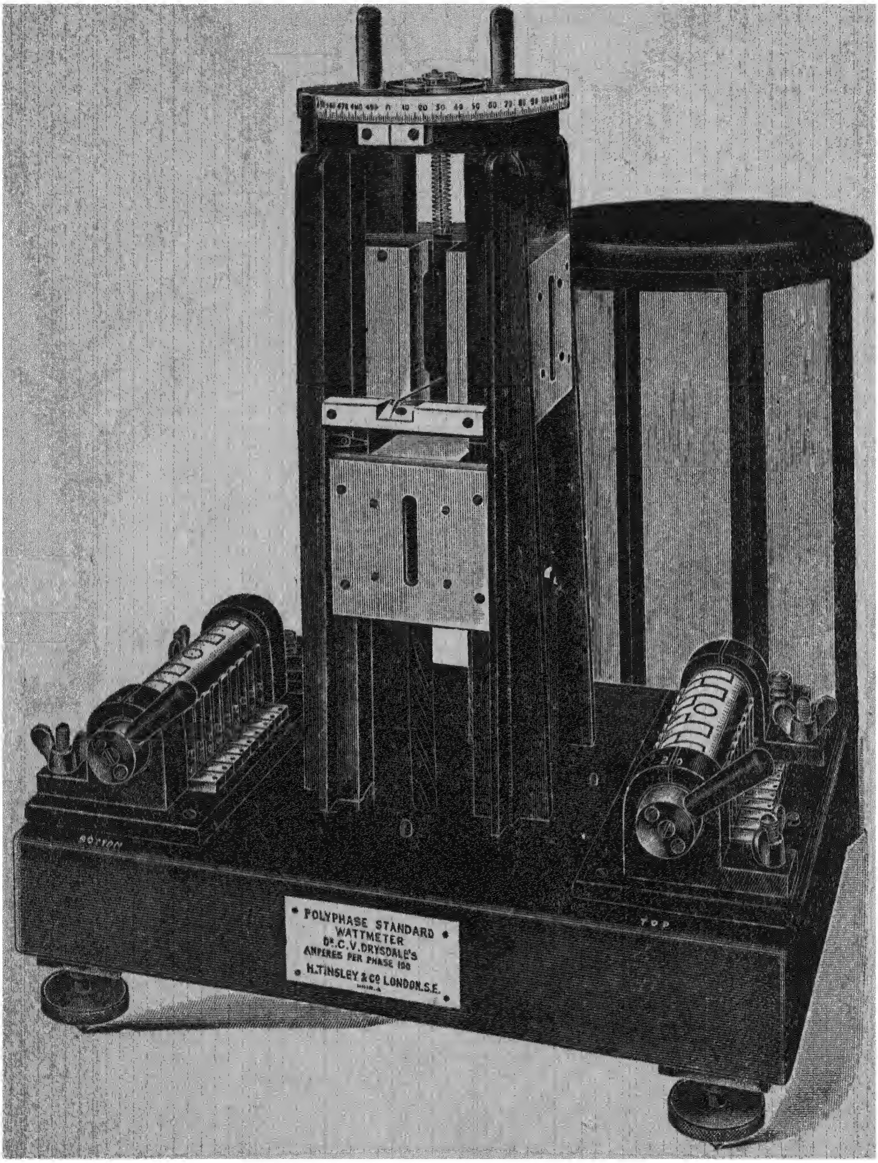


FIG. 125. Drysdale Double Standard Wattmeter.

of a large torsion head, the reading being obtained from a scale on this torsion head. Fig. 125 shows the complete view of the instrument.

The normal current through the pressure coils should not exceed

0·02 ampere, and since the resistance of each is 100 ohms, additional resistance has to be added externally to make it suitable for use on commercial voltages.

The reading will always include the loss in either the pressure coil or the current coil, but in most cases it is best to make it include the former, as this can be easily calculated and deducted from the reading. The swinging coil should always be joined to the nearest point possible of the main current coils, in order that electrostatic forces between the two may be minimised and to obviate any danger of sparking between them.

Supply Meters.—A large number of A.C. watt-hour meters, or *integrating wattmeters*, as they are sometimes called, have been developed, but only the principles of two distinct types will be discussed. Each one of these types has a number of variants, and one example from each type is chosen for illustration.

Induction Motor Meters.—These meters depend for their movement upon the interaction of a magnetic field and a metal disc

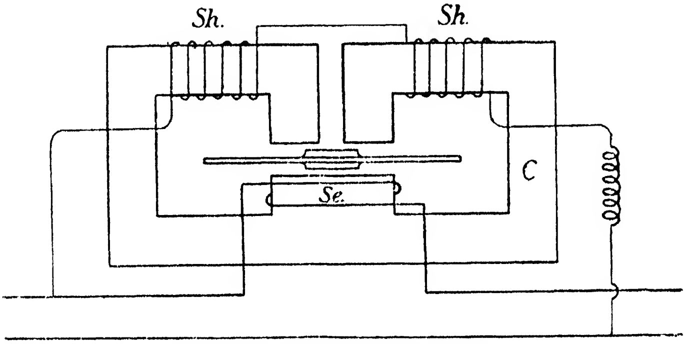


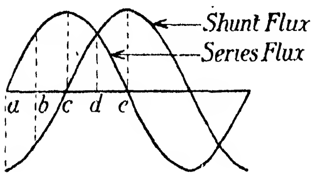
FIG. 126.—Induction Motor Meter.

placed in the field. They consist of four main parts: (1) the driving combination, consisting of a laminated iron core suitably wound with shunt and series coils; (2) the moving system, or rotor; (3) an eddy current brake; and (4) the registering train.

The instruments do not give an indication by means of a pointer, but record the total number of revolutions made, this being an indication of the B.T. units consumed. Fig. 126 represents diagrammatically a meter of this type, manufactured by the British Westinghouse Co., Ltd., *C* being a laminated iron core upon which a shunt coil, *Sh*, and a series coil, *Se*, are wound. The rotor consists of a flat disc or bell of copper or aluminium with a vertical spindle running in a jewelled footstep bearing, friction being reduced to a minimum by making the rotor as light as possible. The brake disc rotates between the poles of a permanent magnet which has been aged artificially to ensure constancy. The registering train consists of a system of gear wheels and indicators simply measuring the total number of revolutions.

The shunt coil is made as inductive as possible, so that the flux lags by practically 90° behind the volts, whilst the current coil is made of low resistance and should be non-inductive, so that the phase of the current is not disturbed and will be determined by the circuit. In this way, two fluxes are produced differing in phase by $(90° - \phi)$, where ϕ is the angle of lag.

These two fluxes can be considered as forming one resultant flux which alters in position and magnitude from instant to instant. Considering the load to be non-inductive, the two fluxes should be in exact quadrature, and referring to Fig. 127 it is seen that at *a* the series flux is zero, whilst the shunt flux is at its negative maximum, as represented in Fig. 127 (*a*). A little later, at *b*, the series flux has assumed a positive value, whilst the shunt flux is still negative, as indicated in Fig. 127 (*b*). At *c*, the shunt flux has fallen to zero, except for a little leakage, whilst the series flux has attained its maximum value [see Fig. 127 (*c*)], whilst *d* and *e* represent conditions occurring still a little later on in the cycle, where the shunt flux reverses and the series flux falls to zero. These events occur every half-cycle, with the result that a torque is developed in the disc. Since the flux moves from right to left in the air gap, the disc tends to follow it, thus producing a rotation.



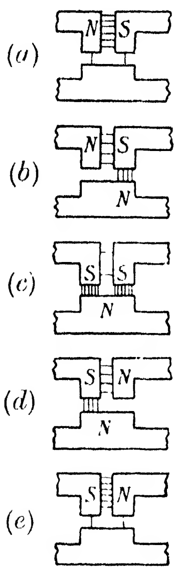


FIG. 127.
Flux Variation in Induction Motor Meter.

If the current in the series coil is given a lag of 90° and the events are again traced out, it will be found that there is no shifting of the flux from right to left, and consequently no rotation is produced. For other angles of lag, the effect is intermediate between these two extremes and a reduced speed of rotation is obtained. The speed obviously depends on the power factor as well as on the magnitudes of the shunt and series fluxes. These latter are proportional to the current and voltage respectively, if the iron be not saturated.

The driving torque is thus proportional to the power, whilst the retarding torque due to the brake wheel is proportional to the speed, for the power due to eddy currents is proportional to the square of the volts, *i.e.* the square of the speed and hence

$$\text{power due to eddy currents} \propto (\text{speed})^2,$$

$$\text{retarding torque} \times \text{speed} \propto (\text{speed})^2,$$

and

$$\text{retarding torque} \quad \ldots \quad \ldots \propto \text{speed}.$$

The speed will adjust itself so that the driving and retarding torques are equal, and when this occurs it follows that the power consumed is proportional to the speed and that the energy consumed is proportional to the total revolutions of the disc. The dials can therefore be calibrated directly in B.T. units.

In actual cases the two fluxes are not in exact quadrature, due to the presence of resistance in the shunt coil and reactance in the series coil. This produces errors in the same way as in wattmeters, and to get over this difficulty a compensating device has to be resorted to. This may be done (1) by means of an auxiliary shunt winding so arranged as to produce a resultant shunt flux lagging slightly behind that produced by the shunt winding itself, or (2) by means of an auxiliary series winding so arranged as to produce a resultant series flux slightly leading that produced by the series winding itself.

The first method is carried out by winding a second shunt winding on the core in opposition to the main shunt winding, a non-inductive resistance being placed in series with it so that the angle of lag is

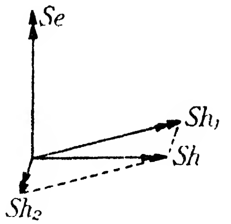


FIG. 128.—Flux Vector Diagram for Shunt Compensation.

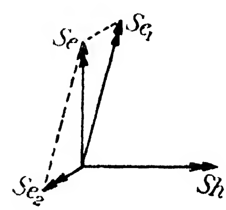


FIG. 129.—Flux Vector Diagram for Series Compensation.

reduced. Fig. 128 shows a flux vector diagram, Se and Sh_1 being the fluxes produced by the series and main shunt coils respectively. The auxiliary shunt winding tends to produce a flux, Sh_2, which when combined with Sh_1 produces the resultant shunt flux, Sh, lagging by exactly 90° behind the series flux, Se. The final adjustment is made by varying the non-inductive resistance in the auxiliary shunt circuit.

In the second method of compensation, an auxiliary series coil is wound in opposition to the main series coil, a certain amount of extra inductance being included in the circuit. The flux vector diagram is now represented by Fig. 129, where Sh is the shunt flux, Se_1 the main series flux and Se_2 the auxiliary series flux. Combining the two latter, the resultant series flux, Se, is obtained, this being in exact quadrature with the shunt flux, Sh.

Thomson Motor Meter.—This meter, manufactured by the British Thomson-Houston Co., Ltd., consists of a little motor having no iron in its construction (see Fig. 130). There are two field coils, *FF*, wound with thick copper wire and connected in series with

each other and one of the mains. In between these two field coils is situated the armature, A, rotating on a vertical spindle. The armature forms the volt coil together with a high non-inductive resistance, R, which is connected in series with it. This makes it carry a small current proportional to the voltage. Connection is made to the armature by means of two light springs pressing on to a small silver commutator, C. Underneath the armature is a thin copper disc, D, attached to the same vertical spindle, this rotating between the pòles of two or more permanent magnets, M. Eddy currents are induced in this disc, causing a retarding torque to be set up proportional to the speed. The driving torque at any instant is proportional to the current $\times$ volts at that instant, the average value of which product is the power. When constant speed is attained the driving and the retarding torques are equal and therefore, since

$$\begin{aligned} &\text{Driving torque} \propto \text{watts,} \\ &\text{Retarding torque} \propto \text{speed,} \\ &\text{Watts} \ldots \ldots \propto \text{speed.} \end{aligned}$$

The total revolutions are therefore proportional to the total energy transmitted, and this is registered on a dial system driven by a train of wheels from the spindle.

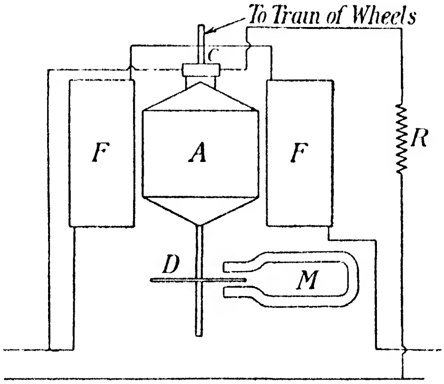


FIG. 130.—Thomson Motor Meter.

In all motor meters a certain starting current is necessary to overcome the statical mechanical friction, and compensation for this is sometimes accomplished by means of an auxiliary winding placed inside the series coil, but connected in series with the shunt coil. The torque thus produced by the shunt circuit alone is arranged to be just sufficient to overcome the statical friction, but it must not cause the meter to register.

Another way of overcoming the starting friction is to connect the pressure coil on to the load side of the current coil. The current through the pressure coil then flows through the current coil also and compensates for the statical friction.

Polyphase Supply Meters.—These are constructed for two and three phase circuits in the same way as wattmeters, viz. two separate instruments operate on the same spindle. Instead of adding the deflections, the speeds are added so that the total revolutions of the disc are proportional to the sum of the B.T. units measured by each portion of the instrument. The connections are the same as have been already shown in Figs. 122 and 123 in connection with wattmeters.

CHAPTER XII

ALTERNATING CURRENT INSTRUMENTS (*continued*)

Galvanometers.—A number of the sensitive and accurate galvanometers which have been devised are not suitable for A.C. work, since their deflections reverse with the current. For A.C. work, a galvanometer must give a deflection which is in some way proportional to the square of the current in order that it shall indicate R.M.S. values. Due to considerations of space, not more than one example of each of the leading types of galvanometer will be described.

Irwin Astatic Dynamometer.—This instrument, as its name implies, is constructed upon the dynamometer principle, there being two fixed coils contained in ebonite boxes, one attached to the frame and the other removable. These coils are connected in opposition, so that the field between them is radial in character (see Fig. 131). The moving system consists of a mica disc on each side of which is a D-shaped coil as shown at A and B. The disc and coils are suspended by a phosphor bronze strip so as to be capable of rotation in the radial field. A phosphor bronze spiral and the suspension serve to lead the current into and out of the moving system. Owing to the peculiar shape of the moving coils, they each tend to move in the same direction across the radial field when the current flows. Moreover, the shape of the moving coil renders inappreciable the effect of stray magnetic fields even when these are not uniform. Efficient air damping is obtained, since the clearance between the edge of the mica disc and the damping chamber is small. The deflection obtained is proportional to the square of the current in accordance with the dynamometer principle. Fig. 132 shows a view of the complete instrument which

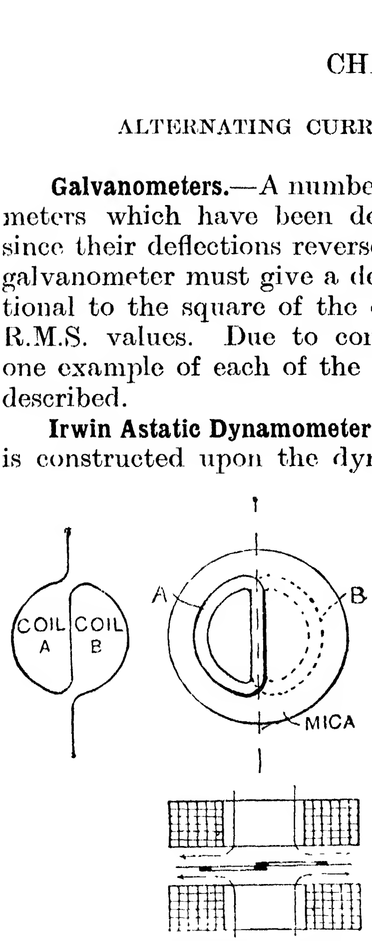


FIG. 131.—Construction of Irwin Astatic Dynamometer.

can be made suitable for the measurement of either current, voltage or power.

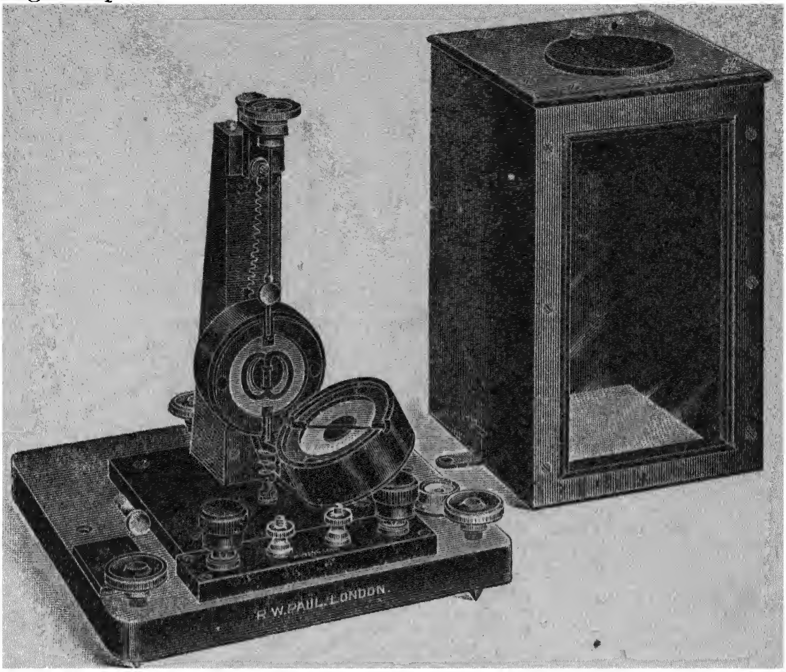


FIG. 132.—Irwin Astatic Dynamometer.

Sumpner Iron Cored Dynamometer.—The principle upon which this instrument works is the same as that in the iron cored voltmeter (see p. 135). The electromagnet is constructed of thin iron laminations, upon which three coils are wound for the purpose of obtaining different ranges, these windings having negligible resistance, so that the flux is proportional to and exactly in quadrature with the applied voltage. The moving coil consists of but a few turns suspended in the two air gaps of the electromagnet, so that both its resistance and inductance are negligible when connected in series with a suitable condenser. The electrical connections are the same as shown in Fig. 109, but these can be varied by means of the different coils so as to obtain different sensitivities. Air damping is provided by means of a vane attached to the moving system and fitted in a closed chamber.

The sensitivity can be increased by separately exciting the magnets from the same source and applying the voltage to be measured to the moving coil with its condenser in series. In this way, it is possible to detect pressures of the order of a micro-volt.

This instrument can be used in a variety of ways; it can be used as a voltmeter over the extensive range between 1 micro-volt and 200 volts, and it can be used as a low reading wattmeter. Also,

if the instrument has been previously calibrated, it can be used to measure very small capacities by inserting these in the moving coil circuit.

Duddell Thermo-Galvanometer.—The operation of this instrument depends upon the measurement of the thermo-electric E.M.F.'s set up, due to the heating of a conductor by the passage of a small current. The working part of the instrument consists of a single loop of silver wire, *L* (see Fig. 133), joined at the lower end to a bismuth-antimony thermo-couple. This loop is suspended between the pole pieces, *NS*, of a permanent magnet by means of a quartz fibre, *Q*, the latter being joined to the loop by means of a glass stem, *G*, carrying the mirror, *M*.

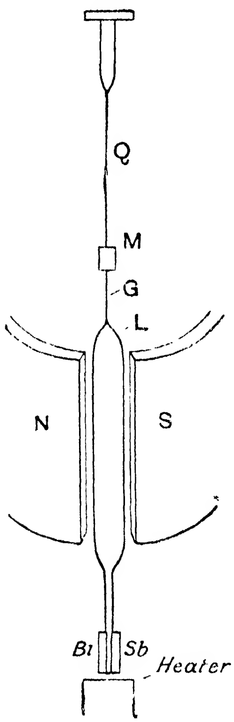


FIG. 133.—Construction of Duddell Thermo-Galvanometer.

A resistance known as the *heater* is fixed immediately underneath the thermo-couple so as to heat it both by radiation and convection. The heater is made in the form of a fine filament of metal wire for resistances up to 40 ohms, and of platinised quartz fibre when high resistances are desired. The E.M.F. generated by the thermo-couple causes a unidirectional current to flow round the moving coil, producing a deflection in the magnetic field. Since the heater consists of a straight filament only 3 or 4 mm. long connected to the terminals by two straight wires, the inductance and capacity are exceedingly small.

It is very necessary to avoid even small changes of temperature, owing to the sensitiveness of the instrument, and for this purpose the suspension, silver loop and heater are enclosed in a heavy brass block of which the front *E* (see Fig. 134) is removable. This brass block slides over the studs *DD*, where it is clamped into position. The heaters are made so that they can be quickly interchanged, the distance from the thermo-couple being adjusted by means of the milled head, *F*, and the position underneath the thermo-couple being adjusted by means of the set screws, *G*.

A pin, *B*, screwing into the brass block, clamps and unclamps the moving system, whilst the zero is adjusted by means of the torsion head, *L*, working in the collar, *K*, the mirror being attached at *H*. A brass cover is held in position over the whole by screws at *AA*, so as to keep the instrument at as uniform a temperature as possible.

This galvanometer is very sensitive and can be used for both C.C. and A.C., frequency and wave form having no effect at all.

On the other hand, the zero is somewhat unstable and should be checked after each observation External magnetic fields have practically no effect at all.

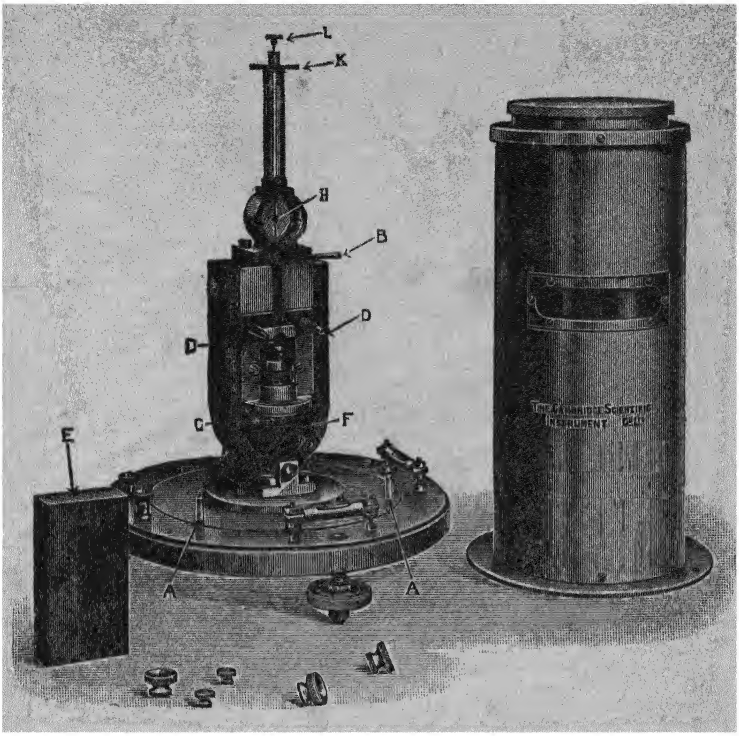


FIG 134.—Duddell Thermo-Galvanometer.

The heat produced per second is proportional to the square of the current, and since the E.M.F. and current in the moving system are practically proportional to the rise of temperature, it follows that the deflection is very nearly proportional to the square of the current.

Drysdale Vibration Galvanometer.—The principle involved in vibration galvanometers is that of resonance, the suspended system of a special moving needle galvanometer being set in vibration by the alternating current in this instance. This needle is controlled by means of a permanent magnet, the strength of the field being adjusted by a magnetic shunt, and by means of the latter the moving system can be brought into mechanical resonance with the electrical circuit. The spot of light then appears as a broad band, the width

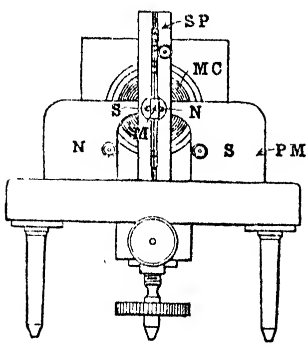


FIG. 135.—Drysdale Vibration Galvanometer. Front Elevation.

of which is taken as the deflection. Figs. 135 and 136 show a front and side elevation of the instrument respectively, whilst Fig. 137

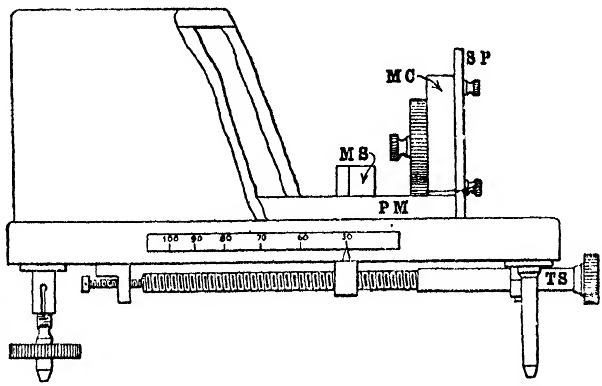


FIG. 136.—Drysdale Vibration Galvanometer. Side Elevation.

shows a general view. The magnetising coil, *MC*, actuates the needle, *NS*, which is supported by the suspension piece, *SP*, the

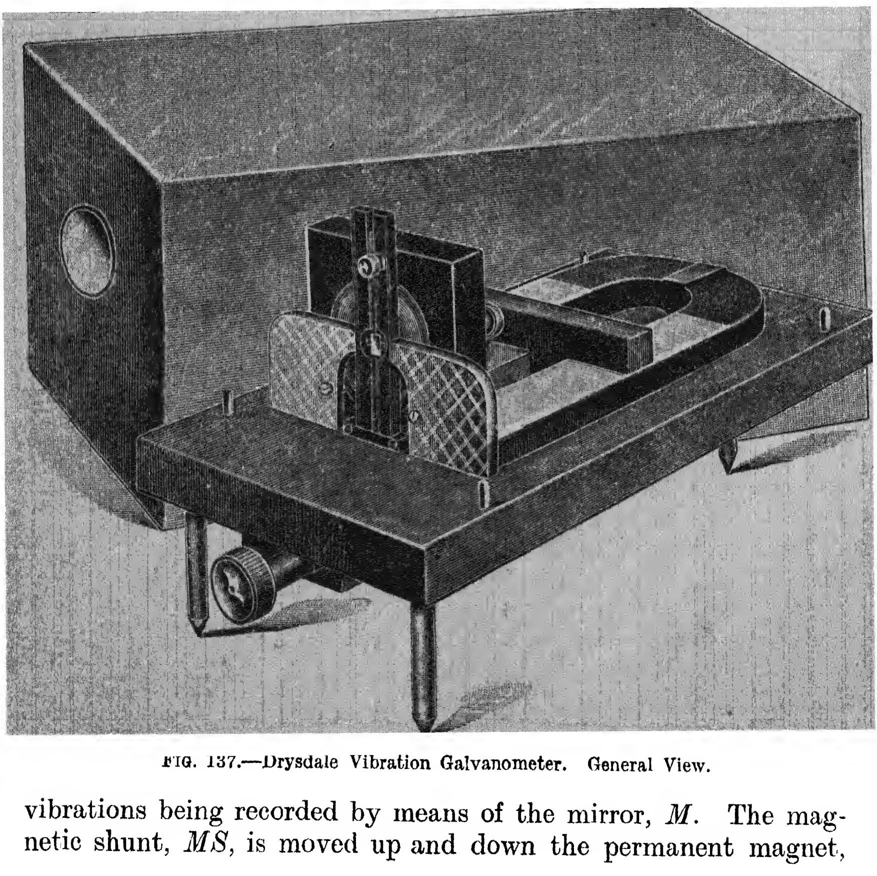


FIG. 137.—Drysdale Vibration Galvanometer. General View.

vibrations being recorded by means of the mirror, *M*. The magnetic shunt, *MS*, is moved up and down the permanent magnet,

PM, by means of the tuning screw, *TS*, which is adjusted for each particular frequency so as to give the maximum deflection. The magnetising coils are removable, thus enabling coils of different resistance to be used, but a coil of about 40 ohms suits most requirements, although they can be constructed over a very wide range. Vibration galvanometers are extremely sensitive to changes of frequency, this practically precluding their use as deflectional instruments, although they are of great value in zero methods of test.

Oscillographs.—These instruments are employed for the purpose of obtaining the exact shape of a pressure or current wave form.

All oscillographs are in reality only galvanometers, the moving systems of which are capable of following the extremely rapid variations of the p.d. or current, enabling these to be reproduced on the screen. There are three main types at present in use operating on the moving coil, moving iron and hot wire principles. These will each be described in turn, but the general arrangements, apart

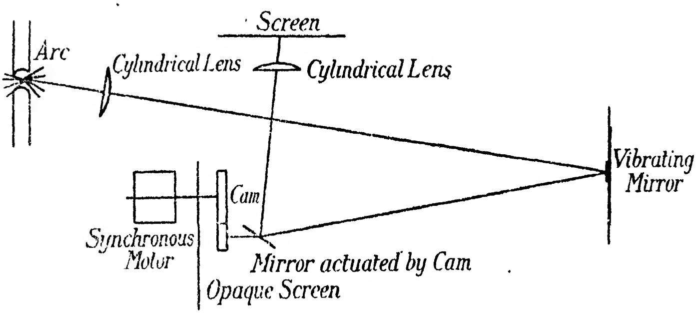


FIG. 138.—Optical Arrangement of Oscillograph.

from the details of construction of the galvanometer part of the oscillograph, are more or less the same for all types.

The galvanometer part gives a rapidly varying deflection which appears as a straight line. The beam of light is then given a movement in a direction at right angles to this, and the spot of light now traces out a curve. This second movement is so arranged that the spot of light is deflected over equal distances in equal times, so that the curve which is plotted automatically is the relation between deflection and time and is, consequently, the wave which it is desired to record. The optical arrangements are shown in Fig. 138, where the light from an arc is passed through a cylindrical lens before impinging on the vibrating mirror of the galvanometer. The light is focussed on to this mirror, and the reflected ray is made to vibrate at right angles to the plane of the paper. The ray of light then impinges on a plane mirror or a totally reflecting prism, which is made to vibrate in such a direction as to give a movement to the ray at right angles to the movement already

impressed on it. The reflected ray then passes through another cylindrical lens before reaching the screen, where it has a movement in the plane of the paper proportional to the time and a movement at right angles to the plane of the paper proportional to the instantaneous value of the current or voltage. The second mirror is actuated by means of a cam driven from a four pole synchronous motor (see Chapter XXI.), which is a motor rotating exactly once for every two periods of the alternating current. The cam is of peculiar design and is so arranged that during one and a half periods the mirror is turning with uniform angular velocity,

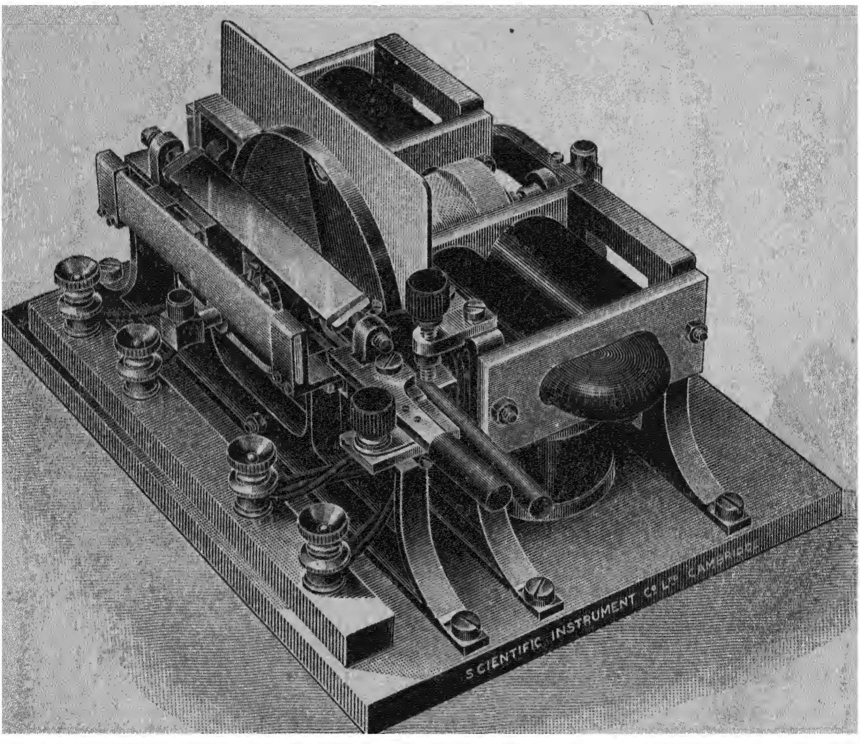


FIG. 139.—Synchronous Motor with Vibrating Mirror.

whilst during the remaining half-period the mirror executes a quick return swing, an opaque screen being interposed to cut off the light and avoid confusing the diagram. An example of such a motor with a vibrating mirror is shown in Fig. 139.

Duddell Moving Coil Oscillograph.—The first instruments of this type were due to M. Blondel, but the successful development has been brought about by the late Mr. Duddell. The oscillograph consists of a powerful permanent electromagnet, *NS* (see Fig. 140), with a narrow air gap in which are stretched two parallel conductors, *ss*, formed of a strip of phosphor bronze bent round an ivory pulley, *P*.

A suitable tension is kept on the strips by means of a spiral spring attached to the pulley. When a current passes through the strips, one is urged forward and the other back, so that the mirror, M, is deflected. Very often two such *vibrators*, as they are called, are placed side by side for the purpose of recording the current and voltage waves simultaneously. The guide piece, L, serves to limit the length of the vibrating portion, which is immersed in an oil-bath for the purpose of damping the movement and making it dead beat. There is also a third mirror for the purpose of recording

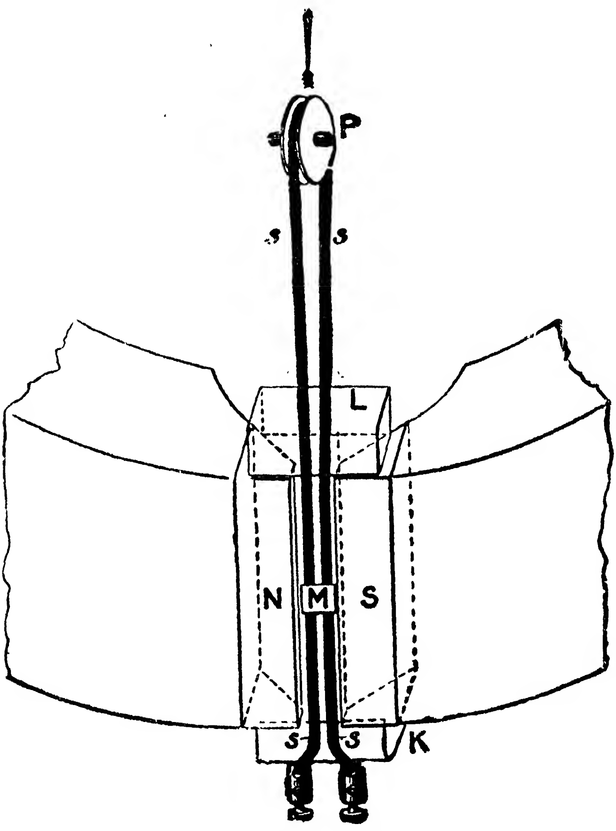


FIG. 140.—Duddell Moving Coil Oscillograph.

the zero, the other mirrors being brought to zero by an adjustment of the guide piece, L. The clearance between the edge of the strip and the walls of the magnet face is very small, varying from 0·04 to 0·15 mm.

The inductance possessed by this type of oscillograph is quite negligible, so that it can be shunted as an ammeter, the safe working current being 0·1 ampere in some instances and 0·5 ampere in others.

Blondel Moving Iron Oscillograph.—A powerful permanent

U-shaped magnet, fitted with laminated tapering pole pieces and an iron core, provides the magnetic field, one or two air gaps being employed, depending upon whether the oscillograph is single or double (see Fig. 141). A very thin and narrow strip of soft iron is placed in each gap so that the lines of force pass from side to side across the width. These strips are kept in tension, so that the natural rate of torsional vibration is very high, reaching at times 40,000 per second. The strips are therefore able to follow the vibrations of lower frequency due to the current the wave form of which is to be determined. Two small coils per gap, one on each side, provide the deflecting force at right angles to the field of the permanent magnet. When a current flows through these coils, the field is distorted and the strip tends to take up a new position along the axis of the resultant field which is produced. The strip is therefore twisted and the mirror, which is attached to the middle of it, indicates a deflection. The twisting strips are immersed in a castor-oil bath so as to damp out any ripples which might appear due to the strips themselves.

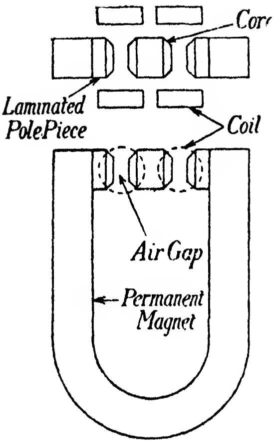


FIG. 141.—Blondel Moving Iron Oscillograph.

Irwin Hot Wire Oscillograph.—The ordinary hot wire galvanometer gives a deflection which is proportional to the square of the current, and arrangements have to be made in this oscillograph in order to produce a deflection which is directly proportional to the first power of the current. This is effected by superposing a continuous current on the alternating current under consideration after the manner shown in Fig. 142. Two hot wires, $S\,S$, are connected in series and form one element, being connected at their centre point to a battery, B, which is also connected to each end by means of two equal resistances, $R\,R$. The currents in the two wires will therefore be the sum of the alternating and continuous currents in one case and the difference in the other. Let i and I represent the instantaneous value of the alternating and the continuous currents respectively. The heating of the two hot wires will be proportional to $(i + I)^2r$ and $(i - I)^2r$, where r is the resistance of one hot wire. The difference of the heating of the two wires will be

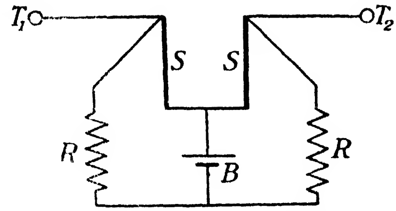


FIG. 142.—Principle of Irwin Hot Wire Oscillograph.

$$(i + I)^2r - (i - I)^2r$$
$$= 4iIr.$$

If, therefore, the continuous current, I, be kept constant, the difference of the heating and consequently the difference of their elongations will be directly proportional to the instantaneous value of the alternating current, and the instrument is designed so that the deflection is proportional to this quantity.

The two hot wires, $ABCD$ and $EFGH$ (see Fig. 143), pass over an ivory pulley and are fixed at their lower ends, AD and EH. Two horizontal wires, BC and FG (shown at the side in the diagram), prevent any horizontal movement, whilst allowing an upward movement when the wires become elongated, due to their rise in temperature. These cross wires, which do not touch each other, also serve to confine the current practically to the lower portions of the hot wires. Further, the two hot wires are tied together at X and Y by means of an insulating loop of thread, so as to avoid any electrical connection, and a mirror is fixed across the wires near these points. The whole system is kept in tension by means of a spiral spring attached to the pulley and is immersed in an oil-bath almost up to the level of the mirror. The two strips are electrically connected together and to one pole of the battery at D and H, the current being led into and out of the system at A and E. Due to the difference of the elongations of the two parts, the system is twisted at XY, thus causing the mirror to deflect.

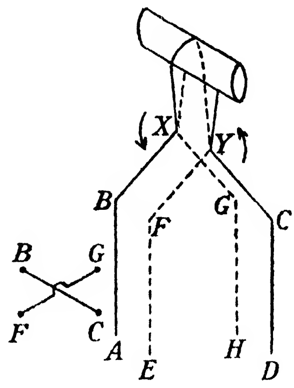


FIG. 143.—Arrangement of Hot Wires in Irwin Hot Wire Oscillograph.

Owing to the time lag which is experienced in hot wire instruments, some method of accelerating the heating has to be employed. The only way of doing this is to advance the phase of the current flowing through it, and this is effected by putting a shunted condenser in series with the hot wires.

Oscillograph Connections.—Fig. 144 indicates the method of connecting up a double strip oscillograph, the current strip, I, together with the resistance, R_2, measuring the p.d. across the known resistance, R_1. The resistance, R_2, is included so as to avoid overrunning the strip and to enable the deflections to correspond to some suitable number of amperes per division. The volt strip, E, is connected in series with a high resistance, R_3, across the mains. A most important point is to have one end of this strip directly connected to the same main as the current one, so as to avoid any large difference of potential between the two strips themselves. If this point is neglected, a short circuit between the two is most likely to occur owing to the small clearances employed. The value of R_3 can also be regulated so as to obtain a convenient volt scale

on the curve. A fuse, f, and a switch, S, are further included in each circuit, whilst the synchronous motor, SM, is run, in series with the resistance R_4, across the mains.

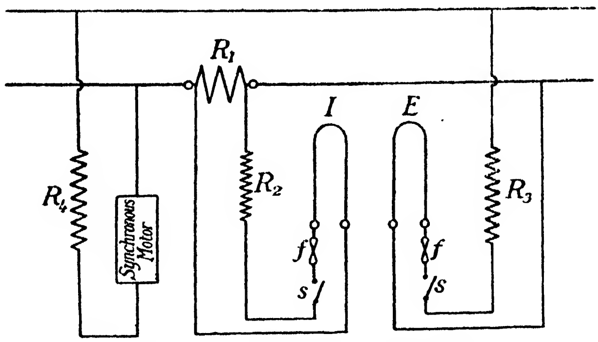


FIG. 144.—Oscillograph Connections.

In some cases the wave is reflected on to a glass screen, where it can be observed by eye or traced by hand by placing a sheet of paper over the screen. Where a photographic record is desired, a falling plate camera is used, the plate falling directly across the path of the ray which is reflected from the oscillograph, the synchronous motor with the vibrating shutter being removed. If the plate falls from the height of a few feet the speed remains very nearly constant during the interval of time in which the record is made.

Apparatus is also made to enable a cinematograph record to be obtained.

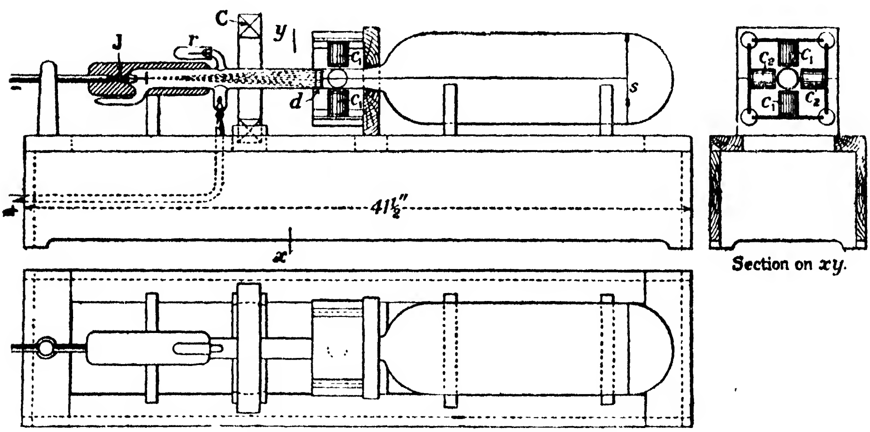


FIG. 145.—Kathode-Ray Oscillograph.

Kathode-Ray Oscillograph.—This instrument does not possess perhaps the accuracy of the types already described, but has a great advantage on the score of cost and simplicity in construction,

and for many purposes is convenient and easy to manipulate. The oscillograph consists of a kathode-ray tube of special design, a pencil of kathode rays producing a bright spot on a fluorescent screen (see Fig. 145). If an alternating current be passed through a pair of coils, C_1 C_1, the rays are deflected and the spot appears as a straight line. Another pair of coils, C_2 C_2, produces a deflection at right angles to the first, and in this way the two axes are obtained. If the two pairs of coils are used together, the spot either traces out an inclined line or a closed curve, from which the wave form is obtained. The divergent beam of kathode rays strikes an aluminium diaphragm, *d*, and by means of the coil, *C*, supplied with continuous current and giving about 2,500 ampere-turns, the spot is focussed on to the screen, *S*. The coil, *C*, requires to be very nicely adjusted to get the best result, and its position has to be found by trial. The

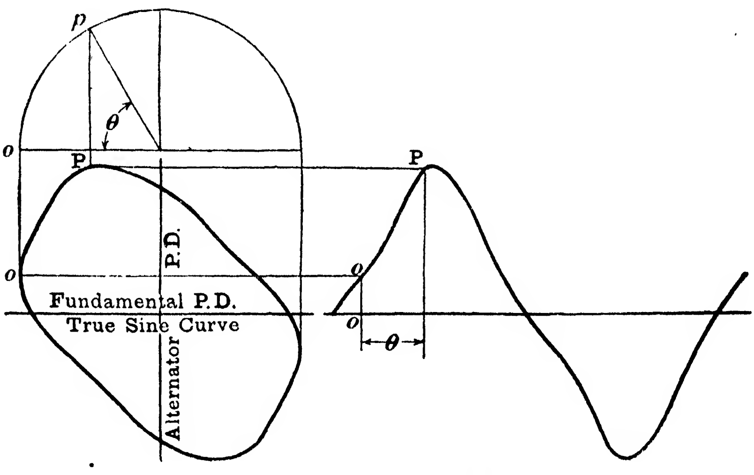


FIG. 146.—Kathode-Ray Oscillogram and Wave Form.

kathode end of the tube, *J*, is thickly coated with paraffin wax, which has the effect of steadying the spot, whilst connection is made to the tube by means of a piece of high tension flexible wire. A palladium wire sealed through the tube at *r* is used to get rid of the "hardening" of the tube, which is done by gently heating the wire.

The kathode-ray tube may be excited either from an influence machine or by means of an induction coil with a rapid make and break.

To determine the shape of a particular wave, it is convenient to apply a sinusoidal E.M.F. to one pair of deflecting coils and the E.M.F. of unknown wave form to the other. The resulting diagram is of the form shown in Fig. 146, which also shows the construction for obtaining the wave form. When the sine wave has zero value,

the magnitude of the unknown is OO and the value of the curve $\theta°$ later is given by the point P.[1]

Idle Current Ammeters and Wattmeters.—These idle current instruments measure the idle component of the current or volt-amperes only, viz. $I \sin \phi$ or $EI \sin \phi$, where ϕ is the angle of lag. Such instruments are useful on the switchboard for noting how near to unity power factor the plant is operating, but the true power factor indicators have largely supplanted them.

The principle of the iron-cored Sumpner instruments (see p. 135) is applied in one form of idle current ammeter. The supply voltage must be kept constant, and is used to excite the shunt electromagnet as before, whilst the moving coil is connected across a low non-inductive resistance placed in series with the mains like an ammeter shunt. If the angle of lag in the main circuit is ϕ, the instantaneous deflecting torque is proportional to

$$F \sin(\theta - 90°) \times I \sin(\theta - \phi)$$
$$= -F \cos\theta \times I(\sin\theta\cos\phi - \cos\theta\sin\phi)$$
$$= -FI(\tfrac{1}{2}\sin 2\theta\cos\phi - \cos^2\theta\sin\phi).$$

The average value of $\sin 2\theta$ being zero and the average value of $\cos^2\theta$ being 0·5, the average value of the torque is proportional to

$$\tfrac{1}{2} FI \sin\phi,$$

and if F is constant the deflection is proportional to $I \sin \phi$, which is the idle component of the current. This assumes a constant maximum flux and a constant voltage. If the latter varies, the instrument measures $EI \sin \phi$, acting as an idle current wattmeter.

Power Factor Indicators.—These instruments can be constructed on the dynamometer principle, the spring control being removed. There is a fixed coil, CC (see Fig. 147), connected in series with the line, and two moving coils, MC_R and MC_L, connected across the mains

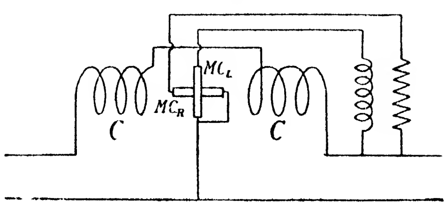


FIG. 147.—Power Factor Indicator.

like pressure coils, the first MC_R having a non-inductive resistance and the second MC_L a choking coil connected in series with it. The magnitude of the resistance and reactance, as well as the turns on the coils, are so adjusted that the ampere-turns of the two coils

[1] *Electrician*, Nov. 7, 1913, p. 172.

are exactly equal. The two moving coils are further rigidly attached at right angles to one another on the same spindle.

If the line current is in phase with the line voltage, the moving coil, MC_L, will endeavour to set itself vertically. The fixed coils, CC, act like choking coils, and consequently produce a flux which is in phase with that produced by MC_L. The combination of these two fluxes, trying to take up as short a path as possible, tends to pull the coil into the vertical position.

When the main current lags behind the volts by 90°, the reverse is the case, and the moving system takes up the position with MC_R vertical. For intermediate angles of lag, the moving system takes up an intermediate position, the pointer (not shown on the diagram) moving over a scale. For leading currents, the moving system would deflect the other way, with the result that these instruments frequently have very long scales.

Instruments working on this principle can also be constructed for three-phase circuits, the moving system being wound with three coils star-connected on to the mains.

Frequency Meters.—The first type which will be described is based upon the fact that when a steel reed is brought near to the poles of an alternating current electromagnet it is set into resonant vibration at one particular frequency, this being independent of the voltage or wave form. In the frequency meter made by Messrs.

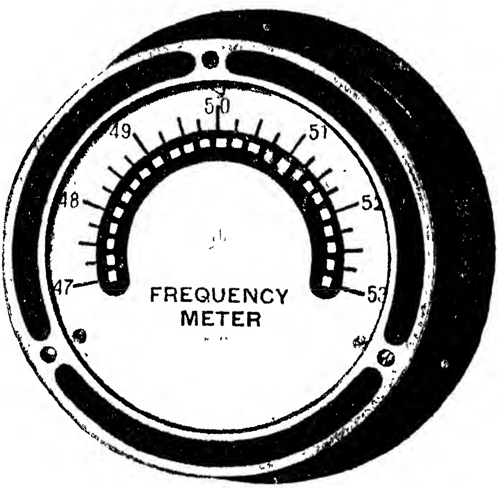


FIG. 148.—Resonance Frequency Meter.

Everett, Edgcumbe & Co., Ltd., there are a number of such reeds, each tuned to vibrate at a particular frequency, the reeds themselves being tipped with a white enamelled flag. These flags appear in an arc of a circle on the dial of the instrument, the frequency corresponding to each reed being marked on the scale opposite to it.

These reeds are very sharply tuned, *i.e.* they only vibrate for the particular frequency which they are supposed to indicate, the indications dying away very rapidly indeed as the frequency is slowly changed.

In most power stations the frequency only varies over very narrow limits and a short range on the scale is all that is desired. Fig. 148 shows such an instrument reading from 47 to 53 at intervals of $\frac{1}{4}$ cycle per second.

If a continuous current equal to, or slightly greater than, the maximum value of the alternating current be passed through the winding at the same time, one half of the wave will be neutralised whilst the other half will be strengthened. The reed which happens to be vibrating will therefore be attracted only once per period instead of twice, and will be thrown into resonance by double the frequency which originally made it indicate. This, therefore, provides a simple means of doubling the range of the instrument.

A frequency meter on a totally different principle is that manufactured by the Weston Electrical Instrument Co. There

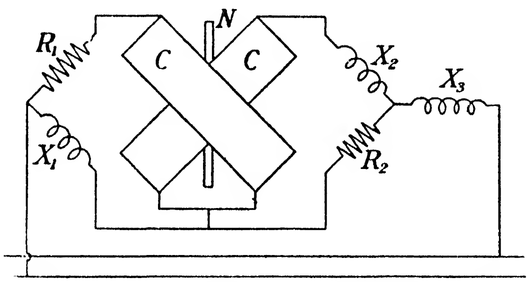


FIG. 149.—Weston Frequency Meter.

are two fixed coils, *CC* (see Fig. 149), one slipped inside the other and at right angles to it. The moving system consists of a needle, *N*, a pointer and a damper, and is free to rotate. The various coils are connected in the form of a Wheatstone bridge, which is balanced at normal frequency. One fixed coil has a resistance, R_1, and the other a reactance, X_2, in series with it, these being placed opposite to the reactance, X_1, and the resistance, R_2, respectively. A third reactance, X_3, is placed in series with the whole, for the purpose of damping out the higher harmonics. When a change in the frequency occurs, the values of the reactances change, causing the system to become unbalanced to a certain extent. This results in an increase in the current in one coil and a decrease in the other. The axis of the resultant field is therefore displaced and the needle takes up a new position, this being an indication of the frequency.

Another type of frequency meter consists of two coils placed vertically over one another, one being connected in series with a

non-inductive resistance, whilst the other is connected in series with a choking coil. The combination is then connected across the mains like a voltmeter. Two pieces of soft iron are attached to a spindle, one opposite to each coil, the resulting pulls tending to make the spindle rotate in opposite directions. At normal frequency the opposing pulls balance, but if the frequency is raised or lowered, the current in the inductive circuit is weakened or strengthened as the case may be, thus causing an unbalanced pull on the spindle which takes up a new position. The pointer is attached to this spindle and indicates the frequency.

Leakage Indicators.—In order to determine the state of the insulation of a system of mains, leakage indicators are installed, these usually taking the form of electrostatic voltmeters connected

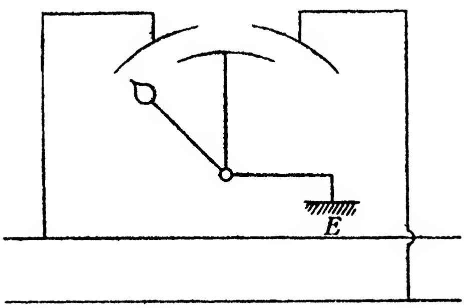


FIG. 150.—Single Phase Leakage Detector.

between each line wire and earth. When high tension circuits are being dealt with, condensers are inserted in series with the voltmeters, so that no part of the instrument shall be directly connected to the high tension line.

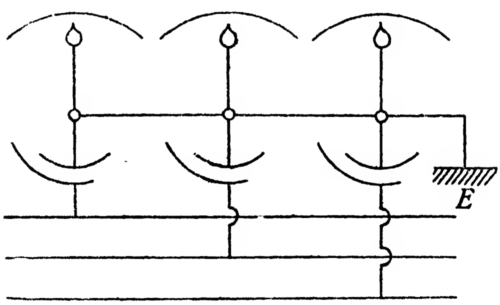


FIG. 151.—Three Phase Leakage Detector.

On single-phase circuits it is common to have two sets of fixed vanes, one set being connected to each line wire. The moving vane is connected to earth (see Fig. 150).

For three-phase systems there are three voltmeters used, all

the moving systems being connected together and earthed. Fig. 151 shows in diagrammatic form such a three-phase instrument, all the three scales being close together to facilitate inspection. When the insulation between one main and earth deteriorates, the leakage goes up and the voltage between that main and earth automatically falls, this being shown by the leakage indicator.

CHAPTER XIII

TRANSFORMERS.—PRINCIPLES AND CONSTRUCTION

General Principle.—When a choking coil is supplied with an alternating voltage, a back E.M.F. is set up, due to the continual rate of change of flux. This E.M.F. is due to the turns of the winding linking with the flux, and if some other turns are placed side by side with the original winding but insulated from it electrically, the turns of the second winding will also have an E.M.F. set up in them, due to the same cause. Neglecting for the moment any losses that might occur, the back E.M.F. in the first or *primary* winding must equal the applied E.M.F. Each turn will provide its own proportion of the total voltage, and if there are T_1 turns on the primary, the back volts per turn will be $\frac{E_1}{T_1}$, where E_1 is the primary applied voltage. Each turn on the other or *secondary* winding will also have the same E.M.F. induced in it, viz., $\frac{E_1}{T_1}$ volts, and if there are T_2 total turns in series on the secondary, the total induced voltage will be $E_1 \times \frac{T_2}{T_1}$. The ratio of the voltages in the two windings is therefore seen to be the same as the ratio of the turns, and a simple means is thus provided of transforming from one voltage to another. Such a piece of apparatus is known as a *transformer*.

Flux in a Transformer.—Consider a rectangular core built up of laminations wound with a primary and secondary winding, shown on opposite limbs in Fig. 152 for the sake of clearness. If the applied voltage follows a sine law, the back voltage, the rate of change of flux and the flux itself must also each follow a sine law. Let the maximum value of the flux induced in the iron core be Φ lines. The total lines cut per cycle by each turn is therefore 4Φ, since the whole flux dies away in a quarter of a period, and the

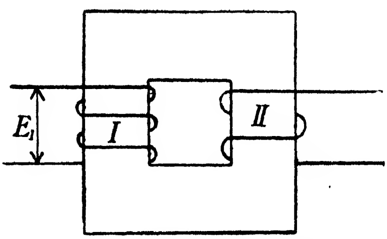


FIG. 152.—Simple Transformer.

total lines cut per second by each turn is $4f\Phi$, where f is the frequency. The *average* E.M.F. induced in each turn is therefore $4f\Phi \times 10^{-8}$ and, since the form factor $\left(=\frac{\text{R.M.S. value}}{\text{average value}}\right)$ is 1·11 for a sine wave, the R.M.S. voltage induced per turn is

$$1{\cdot}11 \times 4f\Phi \times 10^{-8}.$$

The total induced voltage in the primary is

$$E'_1 = 4{\cdot}44 f\Phi T_1 \times 10^{-8} \text{ volts.}$$

Another way of arriving at this result is to consider the maximum rate of change of the flux, which is $2\pi f\Phi$ lines per second, since the flux is sinusoidal in character. The maximum E.M.F. induced per turn is therefore $2\pi f\Phi \times 10^{-8}$ volts, and the total R.M.S. voltage induced in the complete winding is

$$\frac{2\pi}{\sqrt{2}} f\Phi T_1 \times 10^{-8} \text{ volts}$$
$$= 4{\cdot}44 f\Phi T_1 \times 10^{-8} \text{ volts,}$$

the same as before.

This flux links with both the primary and the secondary windings, so that the total R.M.S. voltage induced in the secondary is

$$4{\cdot}44 f\Phi T_2 \times 10^{-8} \text{ volts.}$$

The frequency of the E.M.F. is obviously the same in both cases.

The maximum value of the flux in the ideal transformer can therefore be expressed as

$$\Phi = \frac{E'_1 \times 10^8}{4{\cdot}44 f T_1} = \frac{E_2 \times 10^8}{4{\cdot}44 f T_2}.$$

Effect of Secondary Current.—If the secondary winding be left on open circuit, the primary will act like an ordinary choking coil and will take a small current due to its high impedance, this being called the no-load current. This no-load current will consist of a small power component and a relatively large idle component. The former is necessary to account for the I^2R and iron losses, whilst the latter is due to the interlinking flux making the winding inductive.

Since there is an E.M.F. induced in the secondary winding, a current will flow if the terminals are connected to the ends of a non-inductive resistance. Assuming for the moment that this winding contains no resistance or reactance, the secondary current will be in phase with the secondary E.M.F. But this current tends to produce a flux in the opposite direction to that already existing in the core, and the momentary effect is to reduce the flux, thus

causing a reduction in the primary reactance. This causes an increased current to flow in the primary until a state of equilibrium is attained, when the flux will reach its original value. The secondary ampere-turns must be counterbalanced by an equal and opposite number of ampere-turns in the primary, so that the total primary current will be the vector sum of the no-load current and the additional current required to counterbalance the current in the secondary. Neglecting the effect of the no-load current, it is seen that the ampere-turns of both primary and secondary must be equal, and consequently the currents must be in the inverse ratio of the turns, or, since

$$I_1T_1 = I_2T_2,$$

therefore

$$\frac{I_1}{I_2} = \frac{T_2}{T_1}.$$

This can also be seen from considerations of the conservation of energy, for, since the power factor is unity and there are no losses,

$$E_1I_1 = E_2I_2$$

and

$$\frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{T_2}{T_1}.$$

Effect of Lagging Current in Secondary.—If the secondary winding be closed through a partially inductive resistance, the current will lag behind the secondary E.M.F. by some definite angle, and in order that the ampere-turns thus set up shall be counterbalanced at every instant, it is necessary that the corresponding primary ampere-turns shall have a phase exactly opposite to them. This is apparent when it is considered that for two sine waves to neutralise each other completely at every instant they must be exactly equal and opposite in phase. The effect of a lagging current in the secondary is, therefore, to cause a lagging current to flow in the primary. The actual angle of lag in the primary will usually be greater than that in the secondary, since the no-load current itself has a very large angle of lag, thus tending to make the power factor of the primary a little less than that of the secondary.

In the case of a leading current being taken from the secondary, the primary current will consist of a component leading the primary voltage by the same amount together with the no-load current lagging by an angle approaching to 90°. A certain amount of resonance will therefore be set up in the primary, with the result that the power factor is slightly higher than that in the secondary.

Effect of Ohmic Resistance.—A transformer with its secondary on open circuit may be likened to a choking coil, the applied voltage being divided into two components, overcoming the resistance and reactance respectively. The same thing occurs when the transformer is giving out a secondary current. Part of the primary applied voltage is absorbed by the *IR* drop in the winding, the

remaining component (obtained by vectorial subtraction) producing the flux which generates the voltage in the secondary. When the secondary is on open circuit, the primary current is very small and there is practically no difference between the applied voltage and the voltage producing the flux, particularly since the IR drop, small in itself, is almost in quadrature with the applied voltage as the current lags by nearly 90°. When a current is taken from the secondary, the primary current goes up and the IR drop is increased, so that it may no longer be negligible and must, consequently, be taken into account. But the flux is proportional to the voltage remaining after the IR drop has been allowed for (see p. 84), and, consequently, falls very slightly as the load on the secondary goes up. Neglecting this slight variation, the flux remains constant for all values of the load on the transformer.

In a similar way, due to the ohmic resistance of the secondary winding, the secondary terminal voltage is rather less than the total induced secondary E.M.F. when it is delivering current, and hence the ratio of transformation does not remain strictly constant as the ratio of the turns, but varies somewhat, due to the resistances of the two windings and the currents flowing.

Magnetic Leakage.—The major portion of the magnetic flux produced by the primary current passes through the iron core, but since the air also has a definite permeability, although very much less than that of the iron used, a certain number of lines of force will traverse an air path as indicated in Fig. 153. These air paths are in parallel with the iron paths, but whereas the lines of force traversing the iron cut the secondary winding, those following air paths serve no useful purpose and form a *primary leakage flux*, in contradistinction to the *main flux* following an all iron path and linking with both windings. In a similar way, the secondary current tries to set up a back flux opposing the existing main flux, but the primary automatically takes a larger current to provide sufficient ampere-turns to overcome this effect as far as the main flux is concerned. Notwithstanding this, a certain number of lines of force are set up by the secondary winding following leakage air paths for the most part, and these lines of force constitute what is known as the *secondary leakage flux.*

FIG. 153.—Main and Leakage Fluxes in Transformer.

The effect of the primary leakage flux is to add a certain amount of reactance to the primary winding, which serves no useful purpose and uses up a certain amount of the primary applied voltage in the same way that the resistance of the primary winding does.

The secondary leakage flux acts in much the same manner and

has the effect of diverting more and more of the main flux into leakage paths as the current is increased, for inside the secondary winding the leakage flux is in exact opposition to the main flux. As the currents in the two windings increase, the difference in the ampere-turns of the two windings does not alter very much, the main flux actually decreasing slightly. The leakage fluxes are, however, practically proportional to the currents in the respective windings, and so the total leakage of the transformer increases as the load goes up until on very heavy overloads the majority of the flux has been diverted into leakage paths.

The effect of magnetic leakage upon the ratio of transformation is to reduce the secondary terminal voltage for a given primary applied voltage.

Equivalent Circuits.—A commercial transformer may be represented, for purposes of explanation, as consisting of an ideally perfect transformer having no losses or magnetising current, together with various additions to allow for these effects. In Fig. 154 is shown such an ideal transformer having a resistance, R_1, and an inductance, L_1, in series with the primary winding, representing the

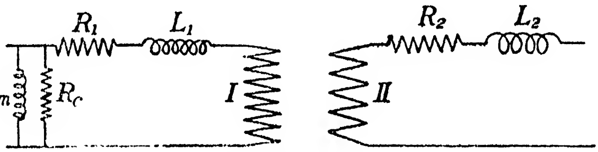


FIG. 154.—Equivalent Transformer Circuits.

resistance and inductance due to leakage of the primary respectively. Another resistance, R_c, and an inductance, L_m, are shown connected in parallel with the primary. The resistance, R_c, is such that when connected to the supply voltage the power absorbed is equal to the core loss in the iron consisting of hysteresis and eddy currents, whilst the inductance, L_m, is such that it takes a purely idle lagging current equal to the magnetising current of the transformer. Further, a resistance, R_2, and an inductance, L_2, are shown in series with the secondary circuit, to account for the resistance and leakage reactance of the secondary winding respectively.

If desired, the resistance and reactance of the secondary can be combined with those of the primary to form one resistance and one reactance. This combination does not consist of simple addition, but necessitates taking into consideration the ratio of transformation. For example, in the case of a step down transformer the impedance of the secondary is made less than that of the primary, because it deals with larger currents, and it must be multiplied by the ratio of transformation in order to refer it to the primary side.

In order to determine the equivalent impedance of a transformer, the secondary is short-circuited and a low voltage applied to the primary. The ratio of the applied voltage to the primary current gives the equivalent impedance referred to the primary side. In order to obtain the equivalent impedance referred to the secondary side, the primary is short-circuited and the voltage is applied to the secondary winding.

Vector Diagrams.—In drawing the vector diagram of a transformer, it is convenient to start with the flux vector, Φ, as being the connecting link between primary and secondary, and for ease of illustration it is convenient to choose a transformer having a ratio of transformation of 1 : 1. Neglecting the resistance and reactance of the two windings, the primary applied voltage, V_P, is exactly the same as the voltage overcoming the induced back E.M.F. in the primary winding, E_1 (see Fig. 155), and leads the flux by 90°. The induced secondary E.M.F., E_2, is 180° out of phase with E_1, and is the same as the secondary terminal voltage, V_S. On no-load, the primary current will consist of a very small power

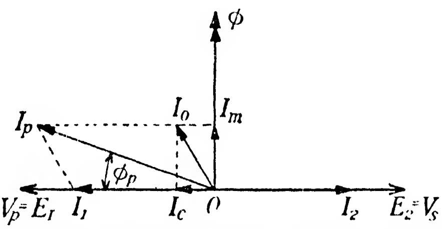


FIG. 155.—Vector Diagram neglecting Resistance and Reactance.

component, I_C, necessary to supply the core loss, consisting of hysteresis and eddy currents, together with a relatively much larger idle component, I_m, necessary for magnetising the core. These two combined form the total no-load current, I_O, this again being small compared with the full load current of the transformer. Assuming that the secondary is delivering current to a non-inductive resistance, the secondary current, I_2, will be in phase with V_S and will cause a corresponding current, I_1, to flow in the primary. Combining this with the no-load current, the total primary current, I_P, is obtained, the angle of lag being ϕ_P. It is thus seen that the primary is operating with a power factor slightly less than unity, although the secondary power factor is exactly unity.

Very frequently in practice transformers are made with a ratio of transformation of the order of 50 or 100 to 1, and if both the primary and secondary sides of the vector diagram were drawn to the same scale, either one would be cumbrously large or the other impracticably small. In order to make both sides of the diagram of approximately equal size, the usual convention is to draw all

voltages on the secondary side to a scale n times that of the voltages on the primary side, n being the ratio of transformation, the reverse being the case with all the current vectors.

Taking into consideration the resistances of the two windings, but not the leakage reactances, the diagram appears in the form shown in Fig. 156, which illustrates the case where the secondary load is non-inductive. The induced secondary voltage, E_2, is still 90° behind the flux, and in phase with E_2 is the secondary current, I_2. The voltage, I_2R_2, is that absorbed by the resistance of the

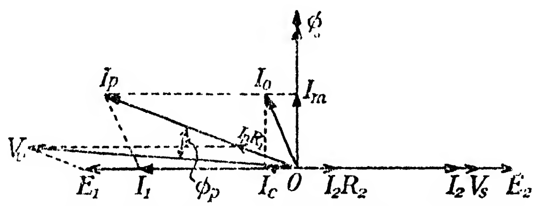


Fig. 156.—Vector Diagram considering Resistance only.

secondary winding, and consequently the secondary terminal voltage, V_S, is the vector difference of E_2 and I_2R_2. The primary current, I_P, is made up as before of I_1 and I_O, but the voltage drop due to the primary resistance, I_PR_1, must be in phase with I_P, so that the primary applied voltage, V_P, must be the vector sum of E_1 and I_PR_1. The angle of lag, ϕ_P, is slightly less than it was previously, and the primary applied voltage, V_P, is no longer exactly 180° out of phase with the secondary terminal voltage, V_S.

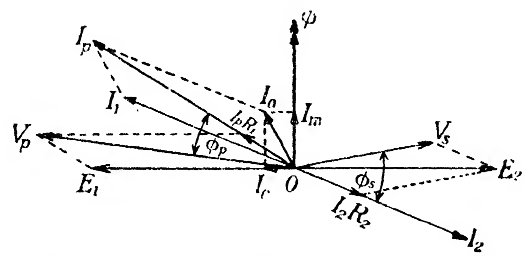


Fig. 157.—Vector Diagram for Lagging Current. Reactance neglected.

Assuming a lagging secondary current and again neglecting the reactance in both windings, the diagram takes the form shown in Fig. 157. Here it is seen that not only is V_S different from E_2 in magnitude, but also in phase, the power factor of the secondary being cos ϕ_S. The primary current, I_P, is, as before, built up of I_1 and I_O, and the primary power factor is given by cos ϕ_P. This time neither V_P nor V_S is in exact quadrature with the flux Φ.

The next case to be considered is the one where both resistance and leakage reactance are taken into account, dealing first with a

non-inductive load. The vector diagram is shown in Fig. 158. Although the secondary load power factor is unity, the secondary terminal voltage, V_S, is not in phase with E_2, for the reason that there is a certain amount of reactance in the winding itself, and consequently the power factor of the complete secondary circuit is less than unity, notwithstanding the fact that the external circuit is non-inductive. The voltage drop in the secondary winding consists of a power component, I_2R_2, to overcome the resistance, and an idle component, I_2X_2, to overcome the leakage reactance. The two combined form the voltage I_2Z_2, overcoming the impedance of the winding, and I_2Z_2 and V_S added vectorially give the total E.M.F., E_2, generated in the secondary. Since I_2R_2 is in phase with I_2, and this phase is not known when commencing to draw the vector diagram, the position of the lines must be found by trial and error, so that I_2 is coincident in phase with V_S. The primary

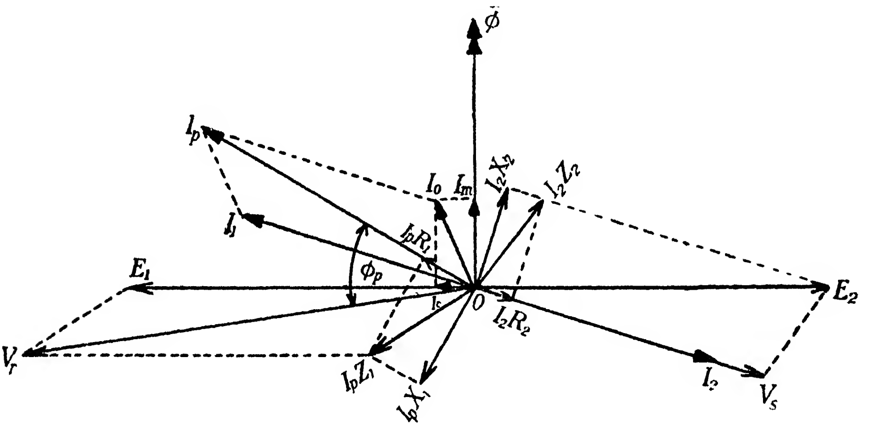


FIG. 158.—Transformer Vector Diagram. Non-inductive Load.

current, I_P, is found in the same way as previously, after which the resistance and reactance drops, I_PR_1 and I_PX_1, can be marked off along I_P and at right angles to it respectively. In this way, the primary impedance drop, I_PZ_1, can be determined, and adding this vectorially to E_1 the primary applied voltage, V_P, is obtained. The primary power factor, cos ϕ_P, is given by the cosine of the angle between V_P and I_P.

As the load increases, both the phase and magnitude of V_S will alter slightly, due to the increase of I_2Z_2; so also will the phase and magnitude of V_P. But the conditions under which a transformer works in practice are that the primary applied voltage is constant, and not the flux, as has been assumed in the above diagram. Since the change is but a small one, the necessary correction can be made by choosing a new scale of voltage for each load, obtained by making V_P represent the constant applied voltage

in each case. This change will not affect the current vectors, but will affect all the voltage vectors and the flux vector proportionally. Thus the flux is seen to decrease slightly as the load goes up, and also to lag a little more than 90° behind the primary applied voltage.

The conditions arising when the load is partially inductive will next be dealt with, Fig. 159 showing the vector diagram, both resistance and reactance being taken into account. This diagram is very similar to the previous one, the secondary power factor being given by cos ϕ_S. One effect of the lag in the secondary circuit is to bring the two currents more nearly into phase opposition and to increase those components of I_PZ_1 and I_2Z_2 which are in phase with V_P and E_2 respectively. This causes a reduction in the secondary terminal voltage as the secondary power factor is decreased, other things being equal, and increases the ratio of

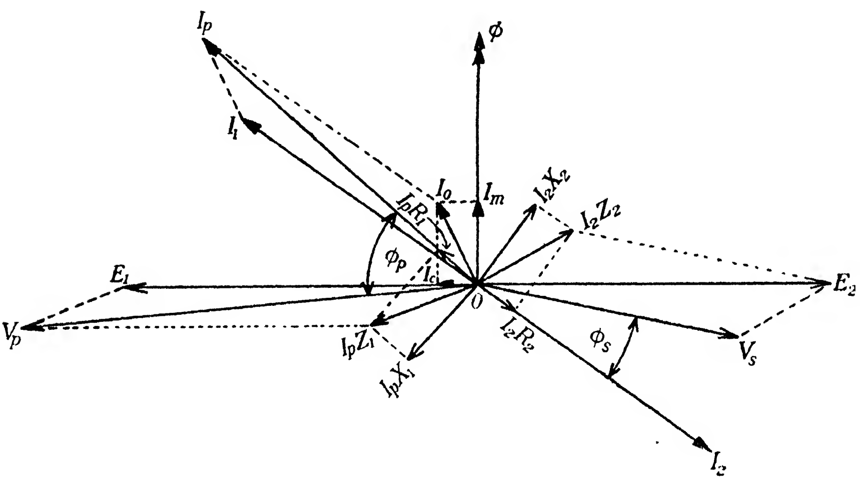


FIG. 159.—Transformer Vector Diagram. Inductive Load.

transformation, this effect being largely due to the presence of magnetic leakage. The primary power factor is always less than that of the secondary, except when it is less than that on no-load, which is not likely to occur in practice. Another point of interest with respect to magnetic leakage is that neither the primary nor the secondary leakage flux is in phase with the main flux, since the leakage fluxes are in phase with the primary and secondary currents which produce them.

The vector diagram for the case where the secondary current is a leading one is shown in Fig. 160. The same construction is adopted as before, but this diagram differs from the previous one inasmuch as the power factor of the primary is higher than that of the secondary, due to the no-load magnetising current neutralising a portion of the capacity current required to balance the secondary.

In fact, for one particular value of the secondary current the primary power factor would be unity, although the secondary current is a leading one. For lower values of the secondary current the primary current would lag, whilst that in the secondary leads.

Another point of note is the fact that the ratio of transformation is much more nearly constant for varying values of the load than was the case when the load was inductive. This is readily understood, because the arithmetical difference between V_P and E_1 and between E_2 and V_S is obviously less in Fig. 160 than in Fig. 159.

Safety Devices—Earthed Shields.—Since the majority of power transformers are used in connection with voltages which are highly dangerous, it is necessary to adopt certain measures for ensuring the safety of the operator, not only for the normal working of the transformer, but also for its abnormal working. Transformers are usually

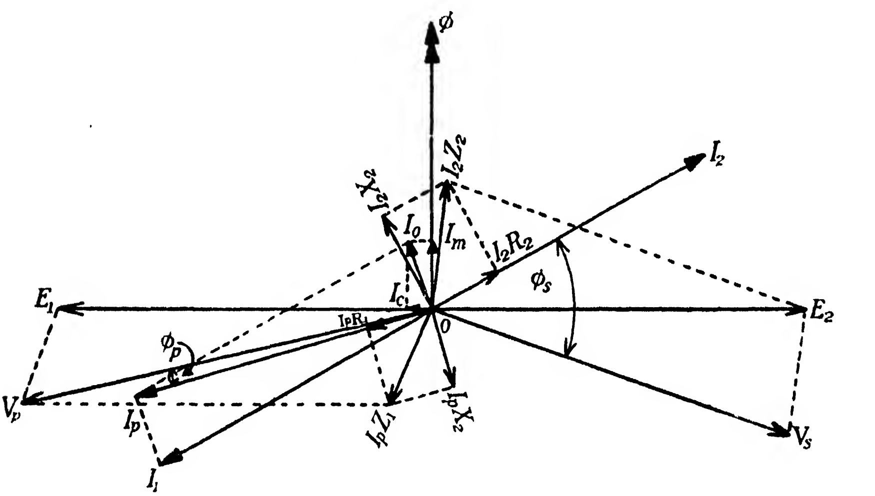
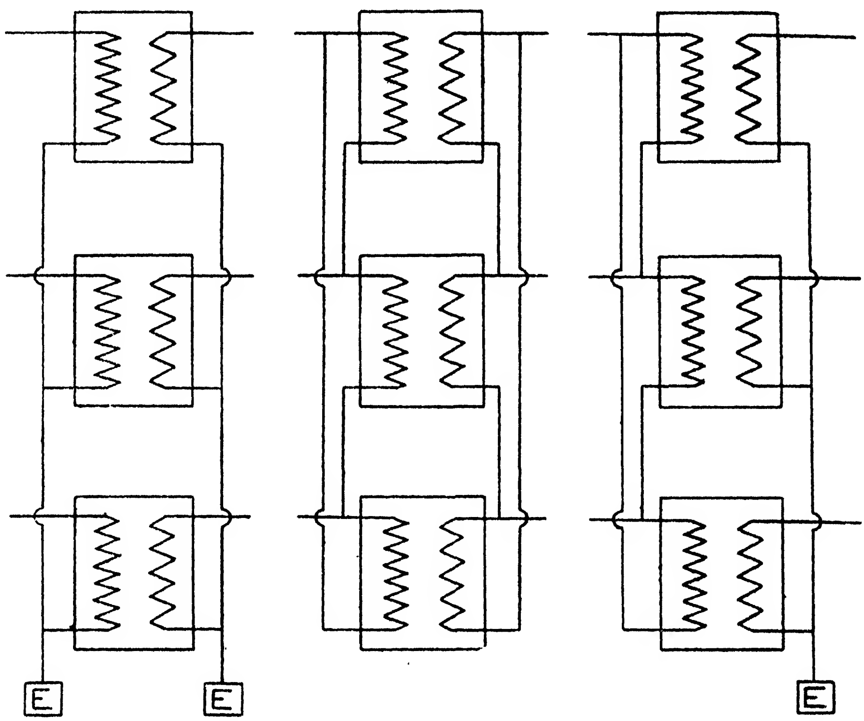


FIG. 160.—Transformer Vector Diagram. Capacity Load.

used for stepping down from a high to a low voltage, the secondary side being connected to the distributing network, and very often the transformer itself is protected from breakdown to earth by being immersed in an insulating oil-bath, the whole being enclosed in a cast iron case. But, in addition to this, it is necessary that any accidental electrical connection between primary and secondary should be avoided, as this may result in a high potential being given to the secondary, which may make its presence felt with disastrous results at any point on the system. High insulation between the windings is therefore adopted as well as high insulation to earth, but if a fault should occur in this the danger is as great as before. To nullify the effect of such a fault, an *earthed shield* is sometimes used. This consists of a layer of copper gauze or thin brass sheet placed between the two insulated windings, this shield being connected to

the transformer case, which is earthed. If a fault on the H.T. (high tension) side should occur, the primary would be earthed before it could do any damage to the secondary, thus calling into play the fuses or circuit breakers on the primary side and cutting out the faulty transformer. But the primary and secondary windings, being separated by a dielectric, constitute a kind of high potential condenser, with the result that electrostatic phenomena may be produced in the secondary, this effect being in no way due to insufficient insulation. This difficulty is overcome by earthing the secondary winding either on one pole or in the middle, and this



Star to Star
Ratio of Transformation $= n$.
(*a*)

Mesh to Mesh
Ratio of Transformation $= n$.
(*b*)

Mesh to Star
Ratio of Transformation $= \frac{n}{\sqrt{3}}$
(*c*)

FIG. 161.—Single Phase Transformers on Three Phase Circuits.

practice removes the necessity of the earthed shield, which weakens the insulation to some extent. On these grounds, the efficient earthing of the secondary winding is preferable to the use of earthed shields and is now largely adopted.

Polyphase Transformers.—Two phase currents can be transformed by means of two similar single phase transformers, the secondaries being independent or interconnected according to choice. Similarly, three phase currents can be transformed by means of three similar single phase transformers, the secondaries either being

independent or connected in star or mesh. It is also possible to connect the primaries in mesh and the secondaries in star, and *vice versa*, the ratio of transformation of the line voltages being, of course, affected by the connections. Fig. 161 (*a*), (*b*), and (*c*) illustrates some of the methods of connection.

If the three primary windings were wound on the same simple core which has been already described, a single phase alternating flux would be produced and the three secondary windings would have equal voltages of the same phase induced in them, so that only a single phase supply could be obtained from their combination. But by adopting a three-limbed core of the type shown in Fig. 162, and winding each phase on to a separate limb, it is possible to build one transformer which will transform three phase currents by itself. The phase of the flux bears the same relationship to that of the voltage in each of the three phases, and consequently the sum total of the flux either upwards or downwards is zero at every instant (see Chap. X). In other words, the flux in any limb is always the sum of the fluxes in the opposite direction in the other two limbs. By winding a secondary coil on to each limb, a three phase supply can be obtained, for in each limb the secondary E.M.F. will lag by 90° behind its respective flux, which lags by 90° behind the impressed voltage.

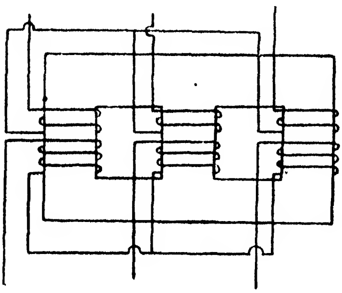


FIG. 162.—Three Phase Transformer.

Single Phase Core Construction.—The simple type of transformer in which a laminated iron core is surrounded by copper coils is called the *Core Type*. In the case of small transformers, the iron core is built up of two shapes of stampings, one set being ⊔-shaped and the other consisting of rectangular strips to close the iron circuit. The thickness of these stampings usually ranges from 0·3 to 0·5 mm. A bundle of stampings is clamped together by bolts or rivets, the latter being insulated from the core in order to reduce the eddy currents. Since it is difficult to make a good magnetic butt joint without introducing additional eddy currents, the plates are sometimes dovetailed into each other, alternate stampings being cut long and short for this purpose. For larger transformers the stamping of the ⊔-shaped pieces involves a considerable waste of material, and the practice is to build up such cores from four bundles of rectangular strips, the thickness of the bundles all being equal (see Fig. 163). These are bolted together between end cheeks at the top and bottom, the latter also being held tightly in position by means of tie rods.

Cross Section of Core.—Since the length of the turn does not influence its flux-producing properties, it is advisable to reduce it

to a minimum, both from the point of view of its resistance and also the amount of material used. But with a fixed magnetic density, the cross section of the iron is fixed and the problem resolves itself into finding a figure which has the minimum periphery for a given cross-sectional area. The ideal figure is the circle, but this involves

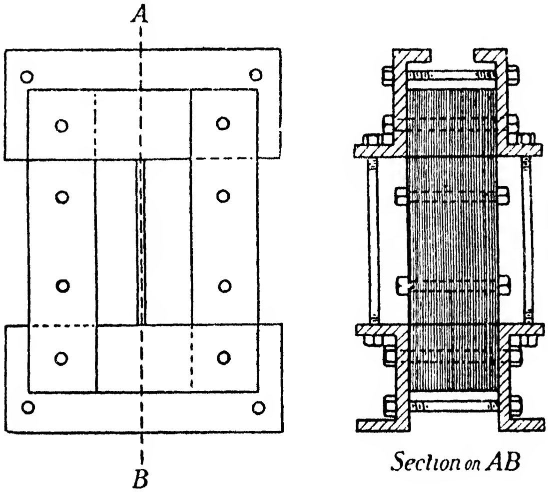


FIG. 163.—Construction of Core Type Transformer.

each stamping having a different width, which is impracticable. By having two or three sizes of stampings, however, a good approximation to the circle can be obtained, the coil itself being made circular, since it does not follow exactly the outline of the core. Figs. 164 and 165 show cross sections of two cores built up of two

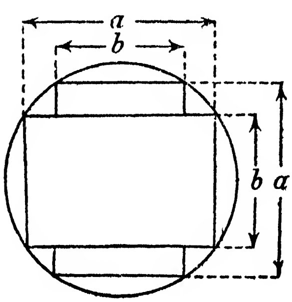


FIG. 164.—Cross Section of Core. Two Sizes of Stampings.

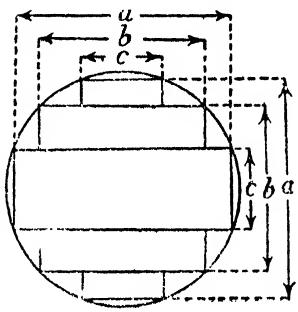


FIG 165.—Cross Section of Core. Three Sizes of Stampings.

and three sizes respectively. The exact relationship between the various dimensions can be worked out mathematically, close approximations being adopted from practical considerations. The presence of vent spaces also affects the ideal dimensions to a slight extent, but if the sizes of these are given the problem is easily

soluble. When no vent spaces are employed, the quantities are connected together as follows :—

Two Sizes of Stampings.

$$a = 1{\cdot}09 \times \sqrt{\text{Area}}$$
$$b = 0{\cdot}67 \times \sqrt{\text{Area}}$$
$$\text{diameter} = 1{\cdot}28 \times \sqrt{\text{Area}}.$$

Three Sizes of Stampings.

$$a = 1{\cdot}11 \times \sqrt{\text{Area}}$$
$$b = 0{\cdot}87 \times \sqrt{\text{Area}}$$
$$c = 0{\cdot}52 \times \sqrt{\text{Area}}$$
$$\text{diameter} = 1{\cdot}22 \times \sqrt{\text{Area}}.\text{[1]}$$

Single Phase Shell Construction.—The core type of transformer already discussed consists of an iron core around which copper coils are wound. The same effect is produced if an iron core is wound round the copper coil, this being the characteristic feature of the shell type transformer. Fundamentally there is no difference between the two types, since the principle involved in each is the

[1] As an example, the relation between a and b for Fig. 164 is worked out as follows :—

Diameter of circle $= d = (a^2 + b^2)^{\frac{1}{2}} =$ constant.

$$\therefore \quad b = (d^2 - a^2)^{\frac{1}{2}}.$$

Total area $= A = ab + 2\frac{a-b}{2}b$

$$= 2ab - b^2$$
$$= 2a(d^2 - a^2)^{\frac{1}{2}} - d^2 + a^2.$$

For maximum area,

$$\frac{dA}{da} = -2a \times \tfrac{1}{2}(d^2 - a^2)^{-\frac{1}{2}} \times 2a + 2(d^2 - a^2)^{\frac{1}{2}} + 2a = 0.$$

Substituting b for $(d^2 - a^2)^{\frac{1}{2}}$,

$$-2\frac{a^2}{b} + 2b + 2a = 0.$$

Rearranging and multiplying by $-\frac{b}{2}$,

$$a^2 - ab - b^2 = 0.$$
$$\therefore \quad a = \frac{b \pm \sqrt{b^2 + 4b^2}}{2}$$
$$= \frac{b}{2} \pm \frac{\sqrt{5}}{2}b = 1{\cdot}62b,$$

since the $-$ sign is inadmissible.

Total area $= 3{\cdot}24b^2 - b^2 = 2{\cdot}24b^2.$

$$\therefore \quad b = 0{\cdot}67 \times \sqrt{\text{area}} \text{ and } a = 1{\cdot}09 \times \sqrt{\text{area}}.$$

linkage of flux with ampere-turns. The appearance of the two types, however, differs considerably, since, in the shell type transformer, the coils are more or less embedded in the iron, which serves also as a mechanical protection.

A common example of the shell type transformer is the one having a three-limbed core shown in Fig. 166. The iron section consists of two E-shaped stampings facing each other so as to form two square holes, through which the coils, both primary and secondary, are threaded. By employing two stampings it is possible to adopt former wound coils, the stampings being cut alternately long and short in order to minimise the effect of the air gap at the joint. There are only three magnetic joints in this form of construction, as opposed to four in the simple core type transformer built up of four rectangular strips, and since the flux is divided outside the central limb, the section of the iron in these parts need only be half that of the central limb.

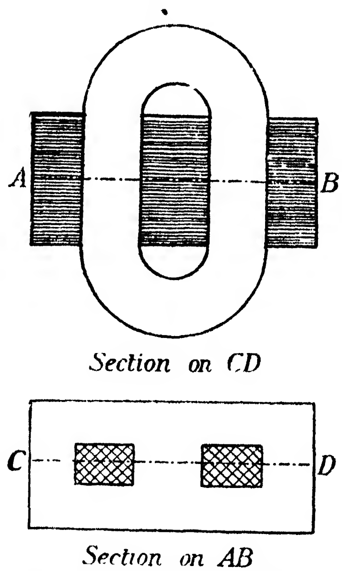


FIG. 166.—Shell Type Transformer.

For large transformers, the core is

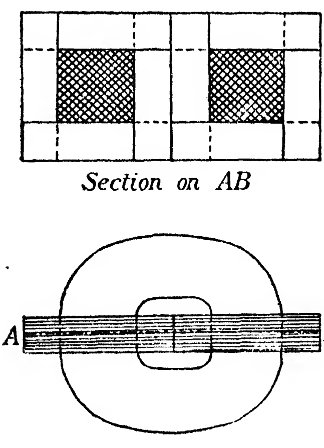


FIG. 167.—Shell Type Transformer.

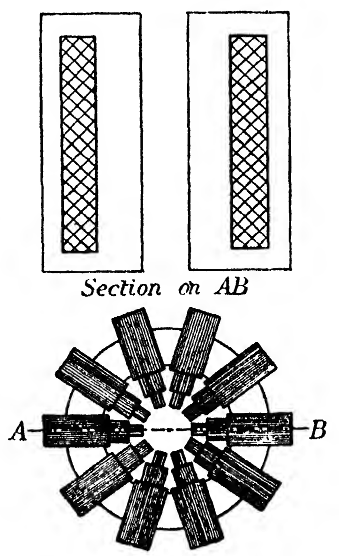


FIG. 168.—Shell Type Transformer with Circular Coil.

often built up from two simple rectangular cores placed side by side (see Fig. 167), the coils being slipped into position before the top cross pieces are added.

Another example of shell construction is that shown in Fig. 168,

where a circular coil, including both primary and secondary, is linked with bundles of stampings which are placed all the way round. Each bundle consists of an upright rectangular strip inside the coil, the magnetic circuit being completed by a ⊔-shaped piece. From the nature of the construction, the stampings leave air spaces between them at their outer edges, whilst they are crowded together in the interior. This ensures a large cooling surface both for the copper and the iron.

Shell type transformers usually provide a shorter magnetic path, and hence the magnetising current is usually less than in the corresponding core type transformer. Also the amount of copper required is less, but, due to the embedding of the coils, the natural cooling is poorer. A point against the shell type transformer is the necessity for dismantling the laminations when the coils have to be withdrawn for repairs, this not being necessary in the core type transformer.

Three Phase Construction.—Theoretically the simplest three phase construction is to have three upright limbs set in a triangle and joined at the top and bottom by a number of plate stampings, as shown in Fig. 169. It is also possible to have the three limbs arranged in a line, as in Fig. 162, without introducing any serious asymmetry.

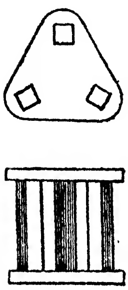
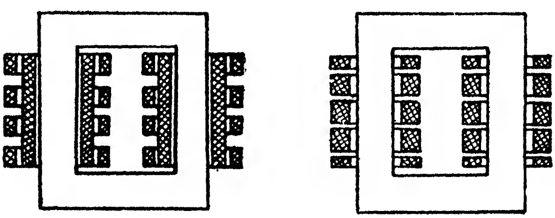


FIG. 169. Three Phase Transformer Construction.

Types of Winding.—It is not the practice to wind one limb with the primary and the other with the secondary, although transformers are shown diagrammatically in this manner for the sake of clearness. On the contrary, a portion of each winding is placed on each wound limb, the L.T. (low tension) side being nearest the core. This only necessitates high grade insulation on the primary (H.T.) side and between primary and secondary, whereas if the H.T. winding were nearest the core there would have to be high grade insulation on the primary itself, between primary and core and between primary and secondary. In addition, the H.T. winding is



(a) (b)

FIG. 170.—Arrangement of Windings.

divided into sections as shown in Fig. 170 (*a*), in order to limit the voltage between adjacent turns.

Another method of winding is to subdivide both primary and secondary and sandwich the sections, as shown in Fig. 170 (*b*). This arrangement requires more insulation than the previous one and is not recommended for voltages above 6,600.

Insulation of H.T. Transformers.—In addition to the high grade insulation on the H.T. windings themselves, it is necessary to guard against a breakdown to earth. A fruitful source of trouble is experienced at the leading-in wires, and to protect these special insulating bushes are adopted. For this purpose frequent use is made of porcelain insulators built up of a number of concentric tubes, each one a little shorter than the next innermost one, as shown in Fig. 171. The outermost tube, which is the shortest of all, is carried by a bush fitting into a hole in the transformer case.

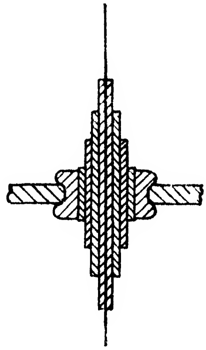


FIG. 171.—Section of H.T. Terminal Bush for Transformer.

Most medium and large sized transformers are immersed in oil, the whole being contained in a cast iron tank. This oil serves a double purpose. It provides a better insulation than is obtained by air and incidentally tends to preserve the insulation on the windings, and, in addition, it acts as a cooling medium. The oil conducts heat better than air and transfers it more readily to the walls of the tank, whence it is dissipated into the atmosphere. The oil which is used is obtained by the fractional distillation of petroleum, and should not have a flash-point of less than 180° C. It is very important that no moisture should be present, as this affects its insulating properties to an enormous extent.

Mechanical Stresses on Windings.—When adjacent primary and secondary coils carry currents, these are opposite in direction, and the two coils consequently exert a repelling force on each other. With normal currents, this effect is not of much account; but in the event of a momentary short circuit on the secondary it may be serious To prevent any movement of the coils, therefore, they are usually braced in position so as to ensure mechanical rigidity. To reduce the magnitude of the mechanical stress set up by a short circuit on a large transformer, the coils are usually subdivided.

Electrical Stresses on End Turns.—When the voltage is first applied to the primary of a transformer, the end turns have to be charged up to the line potential before any current can flow into the remainder of the winding, and since the two extreme turns are separated by a dielectric from the earth and from each other, they constitute a kind of condenser. These extreme turns take, consequently, a small but definite charging current, and then the next turns receive the voltage and are charged up in the same way. All this occurs very rapidly, but the effect is to concentrate the

voltage on to the end turns at the moment of switching on. To avoid possible breakdowns of the insulation, the practice is to put extra insulation on the end turns to enable them to withstand the extra stress to which they are subjected at the moment of switching on.

Artificial Cooling.—For small transformers natural cooling is sufficient, but for the larger sizes artificial means are employed to assist in the dissipation of the heat, since there is no rotating element to set up ventilation. Oil insulation is better than air in this respect, since the oil is a better conductor of heat, but in large transformers the oil itself is sometimes cooled by means of a worm through which a continual stream of cold water is passed. This water is forced through the circulating pipes by means of a small auxiliary motor-driven pump, this system being frequently adopted when a number of large transformers are working together. Another method of cooling consists of an air-blast, which is forced through the windings and ventilating ducts of the core by means of a small air-pump. Air-blast cooling is also used in connection with oil insulated transformers, a forced draught of air being maintained in a number of cooling tubes immersed in the oil.

Polyphase Transformer Connections.—The connections of two phase transformers or two single phase transformers used on a two phase supply are quite simple, each phase being connected up independently and the system linked or not according to desire. With three phase transformers, however, a number of methods of connection are possible, these methods also being applicable when three single phase transformers are used. The several methods employed are as follows :—

(*a*) *Star or Y Connection.*—Both primaries and secondaries are connected in star [see Fig. 172 (*a*)], but if one phase should fail it puts two phases out of action, which practically means a complete shut down of the transformer.

(*b*) *Mesh or* Δ *Connection.*—Both primaries and secondaries are connected in mesh [see Fig. 172 (*b*)], but if one phase winding should fail the other two would continue to supply a true three phase current, although the system would become unbalanced. In this respect the mesh connection is preferable to the star. The new system is called the **V**-connection or open Δ.

(*c*) *V or open* Δ *Connection.*—Only two transformers are needed [see Fig. 172 (*c*)], but since the phase difference between the two secondaries is the same as that between the two primaries, the three phase supply is maintained. The current in the common wire is the vector sum of the currents in the other two, so that the system is unbalanced, but the two transformers should, of course, be similar.

(*d*) *T-Connection.*—Only two transformers are needed in this system of connection [see Fig. 172 (*d*)], but the voltages which are

applied to their terminals are slightly different, that transformer which is connected to the middle point of the other only being supplied with $\frac{\sqrt{3}}{2} = 0{\cdot}866$ times the voltage between line wires. The ratio of transformation of both transformers is the same, so

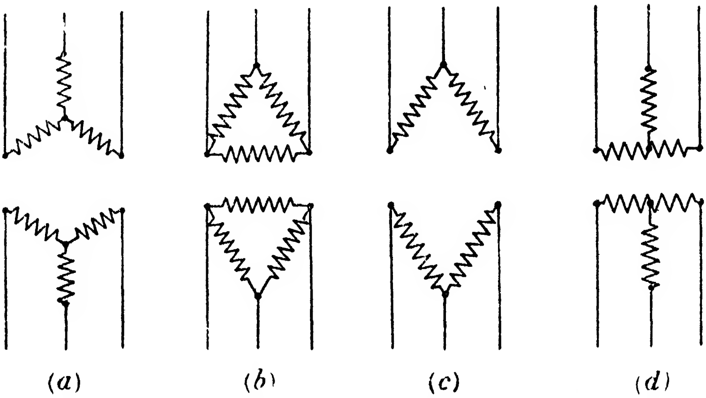


FIG. 172.—Three Phase Transformer Connections.

that a true three phase supply is obtained at the free terminals of the secondaries.

Scott System of Transformation.—The Scott system of transformation is a method whereby the supply may be changed from three phase to two phase, or *vice versa*. Two transformers have their primaries T-connected on to the three phase supply, whilst their secondaries are connected independently and deliver a true two

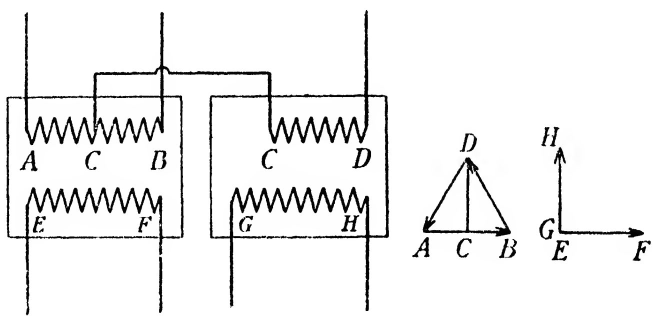


FIG. 173.—Scott System of Transformation.

phase supply (see Fig. 173). The primaries of these two transformers are wound for voltages in the ratio of $1 : \frac{\sqrt{3}}{2}$, since the primary of the second one only receives this fraction of the line voltage. As is seen from the vector diagram, *CD* is 90° out of phase with *AB*, and consequently *GH* is 90° out of phase with

EF. If the ratio of transformation were the same in the two transformers, however, the voltage of the second phase would be too low, and so the voltage given by the secondary of the second transformer is increased relatively to that of the first one in the ratio $\frac{2}{\sqrt{3}}$. This equalises the voltages, which are already 90° out of phase with each other.

Auto-Transformers.—In an ordinary transformer there is no electrical connection between primary and secondary, but since the volts per turn are the same in each case there is no fundamental reason why the two windings should not lie side by side without any intervening insulation. Carrying this idea further, the same wire might be used for carrying the two currents. Since it is presumed that one winding has more turns than the other, this superposition only exists in that part of the winding which is common to both primary and secondary. Fig. 174 illustrates in diagrammatic form the principle of the auto-transformer, the

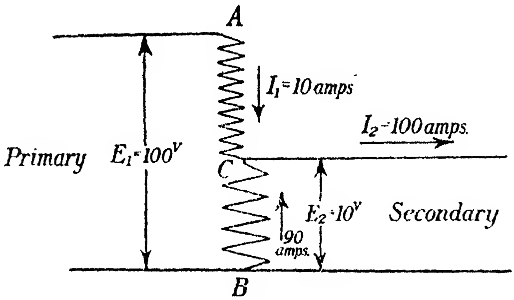


FIG. 174.—Auto-Transformer.

voltage being reduced in this instance, although the reverse is also possible. Since the primary and secondary currents act in opposition, the resultant current in *BC* is the vector sum of the two, this being very nearly the arithmetic difference. Thus suppose the primary voltage and current were 100 volts and 10 amperes respectively and the secondary voltage and current 10 volts and 100 amperes respectively, *AC* would carry 10 amperes whilst *BC* would carry $100 - 10 = 90$ amperes.

The auto-transformer is not merely a potential divider, although it does act in this capacity, but there is true transformer action, for the secondary current exceeds the primary when the ratio of transformation is greater than unity.

There is a distinct saving in copper in an auto-transformer as compared with an ordinary one, which can be shown as follows :—

Let n be the ratio of transformation and

$$E_1 = nE_2, \quad T_1 = nT_2, \text{ and } I_2 = nI_1 \text{ (see Fig. 174).}$$

Current in common portion of winding $= I_2 - I_1 = I_1(n-1)$.

Assuming a constant current density, δ, and a constant length of turn, l, in both auto- and ordinary transformer, the volume of copper in the ordinary transformer would be

$$\frac{I_1}{\delta} \times lT_1 + \frac{I_2}{\delta} \times lT_2$$

$$= \frac{I_1}{\delta} \times lnT_2 + \frac{nI_1}{\delta} \times lT_2$$

$$= 2\frac{I_1}{\delta} \times lnT_2.$$

The volume of copper necessary in the auto-transformer is

$$\frac{I_1}{\delta} l (T_1 - T_2) + \frac{I_2 - I_1}{\delta} lT_2$$

$$= \frac{I_1}{\delta} l (n-1) T_2 + \frac{(n-1) I_1}{\delta} lT_2$$

$$= 2\frac{I_1}{\delta} l (n-1) T_2.$$

Therefore

$$\frac{\text{Volume of copper in auto-transformer}}{\text{Volume of copper in ordinary transformer}} = \frac{2\frac{I_1}{\delta} l (n-1) T_2}{2\frac{I_1}{\delta} lnT_2}$$

$$= \frac{n-1}{n}.$$

If n be given the values 2, 3 and 4, the resultant saving in copper works out at 50 per cent., 33⅓ per cent. and 25 per cent. respectively, but if n is 50 the saving is only 2 per cent., the weight of the active iron being affected in much the same way. Thus the saving is only of importance in the cases where the ratio of transformation is low.

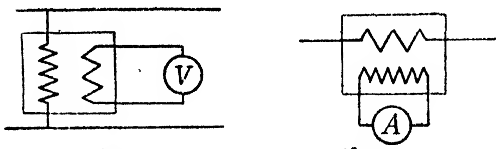
Another great disadvantage of the auto-transformer is that there is a direct electrical connection between the primary and secondary. If the primary is supplied at H.T., there is a possibility of a dangerous voltage reaching the secondary circuit in the event of a fault occurring in the winding, and this precludes the use of an auto-transformer in such circumstances. Its usefulness is therefore limited to cases where the voltages and ratios of transformation are low.

One very useful application is as a starter for induction motors. A number of tappings are made on the winding so as to obtain a variable voltage on the secondary, these tappings being brought out to contacts after the manner of an ordinary C.C. motor starter. Such a piece of apparatus is termed a *Compensator*, for it allows

the motor to take an excess current without putting a heavy overload on the mains. In the majority of small compensators there is only one tapping corresponding to only one starting stop in addition to the full running position.

Potential and Current Transformers.—Small transformers are often used in conjunction with voltmeters for voltage measurements on H.T. lines. The primary is wound with many turns of fine wire, whilst the secondary is designed to give a low voltage which is applied to the terminals of the voltmeter [see Fig. 175 (*a*)], the scale being arranged in most cases to read the voltage on the primary side. Such a transformer is called a *potential transformer* and works under a small constant load due to the voltmeter itself. The ratio of transformation must be determined with the instrument connected in position, for the voltage drop is such that a considerable difference would be made if the voltmeter were removed or if another one of different impedance were substituted.

In a similar way, small instrument transformers, called *current transformers,* are used in conjunction with ammeters. The connec-



(*a*)
Potential Transformer

(*b*)
Current Transformer

FIG. 175.—Potential and Current Transformers.

tions of an ammeter with a current (or series) transformer are shown in Fig. 175 (*b*). In this case the primary current is determined by what current is flowing in the mains rather than by the impedance of the winding, and, with the secondary circuit closed, the secondary current is such that the primary and secondary ampere-turns balance each other, neglecting magnetic leakage. The currents are therefore approximately in the inverse ratio of the turns and the ammeter can be calibrated to read the primary current, provided that the same instrument is always used in conjunction with the same current transformer. This arrangement takes the place of the shunt in C.C. measurements and enables heavy currents to be measured without the trouble of designing instruments with heavy leads, since the instrument transformer, which is only small, can be placed in the path of the main lead, whilst the small secondary leads can be carried to the instrument, which may then be situated wherever convenient.

A.C. wattmeters may also be used in conjunction with both a potential and a current transformer.

Quadrature Transformers.—A quadrature transformer is one in

which the secondary current is 90° out of phase with the primary current instead of the usual 180°. They are used in making measurements with certain A.C. instruments, *e.g.* the Sumpner wattmeter, the connections being those of an ordinary current transformer. A peculiar construction is adopted, a long air-gap being included in the magnetic circuit to make it as leaky as possible. In fact a wholly non-magnetic core might be used, but this tends to increase the size and is not necessary. The secondary circuit has a high non-inductive resistance placed in series with it, so as to cause only a small secondary current to flow, this being in phase with the secondary E.M.F. Since the load on the transformer is low and the no-load ampere-turns are comparatively large, the primary current will lag by practically 90° behind the voltage across its terminals. The secondary current, which is in phase with the secondary voltage, will lag by practically 180° behind the primary voltage, and consequently by 90° behind the primary current.

Boosting Transformers.—Sometimes a transformer is used for boosting up the supply pressure, in which case the primary is connected across the mains, whilst the low voltage secondary is placed in series with the mains as in Fig. 176. A number of tappings are taken from different points along the secondary to the contacts of a multi-way switch, the handle of which is connected to the line. The switch blade is made in two insulated parts connected by a low resistance after the manner of a battery switch, so that in moving from one contact to another the circuit is never opened, nor is one section of the secondary momentarily short-circuited. In this way, a variable boost is obtained depending upon the position of the switch handle. Looked at from another point of view, such a transformer may be regarded as a step-up auto-transformer.

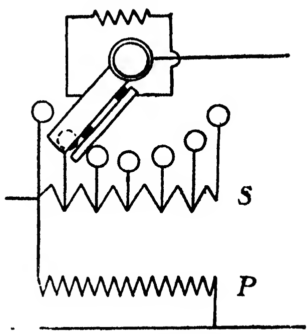


FIG. 176.—Boosting Transformer.

CHAPTER XIV

TRANSFORMERS.—PERFORMANCE AND TESTING

No-load Current.—The no-load current of a transformer consists of a small power component necessary on account of the iron loss in the core and a comparatively large idle component which supplies the magnetising ampere-turns. The total no-load current is the vector sum of these two components. The transformer may be supplied on either the high or low tension side, the two different no-load currents being in the inverse ratio of the number of turns on the two windings. Neglecting the small IR drop, the flux produced is proportional to the applied voltage, so that if the relation between magnetising current and voltage be plotted for various applied voltages, the resulting curve should have the same shape as the B—H curve for the iron. In order to avoid excessive iron losses and magnetising current, transformers are always worked below the knee of the magnetisation curve.

If the applied voltage be kept constant and the frequency varied, it is seen from the general equation that a high frequency corresponds to a low no-load current, for

$$\Phi = \frac{E_1 \times 10^8}{4{\cdot}44 f T_1}$$

(see p. 172). The flux is inversely proportional to the frequency, an increase in which means a reduction of the flux and consequently of the magnetising current.

The hysteresis loss is proportional to $fB^{1{\cdot}7}$, so that an increase in frequency causes $B^{1{\cdot}7}$ to decrease more rapidly than f increases. The hysteresis loss therefore decreases as the frequency is raised. The eddy current loss is proportional to f^2B^2, and the increase in one balances the decrease in the other, the loss being independent of frequency. Thus the total losses are reduced, and the power component of the current as well as the magnetising component decreases with an increase in frequency which enables a smaller core to be used in a transformer for a given duty resulting in a lowering of the first cost.

No-load Losses.—The no-load losses of a transformer consist of a

very small I^2R loss and an iron loss due to hysteresis and eddy currents. The latter can be determined in the way indicated in Chapter VIII by simply measuring the power input to the primary by means of a wattmeter. The iron losses can be separated by the method involving a constant flux obtained by varying the applied voltage and the frequency at the same rate (see p. 86). Ordinarily, the wattmeter reading includes the small I^2R loss, but if it is desired to take account of this, the connections shown in Fig. 177 can be adopted, the pressure coil of the wattmeter being connected across the secondary. The wattmeter reading multiplied by the ratio of transformation gives the iron loss only, for the voltage actually used for transformer action is only what is left after the small IR drop has been subtracted vectorially. The voltage required is therefore the secondary voltage multiplied by the ratio of transformation.

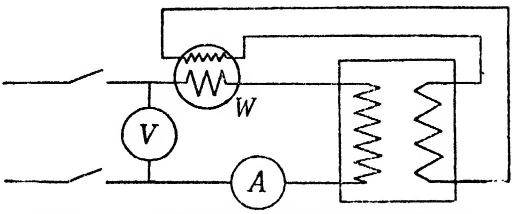


FIG. 177.—Connections for Measuring Iron Loss.

Effect of Wave Form on Iron Loss.—When the applied E.M.F. wave form is not sinusoidal the flux is given by

$$\Phi = \frac{E_1 \times 10^8}{4kfT_1},$$

where k is the form factor. Thus a high form factor causes a reduction in the flux and results in a decrease in the hysteresis loss, for the work done in carrying the iron through a complete magnetic cycle is independent of the rate at which this is done. At first sight it would appear as if the eddy current loss is also reduced, but this is not so, for although the maximum value of the flux is reduced, the slope of the E.M.F. curve must be increased in certain places, since the R.M.S. value is assumed to be unaltered. This tends to increase the eddy current loss by increasing the induced eddy voltage. The net result is that these two effects balance one another and leave the eddy current loss unaffected by wave form. The constancy of this loss can also be seen when it is considered that the R.M.S. value of the eddy voltage is dependent on the R.M.S. value of the applied voltage, which is unaltered.

The total iron loss is therefore slightly reduced when a peaked E.M.F. wave form is employed, since in such cases the form factor is greater than that of a sine wave. Alternatively, a flat-topped wave causes a slight increase in the iron loss.

The disadvantage of a peaked wave is that it imposes a greater strain on the insulation, and, as a compromise, the sinusoidal wave form is usually aimed at.

Abnormal Current Rushes when Switching on.—When a choking coil is connected to the supply, the current lags normally by nearly 90° behind the voltage, but at the moment of switching on the current must start from zero. Neglecting the power component of the current, this corresponds to the maximum value of the volt wave in normal conditions, and if the switch is closed at this instant, the current would commence in a normal manner. But if the voltage happens to be switched on at the instant when it passes through zero, certain abnormal conditions are momentarily set up. At every instant the applied voltage must be balanced by an equal and opposite induced voltage which is proportional to the rate of change of flux, which is, in its turn, proportional to the rate of change of the current if there is no iron in the core of the choking coil. As long as the applied voltage is in the positive direction, the rate of change of the current must be positive, which means that the current must be increasing. But the voltage is positive for a complete half-period, and so the current will only reach its maximum value in the time taken to go through half a period instead of a quarter, as is the case in normal conditions. The current in the first half-period will rise, therefore, to double its normal maximum. If the switch is closed at a point in between the zero and the maximum value of the volt wave, the first half-wave of the current will rise to a maximum value which is less than twice the normal maximum.

A transformer on no-load corresponds, however, to a choking coil with an iron core, and in this case the effect is accentuated. The instantaneous applied voltage must still be balanced at every instant by an equal and opposite induced voltage which is proportional to the rate of change of flux, but, due to the presence of the iron, this is no longer proportional to the rate of change of the current. In fact, when the iron is approaching saturation the rate of change of the current has to increase greatly due to the decrease in the permeability. This means that the first half-period of the current wave is still further increased.

There is yet another factor which helps to cause these current rushes. The core may have been left strongly magnetised from the time when it was last switched off, thus increasing the flux density and decreasing the permeability.

These current rushes consist of very large positive half-waves, which may rise to ten or twenty times the normal maximum, followed by very small negative half-waves, each succeeding wave approaching more nearly to the normal until, after a few cycles, the two half-waves become equal.

This effect may be considered as being due to the fact that the

voltage and current waves commence at the same instant and that the voltage has got to gain a quarter of a period on the current before normal conditions are set up.

These current rushes are not accompanied by any pressure rises, but they may occasion large mechanical stresses between adjacent coils, which ought to be avoided.

Regulation of a Transformer.—By the regulation of a transformer is meant the drop in the secondary terminal voltage experienced when full-load current is taken from the transformer and is usually

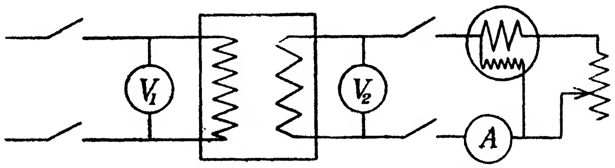


FIG. 178.—Regulation Test of a Transformer.

expressed as a percentage of the open circuit secondary voltage. It is usual, also, to specify the power factor of the load, as this has a most important effect, the voltage drop being considerably greater for low power factors. At no-load the ratio of transformation is practically equal to the ratio of the number of turns on the two windings, but as the load comes on, the secondary terminal voltage drops, the applied primary voltage being assumed constant. The ratio of transformation, therefore, is no longer the exact ratio of the turns, but becomes slightly larger, due to the presence of resistance and reactance in both windings. Referring to the

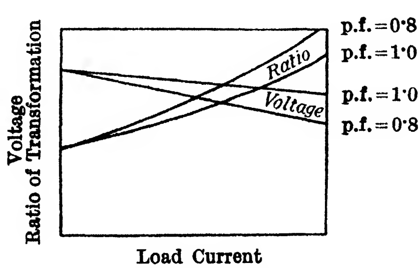


FIG. 179.—Regulation Curves.

vector diagram in Fig. 159, it is seen that the drop in voltage on the secondary side is the numerical difference between E_2 and V_S, and that this is *not* the same as the voltage absorbed in overcoming the secondary impedance, which is I_2Z_2. The same thing occurs on the primary side, so that in general the voltage drop is less than the voltage absorbed by the impedance of the windings.

The regulation can be determined experimentally by connecting the transformer as shown in Fig. 178. The primary voltage should be kept constant at its normal value. If the load is non-inductive,

the wattmeter may be omitted, but otherwise it is necessary, in order to determine the power factor. Fig. 179 illustrates the type of results which may be expected.

Since the drop is measured by the difference of two very similar voltages, it is important that great accuracy in the voltage measurements is necessary in order to ensure a moderate accuracy in the result. This disadvantage is overcome in the back to back method, which will next be described.

Back to Back Regulation Test.—For this test two approximately similar transformers are required. The primaries are connected in parallel across the mains as shown in Fig. 180, whilst the secondaries are also connected in parallel, but with a low reading voltmeter inserted in one lead. It is important that the two secondaries should be in opposition and not aiding one another, as in the latter case the low reading voltmeter would be burnt out. In order to test this before the voltmeter is inserted, the voltage across the free ends should be tested by means of another voltmeter or lamp

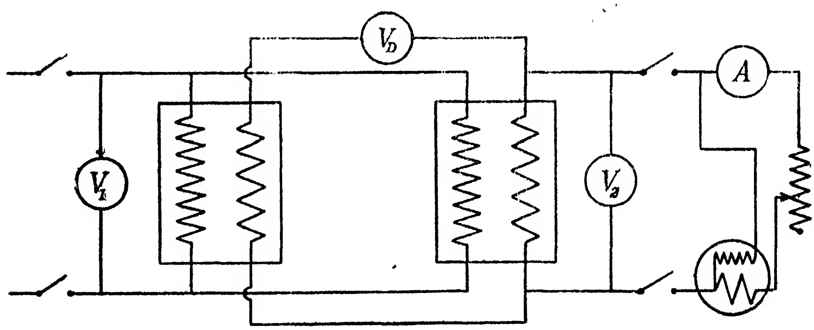


FIG. 180.—Back to Back Regulation Test.

capable of standing double the secondary voltage of either transformer. If a high voltage is observed, the connections to one transformer must be interchanged. Having arranged this satisfactorily, a load circuit is connected to one secondary as shown in the diagram. The other transformer will not contribute anything towards the load current, for it has the voltmeter V_D directly in series with it. If the transformers are similar, the voltmeter V_D should indicate no voltage at all when the load circuit has the switch open. If there is a slight voltage indicated, it means that the two transformers are not exactly similar. When a load is applied the terminal voltage of the loaded transformer will fall slightly, and V_D will give a small indication. The increase of the reading of V_D gives the voltage drop of the loaded transformer. This can be repeated for all loads up to full load and further. If a partially inductive load is used, a wattmeter will be necessary in the secondary circuit, the same as before.

A development of this test due to Shane[1] enables the voltage absorbed by the impedance of the windings to be determined in addition to the regulation. The necessary connections are those shown in Fig. 181, two similar transformers being used in addition to an auxiliary transformer, T_A, with a variable secondary which is used for introducing a voltage into the secondary circuit of the other two transformers. The action can be best understood by considering the vector diagram in Fig. 181, which refers to the secondary side only. OA and OB represent the terminal voltages of the unloaded and loaded transformer respectively. The drop is the numerical difference between OA and OB, but the voltage absorbed is given by BA. The auxiliary transformer being practically on no-load introduces a voltage having the same phase as OA, but is connected so as to be in opposition to OA. If the magnitude of this voltage,

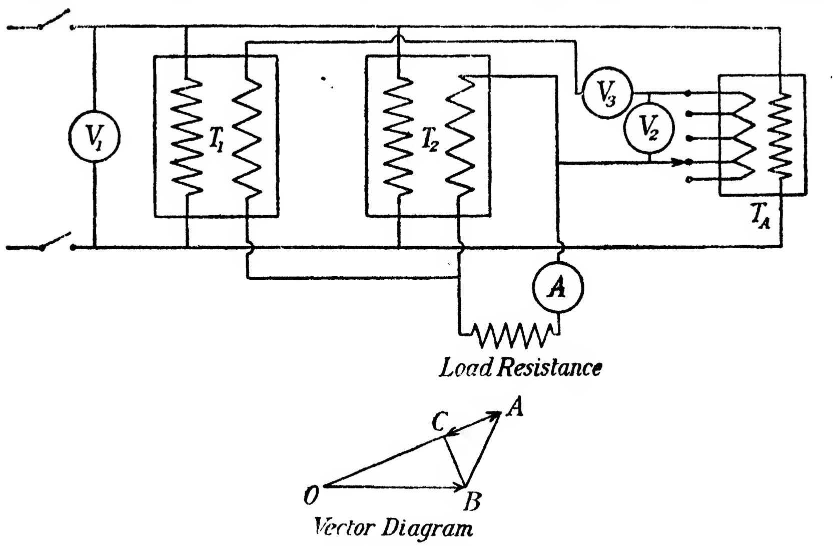


FIG. 181.—Shane's Method.

which is measured by V_2, is made equal to AC, the voltmeter V_3 should indicate the vector difference between OC and OB, *i.e.* BC. But BC has its minimum value when BC is at right angles to AC, and in that case AC is equal to the voltage drop, for OB and OC are approximately equal. The voltmeter V_2 therefore records the voltage drop when V_3 gives its minimum reading, and the voltage absorbed by the impedance is given by $BA = \sqrt{V_2^2 + V_3^2}$.

A disadvantage of this test is that the auxiliary transformer must have a number of separate tappings for the purpose of varying the voltage, as it is not permissible to do this by resistance regulation, since this would involve altering the phase of AC.

Short Circuit Test.—The object of this test is to enable the

[1] *Proc. Amer. Inst. Elec. Eng.*, vol. xxix., p. 1089.

regulation of a transformer to be determined without actually putting it on load. The secondary is short-circuited through an ammeter capable of reading the full load current, whilst the applied primary voltage is measured by the voltmeter V (see Fig. 182). The applied voltage must be very low. Usually it is necessary to have a considerable resistance in series with the supply for the

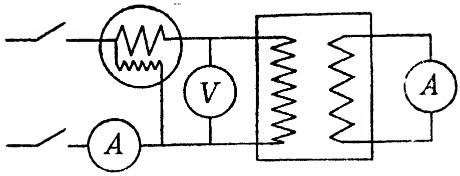


FIG. 182.—Short Circuit Test.

purpose of cutting down the voltage. The addition of the wattmeter enables the copper losses to be determined. If the secondary current be adjusted to its full load value, and then, without altering the applied voltage, the secondary ammeter be replaced by a low reading voltmeter, the latter will record the open circuit voltage. The whole of this was absorbed previously by the impedance of the two windings, and the ratio of this voltage to the secondary current gives the equivalent impedance referred to the secondary side. But on open circuit $V_S = V_P \times \frac{T_2}{T_1}$, so that the equivalent impedance is equal to $\frac{V_P}{I_2} \times \frac{T_2}{T_1}$. The total power component of the voltage drop, referred to the primary side, is $\frac{P}{I_1}$, where P is the total power registered by the wattmeter. When this quantity is referred to the secondary side it becomes $\frac{P}{I_1} \times \frac{T_2}{T_1} = \frac{P}{I_2}$, so that the equivalent resistance is $\frac{P}{I_2{}^2}$. The equivalent reactance referred to the secondary side is, therefore,

$$\sqrt{\left(\frac{V_P T_2}{I_2 T_1}\right)^2 - \left(\frac{P}{I_2{}^2}\right)^2}$$

$$= \frac{1}{I_2}\sqrt{\frac{V_P{}^2 \times T_2{}^2}{T_1{}^2} - P^2}.$$

The determination of the regulation, using these results, is explained in the next paragraph.

Fig. 183 shows the vector diagram for this short circuit test. Due to the low applied voltage, the flux is very small, so that the open circuit current is negligible and the primary and secondary currents are practically in phase opposition. I_2 and I_1 represent the secondary and primary currents respectively. In order to

generate I_2, a voltage E_2 is necessary, having for its power and idle components I_2R_2 and I_2X_2 respectively. On the primary side there are the corresponding voltages I_1R_1 and I_1X_1, the total primary applied voltage consisting of the vector sum of E_1, I_1R_1 and I_1X_1. The small flux vector Φ is at right angles to E_1 and E_2.

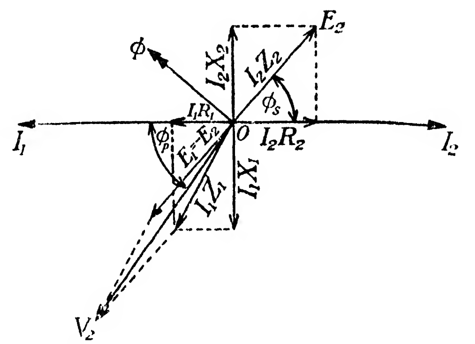


FIG. 183.—Vector Diagram for Short Circuit Test.

Since the flux is very small, the iron losses are negligible, so that the wattmeter reading practically consists of the copper losses of both windings, and this forms a very convenient method of measuring them. If the resistances are measured with continuous currents, the results obtained are frequently too low, due to the fact that with alternating currents the current distribution is often unequal, resulting in an increased power loss.

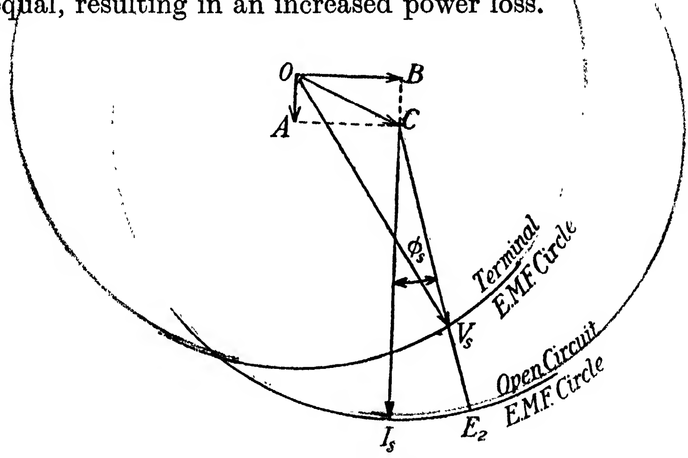


FIG. 184.—Kapp's Regulation Diagram.

Kapp's Regulation Diagram.—By means of a graphical construction proposed by Kapp, the voltage drop for a given load can be determined for all power factors. For this purpose it is necessary to know the equivalent resistance and the equivalent reactance, both referred to the secondary side. In Fig. 184 *OA* and *OB*

represent the power and idle components respectively of the equivalent impedance voltage OC. Taking the secondary current line, CI_S, as the axis of reference, and assuming an angle of lag ϕ_S in the load circuit, the secondary terminal voltage will lie along a line CV_S. The open circuit voltage, which is kept constant, will be given by OV_S, being the vector sum of OC and CV_S. The point V_S will, therefore, always lie on an arc of a circle of radius OV_S and centre O. Another arc of a circle having the same radius is now drawn with C as centre, and if CV_S be produced until it meets the other circle at E_2, then V_SE_2 represents the voltage drop at the load corresponding to the secondary current CI_S and the power factor $\cos\phi_S$. When V_S is to the right of CI_S it corresponds to a lagging current, and when it is to the left it corresponds to a leading current. All that is necessary, then, in order to determine the voltage drop at any power factor is to draw a line CE_2 making an angle ϕ_S with CI_S such that $\cos\phi_S$ is the power factor, when V_SE_2 can be read directly from the diagram. It is seen that the more the current lags in the secondary circuit the greater is the voltage drop until OV_S and OC are in phase. For leading currents the voltage drop decreases until it finally becomes zero. If the current is made to lead still further, the terminal voltage on load is actually greater than it is on open circuit.

Efficiency Test.—The simplest method of measuring the efficiency of a transformer consists in measuring the output and input by means of two wattmeters and is usually stated for a particular load which is given by an ammeter in the secondary main circuit. If the load is non-inductive, a voltmeter and ammeter are sufficient to measure the power, but in most cases it is desirable to check the power factor by means of wattmeter observations. In any case, a wattmeter must be included in the primary circuit, since there is a quite appreciable idle component of the primary current. The efficiency is, of course, given by the ratio of the output to the input. This method is a wasteful one, as it necessitates the whole of the power used being dissipated in some form or other.

An indirect method of determining the efficiency consists in measuring both the iron loss and the copper loss separately. The power input at no-load is practically all iron loss, and this remains substantially constant for all loads. The copper loss is obtained from the short circuit test, as already explained. Adding these losses to the output at any load, the input is obtained, knowing which, the efficiency may be calculated.

The direct and indirect methods explained above are equally applicable in the case of three phase transformers, the only difference being that three phase wattmeters are necessary, or, if these are not available, single phase instruments may be used for measuring the three phase power, as explained in Chapter X.

Sumpner's Test.—This test is carried out on two similar trans-

formers, and not only enables the efficiency to be determined, but, in addition, measures the copper and iron losses separately. The first transformer supplies the second, which is loaded back on to the mains after the fashion of the Hopkinson test with continuous current machines. The net power taken from the mains is therefore only that required to make up the losses of the two transformers and the various instruments. The two transformers are connected back to back as described on p. 198, the second one being worked the reverse way, so that the winding which is normally the secondary now acts as the primary. If there were no losses in the circuit, the acting secondary (real primary) of this transformer would deliver back to the mains a current equal to that taken by the first transformer. In order to enable this action to take place in an actual case, the primary of the first transformer must have its applied voltage boosted up a little, this being done by means of a small auxiliary transformer the secondary of which is connected in series

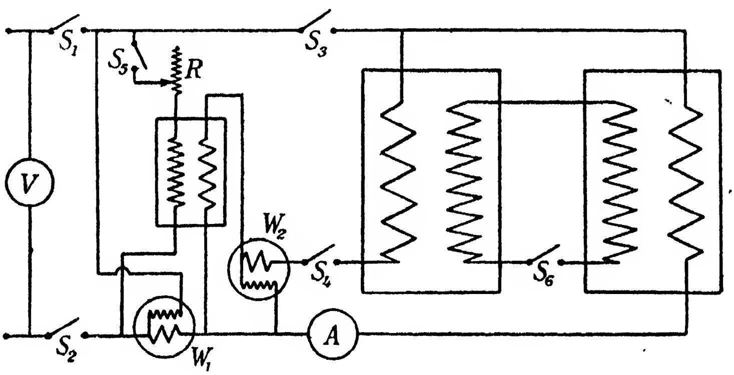


FIG. 185.—Sumpner's Test.

with the primary of the transformer which requires the boost The full diagram of connections necessary for this test is shown in Fig. 185.

If the primary of the auxiliary transformer be open-circuited, the secondary will act merely like a high impedance in series with the primary of the main transformer. The magnetising current of the latter will therefore be supplied from the second transformer and the ammeter, A, will register the total magnetising current of both. The wattmeter W_1 will register the total power supplied, and this represents the iron loss of the two transformers. If the wattmeter still gives an indication on opening S_3, this will be due to instrument losses and should be deducted from the former reading. When S_5 is closed the auxiliary transformer supplies an additional voltage which can be regulated by means of the adjustable resistance, R. The extra power supplied in this manner is measured by the wattmeter W_2, and since this causes a circulating current to be set

up between the two transformers and loads them up, the extra power supplied must be on account of the copper losses. The reading of W_2 therefore indicates the total copper loss. This will not cause the reading of W_1 to go up by a like amount, for the power supplied by the auxiliary transformer is received in its primary, and this primary current does not go through the wattmeter W_1, which will therefore continue to register the total iron loss. By varying the value of R, the load may be varied over a wide range, enabling the efficiency to be determined from no-load up to any desired overload. If the instrument losses are appreciable, the switches S_3 and S_6 may be opened, when the wattmeters will register their value. These losses may be taken as being proportional to the square of the current.

If the auxiliary transformer acts in opposition to the supply voltage, it reduces the terminal voltage on the test transformer, but this does not matter, as it simply means that, instead of supplying the second test transformer, it is supplied from it.

The output of each transformer may be taken as the product of the current read on the ammeter A and the voltage read on the voltmeter V, whilst the total input may be obtained by adding the copper loss and the iron loss to the output. The individual efficiency of each transformer is obtained by extracting the square root of the joint efficiency.

Efficiency Test by means of a Drysdale Double Wattmeter.—If one half of a Drysdale double wattmeter (see p. 146) be connected so as to read the input and the other half the output of a loaded transformer, and the connections arranged so that the two halves are in opposition, the indication of the instrument will represent the total losses of the transformer. On open-circuiting the volt coil connected to the primary, the instrument will record the output of the transformer, enabling the efficiency to be determined. It is convenient to make the wattmeter read backwards for the loss measurement, as it will then read forwards for the relatively large output measurement. The power factor of the load may be either unity or less, as desired, or some arrangement may be made for loading back on to the mains if convenient.

Separation of Losses.—The total copper loss and the total iron loss may be determined separately or from the Sumpner efficiency test. The copper loss is divided between primary and secondary, and the watts wasted in each can be determined from a knowledge of their relative resistances and the current flowing in each, and will usually be somewhere about the same. The iron loss can be separated into hysteresis and eddy current loss by the method explained on p. 86, where the flux is kept constant by varying the frequency at the same rate as the voltage. In this test, the secondary is left entirely disconnected.

All-Day Efficiency—Since some transformers, notably those used

for lighting purposes, work for very considerable periods every day at loads much less than full load, it is advisable to take this into account when considering the suitability of a transformer for a particular duty. In this connection, an expression termed the *all-day efficiency* is introduced, this being defined as

$$\frac{\text{Kilowatt-hours output per 24 hours}}{\text{Kilowatt-hours output per 24 hours} + \text{Kilowatt-hours wasted per 24 hours}}.$$

In order to get a high value for this ratio, it is necessary to have the maximum efficiency in the neighbourhood of the load at which the transformer works for the major portion of the time. But the maximum efficiency occurs when the copper loss equals the iron loss,[1] so that in a case of a lighting transformer the efficiency is designed to reach a maximum at very much less than full load, being usually about half full load. This means that at full load the copper losses will considerably exceed the iron losses, whilst in a transformer designed to have its maximum efficiency at full load these two would be equal. In order to reduce the iron loss, the flux density must be reduced, and this can be brought about either by increasing the cross-sectional area of the core or by increasing the number of turns. The former method involves an increased length of mean turn, resulting in an increased copper loss, whilst the latter method also increases the resistance of the windings. There is a practical limit to this, since an increase in the copper losses makes the regulation of the transformer worse, and after a certain point this becomes prohibitive.

As an example, consider the case of two 10 k.W. transformers, the first having a maximum efficiency of 97 per cent. occurring at full load and the second having the same maximum efficiency occurring at half full load. The full load losses of the first are 300 watts, divided equally between the iron and the

[1] This can be proved as follows :—

Let output $= EI \cos \phi$ and input $= EI \cos \phi + P_i + I^2R$, where P_i represents the constant iron loss and I^2R the copper loss.

$$\text{Efficiency} = \eta = \frac{EI \cos \phi}{EI \cos \phi + P_i + I^2R},$$

$$y = \frac{1}{\eta} = 1 + \frac{P_i}{EI \cos \phi} + \frac{I^2R}{EI \cos \phi}.$$

For $\eta = \text{max.}$ or $\frac{1}{\eta} = \text{min.}$, $\frac{dy}{dI} = 0$.

$$\therefore \quad \frac{dy}{dI} = 0 - \frac{P_i}{EI^2 \cos \phi} + \frac{R}{E \cos \phi} = 0,$$

$$\frac{R}{E \cos \phi} = \frac{P_i}{EI^2 \cos \phi},$$

$$R = \frac{P_i}{I^2} \text{ and } I^2R = P_i.$$

copper. At one-quarter full load the copper loss will be $\frac{150}{16} = 9{\cdot}4$ watts and the efficiency $\frac{2500}{2500 + 150 + 9{\cdot}4} = 94{\cdot}0$ per cent., the no-load loss being 150 watts. The losses of the second transformer at half full load are $\frac{3}{100} \times 5000 = 150$ watts. The iron loss is 75 watts and the full load copper loss is $4 \times 75 = 300$ watts. The full load efficiency is therefore $\frac{10000}{10000 + 75 + 300} \times 100 = 96{\cdot}4$ per cent. The efficiency at one-quarter full load is

$$\frac{2500}{2500 + 75 + 18{\cdot}75} \times 100 = 96{\cdot}4 \text{ per cent.}$$

Now assume that each transformer is kept on full load for six hours, one-quarter full load for twelve hours, and on no-load for six hours every day. The total energy output during a day is

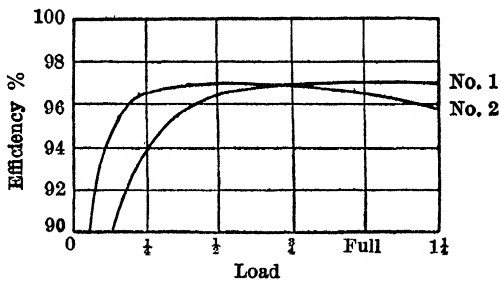


FIG. 186.—Efficiency Curves of Two Transformers.

$6 \times 10 + 12 \times 2{\cdot}5 = 90$ k.W.h. The total energy losses of the first transformer are $24 \times 0{\cdot}15 + 12 \times 0{\cdot}0094 + 6 \times 0{\cdot}15 = 4{\cdot}61$ k.W.h. The all-day efficiency is therefore $\frac{90}{90 + 4{\cdot}61} \times 100 = 95{\cdot}1$ per cent. The total energy losses of the second transformer are $24 \times 0{\cdot}075 + 12 \times 0{\cdot}01875 + 6 \times 0{\cdot}3 = 3{\cdot}825$ k.W.h. The all-day efficiency of this transformer is $\frac{90}{90 + 3{\cdot}825} \times 100 = 95{\cdot}9$ per cent., and is an improvement on the other. If a greater period of the time were spent on no-load, the improvement would be still more marked. Fig. 186 shows the efficiency curves of these two transformers.

For power work, transformers are mostly operated in the neighbourhood of full load and the efficiency is designed to be a maximum at this point.

Heat Tests.—When a transformer is run on a constant load its temperature gradually rises to a maximum value, which depends

upon the total losses which have to be dissipated. If the heat were produced uniformly throughout the transformer and also dissipated uniformly from its surface, the temperature rise of all the parts would be equal. In this case, the rate of production of heat is constant, whilst the rate of dissipation of heat is proportional to the temperature rise at that moment. The instantaneous temperature rise, θ, is then given by the equation

$$\theta = \theta_m (1 - \epsilon^{-\frac{t}{T}}),$$

where θ_m is the maximum temperature rise and T is a constant called the heating time constant.[1]

This latter quantity is the time taken to reach the final temperature if the initial rate of increase of temperature were maintained and is

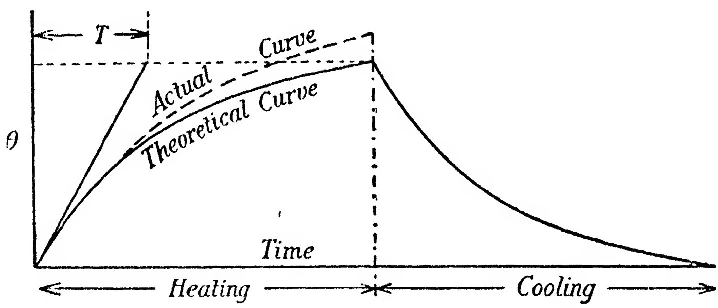


FIG. 187.—Heating and Cooling Curve.

indicated in Fig. 187, which shows the general form of the temperature rise curve. The cooling curve is the same shape as the heating curve, except that it is inverted.

[1] This law can be worked out as follows :—

Let H and $\frac{1}{T}\theta$ be the rate of production and of dissipation of heat respectively.

Then $$\frac{d\theta}{dt} = H - \frac{\theta}{T} \text{ and } \frac{d\theta}{H - \frac{\theta}{T}} = dt.$$

Integrating, we get $$\int \frac{d\theta}{H - \frac{\theta}{T}} = \int dt \text{ or } -T \log\left(H - \frac{\theta}{T}\right) = t + k.$$

When $t = 0$, $\theta = 0$ and $k = -T \log H$.

Therefore $$t = T \log H - T \log\left(H - \frac{\theta}{T}\right) = T \log \frac{H}{H - \frac{\theta}{T}}.$$

$$\epsilon^{\frac{t}{T}} = \frac{H}{H - \frac{\theta}{T}} \text{ and } \theta = TH\left(1 - \epsilon^{\frac{-t}{T}}\right).$$

But $\theta = \theta_m$ when $t = \infty$ and $\epsilon^{\frac{-t}{T}} = 0$, therefore $\theta_m = TH$ and $\theta = \theta_m (1 - \epsilon^{\frac{-t}{T}})$.

Unfortunately, the heating is not uniform in an actual case ; the iron may have more watts to dissipate than the copper and may have less cooling facilities, or *vice versa.* The different parts of a transformer, therefore, may heat up unequally, and in any case the heating curve, in the majority of instances, does not exactly agree with the theoretical curve, the continued increase in temperature after the bend has been passed being slightly greater in an actual case than is inferred from the theoretical curve.

A peculiar point sometimes observed is that on switching off the load the copper continues to rise in temperature for a short time. This is due to the fact that the iron is hotter than the copper, and, when cooling commences, the iron gets rid of some of its heat to the less hot copper and temporarily raises its temperature.

To carry out such a heating test, the transformer must be run on full load until it reaches its final temperature, which will usually not be for several hours. In order to reduce the wastage of energy, two transformers are tested at the same time, the second one loading back on to the mains. It is preferable to determine the temperature of the copper by the increase of resistance method, as this measures the average temperature of all the turns and not merely that of the surface. If this is done by means of a continuous current, it involves a temporary interruption of the test, but it is possible to avoid this by determining the equivalent resistance of the transformer whilst still working. In order to prevent damage to the windings the maximum temperature rise is usually limited to 50° C.

Short Time Heat Tests.—In a number of cases it is undesirable to carry out a heat test for the length of time necessary to arrive at a constant temperature. But running the transformer for a shorter period than this enables the initial part of the temperature rise curve to be determined, and knowing the law which is followed the final temperature can be calculated. One method of estimating this, due to the late Prof. S. P. Thompson, is to determine graphically the point at which the slope of the curve has been reduced to half its initial value. θ_m is then twice the temperature rise at that point.[1] Unfortunately, this method is very often found to be too inaccurate even for practical purposes. One reason for this is that it depends upon the initial rate of rise of temperature, and this is the most irregular part of the curve in the majority of actual cases.

Another method of estimating the final temperature rise, due

[1] $\frac{d\theta}{dt} = \frac{\theta_m}{T}\epsilon^{\frac{-t}{T}}$, and this is equal to $\frac{\theta_m}{2T}$.

$$\therefore \quad \epsilon^{\frac{-t}{T}} = \tfrac{1}{2} \text{ and } \theta = \theta_m(1 - \tfrac{1}{2}) = \tfrac{1}{2}\theta_m.$$

to Prof. C. H. Lees, is as follows : If θ_1, θ_2 and θ_3 are three rises of temperature observed successively at equal intervals of time, then

$$\frac{1}{\theta_m - \theta_2} = \frac{1}{\theta_3 - \theta_2} - \frac{1}{\theta_2 - \theta_1},$$

or

$$\theta_m = \theta_2 + \frac{(\theta_3 - \theta_2)(\theta_2 - \theta_1)}{(\theta_2 - \theta_1) - (\theta_3 - \theta_2)}.$$

Since this formula involves the difference of two differences, the points selected should be sufficiently far apart on the curve to avoid the production of a large error in the result due to small errors in the observations.

None of these methods, however, is as accurate as the direct method of measurement. Their chief advantage lies in the fact that they cause a great reduction in the amount of energy consumed and the time taken to complete the test.

CHAPTER XV

TRANSFORMERS.—DESIGN

Usual Shapes of Core.—The two most usual shapes for the cores of single phase transformers are those known as the core and shell types respectively. Having decided upon the type to be employed in a particular design, a provisional estimate of the sectional area of the magnetic circuit can be obtained from Fig. 188, which gives the relation between the output and the total gross cross-sectional area of *all* the limbs of the core. A very rough approximation to this curve is given by the relation

$$\text{Area of all limbs in sq. in. (net)} = 10\sqrt{\text{k.V.A.}}$$

$$\text{Area of all limbs in sq. cm. (net)} = 65\sqrt{\text{k.V.A.}}$$

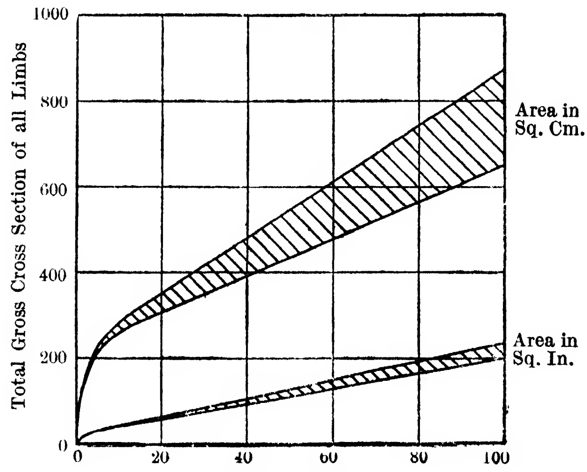


FIG. 188.—Total Cross Section of All Limbs.

The area per limb having been decided upon, the shape of the magnetic circuit can be determined from a series of relations given by Prof. Kapp which lead to the approximately best dimensions (see Fig. 189).

These relations are given in the following tables :—

CORE TYPE.

	Inch Dimensions.	Cm. Dimensions.
A	$0{\cdot}4 + 3{\cdot}2\,F$	$1{\cdot}0 + 3{\cdot}2\,F$
B	$4 + 4{\cdot}6\,F$	$10 + 4{\cdot}6\,F$
C	$0{\cdot}4 + 1{\cdot}2\,F$	$1{\cdot}0 + 1{\cdot}2\,F$
D	$4 + 2{\cdot}6\,F$	$10 + 2{\cdot}6\,F$
E	F to $2\,F$	F to $2\,F$

SHELL TYPE.

	Inch Dimensions.	Cm. Dimensions.
A	$3{\cdot}2\,F$ to $3{\cdot}4\,F$	$3{\cdot}2\,F$ to $3{\cdot}4\,F$
B	$2{\cdot}2\,F$ to $2{\cdot}4\,F$	$2{\cdot}2\,F$ to $2{\cdot}4\,F$
C	$0{\cdot}6\,F$ to $0{\cdot}7\,F$	$0{\cdot}6\,F$ to $0{\cdot}7\,F$
D	F	F
E	$2\,F$ to $4\,F$	$2\,F$ to $4\,F$

The proportions given in the above tables are not rigid, and may be departed from considerably without any very great disadvantages. In fact, several shapes of stampings are often used for the purpose

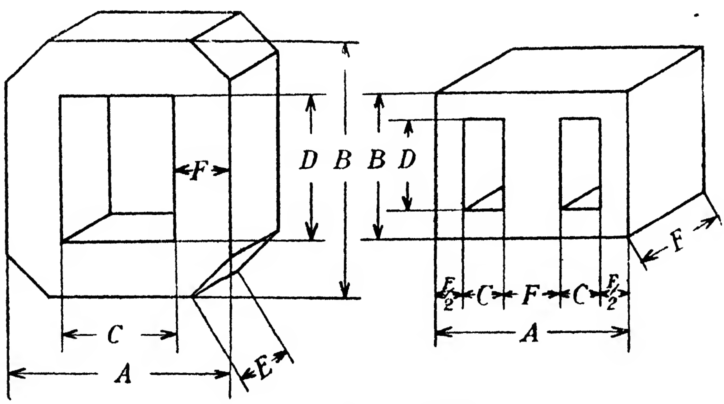


FIG. 189.—Kapp's Relations.

of obtaining an approximation to a circular cross-section, whilst the dimension C is often reduced at the expense of an increase in the dimension D, the object being to obtain a reduction in the floor space required.

Allowable Losses.—The allowable losses are determined by the efficiency at which it is desired to work, and this is obtained by comparison with existing transformers. Figs. 190 and 191 show the relation between efficiency and output which might be expected

in a normal case, the two curves on each diagram showing the upper and lower limits within which the transformer should work. The full load losses can thus be found, and if it is desired

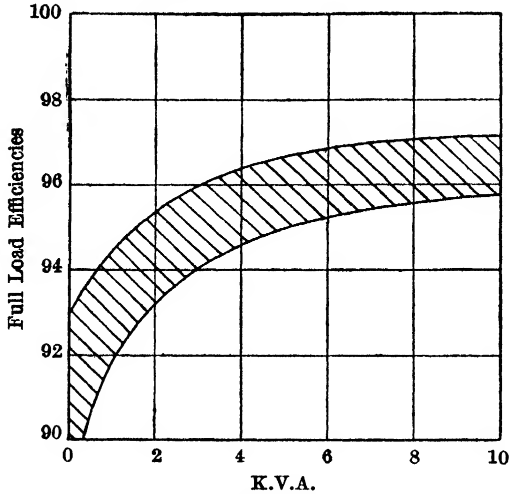


FIG. 190.—Full Load Efficiencies. 0–10 k.V.A., p.f. = 1.

to make the efficiency a maximum at full load, the core loss should be half the total loss. If the maximum efficiency is to occur at some other load, the ratio of the two losses will be altered

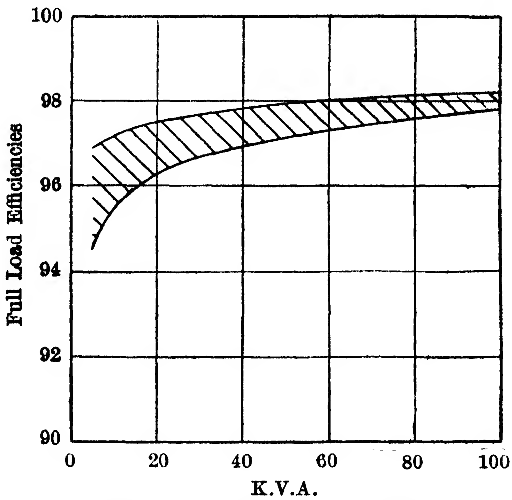


FIG. 191.—Full Load Efficiencies. 0–100 k.V.A., p.f. = 1.

accordingly. For example, if the maximum efficiency is to occur at half full load, the iron loss will be one-fifth of the full load losses instead of one-half. As a check on this value, reference

may be made to Fig. 192, which shows the relation between the core loss and the full load output.

Flux Densities.—Having obtained an approximate idea of the

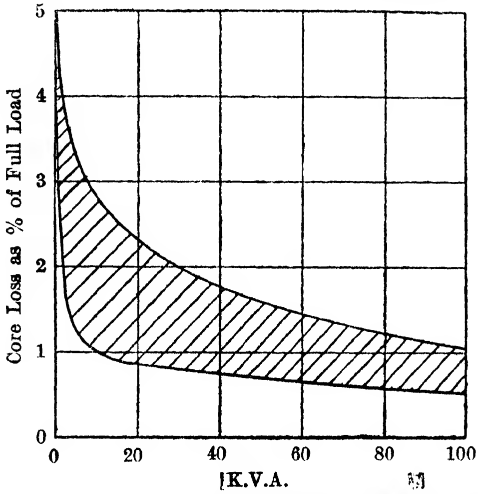


FIG. 192.—Core Loss as Percentage of Full Load.

shape and volume of the core and a knowledge of the core loss, the maximum flux density allowable can be calculated. The first

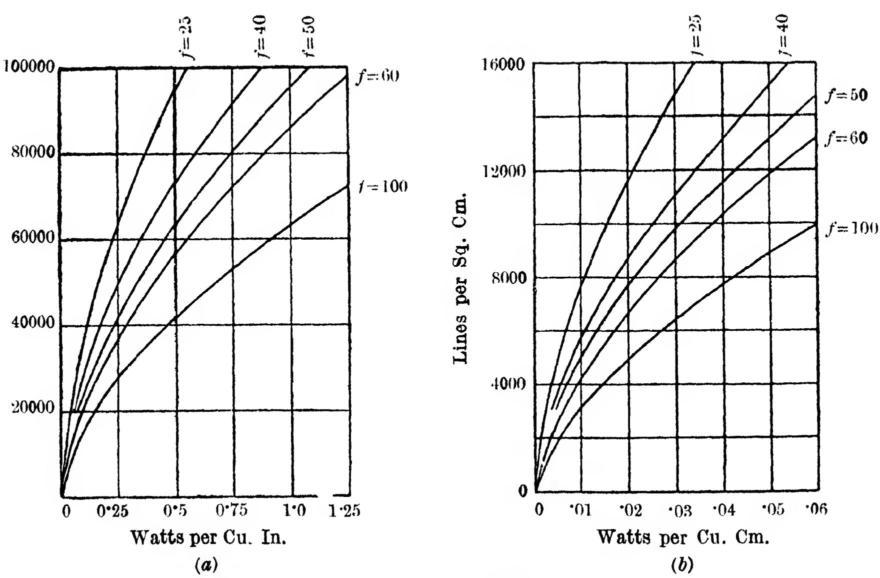


FIG. 193.—Flux Densities.

step is to obtain the core loss per cubic inch or per cubic cm. With a given frequency this settles the maximum flux density, which can be read off the curves in Fig. 193 (*a*) or (*b*).

For strict accuracy, separate curves should be drawn for the hysteresis and the eddy current loss, but the above combination is sufficiently accurate for the present purposes.

The total flux can now be obtained by multiplying the cross section of the core by the flux density to be used.

As a check on the value of the flux density the following table may be consulted, which refers to ordinary transformer iron, somewhat higher values being permissible with the special brands of alloyed iron.

Kilowatts Output.	Maximum Flux Density.			
	$f=40-60$.		$f=80-100$.	
	Lines per sq. in.	Lines per sq. cm.	Lines per sq. in.	Lines per sq. cm.
Up to 3 ...	65000	10000	42000	6500
,, ,, 10 ...	52000	8000	32500	5000
,, ,, 20 ...	45000	7000	26000	4000
,, ,, 100 ...	32000	5000	22500	3500

Copper and Iron Space Factors.—Two very important ratios are the copper and the iron space factors, which are defined as follows:—

$$\text{Copper space factor} = \frac{\text{Total net copper section}}{\text{Total available winding space}}.$$

$$\text{Iron space factor} = \frac{\text{Total net iron section}}{\text{Total gross cross section of core}}.$$

The former largely depends upon the voltage dealt with on the H.T. side, and is also affected by the method of cooling and by the use of oil insulation instead of air. Roughly speaking, the copper space factors of air-insulated transformers are about 20 per cent. poorer than those of corresponding oil-insulated ones. Fig. 194 gives the copper space factors for 10 k.V.A. and 1,000 k.V.A. transformers for voltages up to 30,000 volts. The values for other outputs can be obtained by interpolation.

The iron space factor depends upon the thickness of the laminations and the number of ventilating ducts. The thickness of the laminations usually varies from about 0·25 mm. to 0·5 mm., the insulation being obtained by means of a thin layer of varnish or a very thin sheet of tissue paper. About 15 per cent. of the gross iron length is used up in this way, the remaining 85 per cent. being solid iron. The number and size of the ventilating spaces further reduce the iron space factor, the values given in Fig. 195 being representative of average practice.

Dimensions of Core.—As a check on the core dimensions already worked out, a number of equations have been given by Catterson-

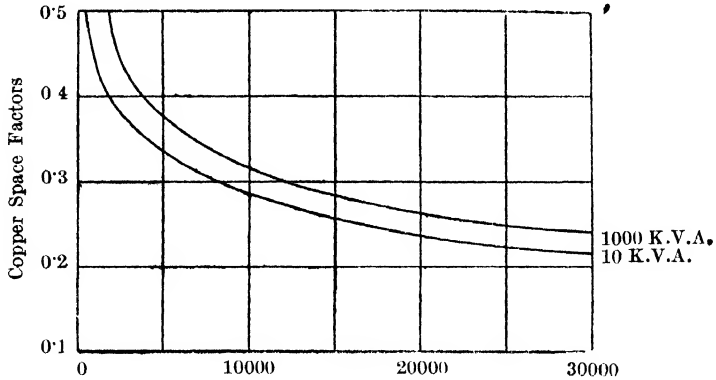


FIG. 194.—Copper Space Factors.

Smith[1] connecting the output with the dimensions. The equations in question are

$$CD \times FE = \frac{\text{k.V.A.} \times 10^{11}}{f \times S_i S_c \times d_i d_c} \times \text{constant (see Fig. 189)},$$

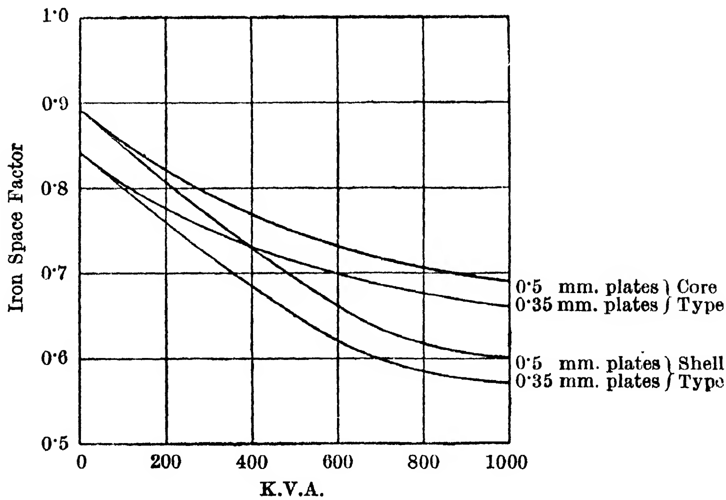


FIG. 195.—Iron Space Factors.

where the constant

$= 0{\cdot}45$ for single phase core type and shell type, as shown in Fig. 189,

$= 0{\cdot}30$ for three phase combined core type,

$= 0{\cdot}15$ for three phase combined shell type,

$= 0{\cdot}45\left(1 - \frac{1}{n}\right)$ for single phase auto-transformers,

[1] J. K. Catterson-Smith, *Electrician*, Jan. 3, 1913.

where S_i and S_c are the iron and copper space factors respectively, d_i and d_c are the flux and current densities respectively, and n is the ratio of transformation, all the dimensions being in cm. measure. The iron and copper space factors can be taken from Figs. 195 and 194 respectively, whilst the flux density can be taken from the table on p. 214. The current density will be approximately the same in both windings and may be taken from Fig. 197, which represents average practice. As it is only a rough preliminary estimate of the core which is given, it is only necessary to know these figures approximately, and so it will be quite sufficient to read the various values from the curves without direct reference to the transformer to be designed at all.

Number of Turns.—When the flux is decided upon, the number of turns on both primary and secondary can be settled from the equation

$$\text{Total primary turns} = \frac{E_1 \times 10^8}{\Phi \times f \times 4{\cdot}44}.$$

If the wave form on which the transformer is to work differs considerably from a sine wave, the constant 4·44 must be replaced by $4k$, where k is the form factor. The value 4·44 is, however, sufficiently accurate for most practical cases.

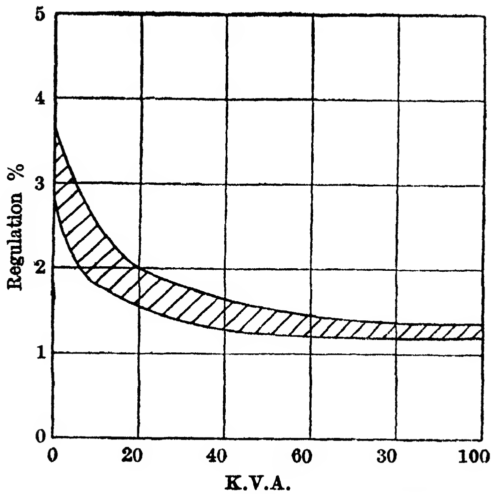


FIG. 196.—Regulation %. p.f.=1.

In order to determine the number of turns required on the secondary, it is necessary to know the voltage drop on full load. An approximate figure for this can be obtained by referring to Fig. 196, which gives the average regulation which can be expected from normal transformers up to 100 k.V.A. In this way the allowable total voltage drop can be obtained, reduced to the

secondary side, and the secondary turns can be calculated from the formula

$$\text{Total secondary turns} = \text{Total primary turns} \times \frac{E_2 + \text{total drop in volts}}{E_1}.$$

The question of the best number of turns and the best flux to adopt depends upon the relative cost of copper and iron. If the price of copper goes up, that of iron remaining constant, it may pay to reduce the number of turns on both primary and secondary and use a bigger flux necessitating more iron in the magnetic circuit.

Size of Wire.—The best distribution of the copper losses is obtained when the current density in primary and secondary is the same, and this means that the cross sections of the two windings should be proportional to their respective currents. The usual current densities adopted vary considerably with the output of the transformer and are given in Fig. 197. The sizes of the primary and

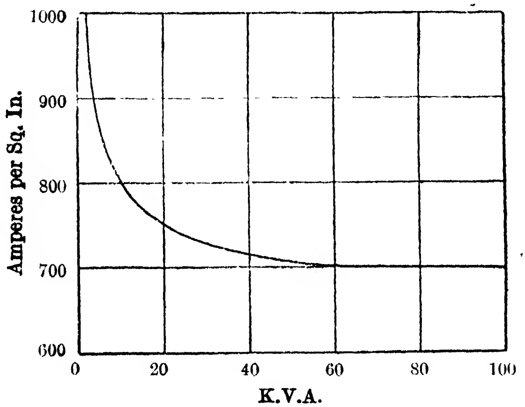


FIG. 197.—Current Densities.

secondary wires can now be settled, since the full load currents in the two windings are known. If the transformer is of considerable size the windings may have to be split up into several coils, as shown on p. 186.

The number of turns, size of wire and copper space factor being known, reference should be made to the core dimensions to see that it is possible to get the two windings into position.

Also the length of a mean turn on both primary and secondary can now be estimated by assuming a probable depth of winding, and hence the resistances of the two windings can be calculated. In this way an estimate of the total copper loss can be arrived at, which should agree substantially with the copper loss allowed in the first instance. In large transformers strip may be used in place of wire, and then, due to an uneven distribution of current, the

resistance may be greater when supplied with A.C. than when supplied with C.C.

Cooling Surface and Temperature Rise.—The various losses should now be re-calculated from the exact data available, so as to determine the total losses and efficiency at which the transformer will work under full load. The temperature rise may then be estimated when the cooling surface has been worked out, and the latter may be taken as

$$1{\cdot}5\{2AB + 2E(A + B)\} \text{ sq. in. or sq. cm. (see Fig. 189),}$$

depending upon the unit used. The constant 1·5 is introduced to take into account the fact that the outer surfaces are more or less rounded and not flat. If a drawing of the transformer is available these areas may be calculated more accurately, the total external surface from which cooling can take place being taken. If the

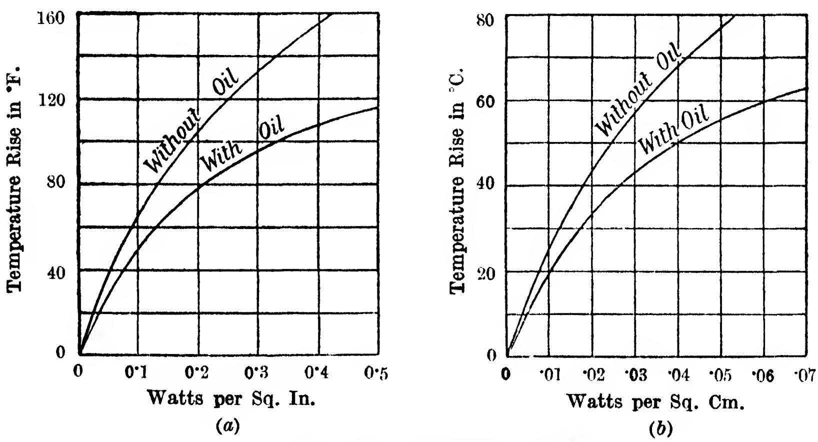


FIG. 198.—Temperature Rises.

core contains ventilating ducts, these should also be taken into account by including the area of both sides of the walls in the total cooling surface, and the same thing applies to any internal part to which the air or oil has free access.

The total watts wasted should lie between 0·1 and 0·2 per sq. in. or 0·015 and 0·03 per sq. cm. Fig. 198 shows a curve given by Prof. Kapp, in which the temperature rise is represented as a function of the watts per unit area of cooling surface for transformers completely enclosed in cast iron cases, both with and without oil, the temperatures being the probable maximum temperature rise when run continuously on full load. Where artificial cooling is employed, twice or three times the above watts may be dissipated for the same temperature rise, depending upon the method and thoroughness of the cooling arrangements.

Predetermination of No-load Current.—The no-load current

consists of a power and a no-load component, the former of which supplies the losses and the latter the magnetising current necessary to force the flux round the magnetic circuit. The relative magnitudes of these two components will be such that the resultant no-load current will usually lag behind the voltage by an angle of about 45° to 60°. The power component is obtained by dividing the total losses in watts by the primary applied voltage in a single phase case, whilst for a polyphase transformer each phase is supposed to take its proper share of the watts, and the current per phase is obtained by dividing this value by the volts acting across each phase, *e.g.* $\frac{1}{\sqrt{3}}$ times the line voltage in the case of a star connected three phase transformer.

In calculating the purely magnetising current, the total reluctance of the magnetic circuit is required, and this usually includes a number of joints of somewhat indefinite reluctance. As an approximation the reluctance of these may be taken as being equal to the reluctance of the remainder of the iron circuit. From the B—H curve of the iron the number of ampere-turns per cm. may be obtained, corresponding to the maximum flux density in the core. This value is multiplied by the total length of the magnetic path and then doubled to allow for the joints giving the total maximum ampere-turns required. Dividing by the number of primary turns and by $\sqrt{2}$ to obtain the R.M.S. value, the magnetising current is obtained. The formula to be used is, therefore,

$$\text{Magnetising current} = \frac{\text{Ampere-turns per cm.} \times \text{length in cm.} \times 2}{\text{primary turns} \times \sqrt{2}}.$$

In three phase transformers the primary turns per phase are taken, giving as the result the magnetising current per phase.

The total no-load current is then given by

$$\text{No-load current} = \sqrt{(\text{magnetising current})^2 + (\text{iron loss current})^2},$$

and the angle of lag, ϕ, is equal to

$$\tan^{-1} \frac{\text{magnetising current}}{\text{iron loss current}}.$$

Predetermination of Regulation.—The voltage drop at the terminals of the secondary winding when on full load is due to the combined effect of the resistance and the reactance of the two windings. The total drop due to the resistance, reduced to the secondary side, is

$$I_1R_1 \times \frac{T_2}{T_1} + I_2R_2 \text{ volts.}$$

The calculation of the total drop due to the reactance is more complicated and is given by a formula due to Prof. Kapp,[1] based upon the dimensions of the transformer, as follows :—

Pressure drop as a percentage of the open circuit voltage

$$= \frac{AT \times l}{\Phi L N^2}(3Nt + D)C,$$

where AT = Total ampere-turns per limb (primary and secondary),
l = Mean length of turn (average of primary and secondary) in cm.,
Φ = Total flux at no-load,
L = Length of winding space on one limb in cm.,
N = Number of primary or secondary coils per limb (using the larger number if they are different),
t = Thickness of insulation between each pair of primary and secondary coils in cm.,
D = Depth of winding (primary + secondary) on one limb in cm.,
C = A constant
= 16 for core type transformers,
= 22 for shell type transformers.

The impedance voltage triangle can now be calculated and the full load voltage drop obtained from Kapp's regulation diagram shown on p. 201.

Predetermination of Power Factor.—The primary power factor can be predicted by graphical means on drawing the vector diagram. Its value will depend upon the magnitude of the load and the power factor of the secondary circuit. If the secondary load is non-inductive, the primary current, I_P, is given by

$$I_P = \sqrt{(\text{power component})^2 + (\text{magnetising component})^2}.$$

The magnetising component has already been estimated, so that the power component, I', can be calculated for a given value of I_P.

In a similar way, the primary applied voltage E_P is given by

$$E_P = \sqrt{(\text{power component})^2 + (\text{idle component due to reactance})^2}.$$

The volts lost due to reactance have been calculated, so that the power component, E', can be calculated for a given applied voltage, E_P. The primary power factor can then be obtained from the expression

$$\text{primary power factor} = \frac{E'I'}{E_P I_P}.$$

[1] *E.T.Z.*, April 14, 1898, p. 244.

This method of treatment can also be extended to apply to the case when the secondary power factor is not unity.

For modern transformers on non-inductive load the power factor is practically unity from quarter load and upwards.

Example of Design.—As an example, a design will be worked out for a 50 k.V.A. single phase transformer working from 6,600 to 220 volts on a frequency of 50 cycles per second.

The transformer will be of the core type and will have its maximum efficiency at full load.

From Fig. 188 the gross area of all the limbs lies between 430 and 540 sq. cm.

Cross section of limb = 215 to 270 sq. cm.

From the formula

$$\text{Area of all limbs (net)} = 65\sqrt{\text{k.V.A.}}$$

the area is given as $65 \times \sqrt{50} = 460$ sq. cm.

Allowing an iron space factor of 0·85 this gives the gross cross section of the limb as $\frac{460}{2 \times 0{\cdot}85} = 270$ sq. cm.

This latter figure will be assumed as a tentative value for the cross section of the core, and, assuming this to be square, the length of the core is given as $\sqrt{270} = 16{\cdot}4$ cm. The other dimensions of the core are given by Kapp's relations (see Fig. 189), whereby the following values are obtained :—

$$\begin{aligned}
A &= 1{\cdot}0 + 3{\cdot}2F = 1{\cdot}0 + 3{\cdot}2 \times 16{\cdot}4 = 53{\cdot}5 \text{ cm.} \\
B &= 10 + 4{\cdot}6F = 10 + 4{\cdot}6 \times 16{\cdot}4 = 85{\cdot}4 \text{ ,,} \\
C &= 1{\cdot}0 + 1{\cdot}2F = 1{\cdot}0 + 1{\cdot}2 \times 16{\cdot}4 = 20{\cdot}7 \text{ ,,} \\
D &= 10 + 2{\cdot}6F = 10 + 2{\cdot}6 \times 16{\cdot}4 = 52{\cdot}6 \text{ ,,} \\
E &= F = 16{\cdot}4 \text{ cm.}
\end{aligned}$$

Ultimately a different cross-section of the core will be adopted, two sizes of stampings being used, as indicated on p. 183, in order to get a shorter length of mean turn for the winding.

The maximum efficiency lies between 97·1 per cent. and 98·0 per cent., according to Fig. 191, and occurs at full load. Thus the allowable losses at this load lie between $\frac{2{\cdot}9}{100} \times 50000 = 1450$ watts and $\frac{2{\cdot}0}{100} \times 50000 = 1000$ watts, giving a core loss ranging from 500 to 725 watts. According to Fig. 192, the core loss is given as 350 to 800 watts, the lower figure referring to transformers which reach their maximum efficiency at less than full load.

The approximate net volume of the iron core is

$$270 \times 0{\cdot}85\,(2 \times 53{\cdot}5 + 2 \times 52{\cdot}6) = 48700 \text{ cu. cm.}$$

The core loss per cu. cm. is, therefore,

$$(a) = \frac{500}{48700} = 0{\cdot}0103 \text{ watt.}$$

$$(b) = \frac{725}{48700} = 0{\cdot}0149 \text{ watt.}$$

These figures correspond to flux densities of 5,000 and 6,500 lines per sq. cm. respectively [see Fig. 193 (*b*)], which agree very well with the densities given on p. 214.

The dimensions of the core may now be checked by means of the Catterson-Smith formulæ. For this purpose the copper space factor may be taken as 0·33 from Fig. 194 and the iron space factor as 0·85. A tentative value for the current density in the windings is 110 amperes per sq. cm. taken from Fig. 197.

Then

$$(a)\ CD \times FE = \frac{\text{k.V.A.} \times 10^{11} \times 0{\cdot}45}{f \times S_i S_c \times d_i d_c}$$

$$= \frac{50 \times 10^{11} \times 0{\cdot}45}{50 \times 0{\cdot}85 \times 0{\cdot}33 \times 6500 \times 110}$$

$$= 225000.$$

$$(b)\ CD \times FE = \frac{50 \times 10^{11} \times 0{\cdot}45}{50 \times 0{\cdot}85 \times 0{\cdot}33 \times 5000 \times 110}$$

$$= 292000.$$

The value of $CD \times FE$ according to the figures already obtained is

$$20{\cdot}7 \times 52{\cdot}6 \times 16{\cdot}4 \times 16{\cdot}4 = 293000.$$

The higher figures correspond to the higher efficiencies and necessitate larger cores.

The section of the core may now be decided upon, the dimensions of the two sizes of stampings being as follows:—

$$a = 1{\cdot}09 \times \sqrt{270} = 17{\cdot}9 \text{ cm.,}$$

say 18·0 cm. (see p. 184).

$$b = 0{\cdot}67 \times \sqrt{270} = 11{\cdot}0 \text{ cm.}$$

With two 0·5 cm. air ducts and 0·5 mm. plates with 0·05 mm. insulation (see Fig. 199) the net iron section is

$$18 \times 11 \times \frac{0{\cdot}5}{0{\cdot}55} + 6 \times 11 \times \frac{0{\cdot}5}{0{\cdot}55} = 240 \text{ sq. cm.}$$

Adopting a flux density of 6,000 this gives a total flux of $240 \times 6000 = 1{\cdot}44 \times 10^6$ lines.

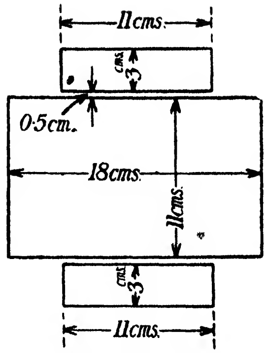


FIG. 199.—Cross Section of Core.

The total primary turns are

$$T_1 = \frac{E_1 \times 10^8}{\Phi \times f \times 4{\cdot}44}$$

$$= \frac{6600 \times 10^8}{1{\cdot}44 \times 10^6 \times 50 \times 4{\cdot}44}$$

$$= 2060 \text{ turns.}$$

The total secondary turns are

$$T_2 = T_1 \times \frac{E_2 + \text{total drop in volts}}{E_1}.$$

The total drop in volts is given as

$$0{\cdot}018 \times 220 = 4 \text{ volts from Fig. 196.}$$

Therefore

$$T_2 = 2060 \times \frac{220 + 4}{6600} = 70 \text{ turns.}$$

The full load primary current on unity power factor is

$$\frac{50000}{6600} = 7{\cdot}6 \text{ amperes}$$

or 7·8 amperes allowing for the efficiency.

The full load secondary current is then

$$\frac{50000}{220} = 227 \text{ amperes.}$$

With a current density of 110 amperes per sq. cm. this gives as the sections of the two conductors

$$\text{Primary} = \frac{7{\cdot}8}{110} = 0{\cdot}07 \text{ sq. cm.}$$

$$\text{Secondary} = \frac{227}{110} = 2{\cdot}06 \text{ sq. cm.}$$

The secondary winding will be in the form of strip, and in order to reduce the eddy current loss in the winding will be subdivided into three strips in parallel laid over one another, the section of each being 0·69 sq. cm.

Increasing the secondary turns to 36 per limb and having 9 turns per layer with 12 layers (3 in parallel) a suitable size of conductor is found to be 4 × 0·175 sq. cm.

The nearest wire to 0·07 sq. cm. is 11 S.W.G. having a cross-sectional area of 0·068 sq. cm. and a diameter of 0·295 cm. The wire is double cotton covered, each layer being further insulated

by a continuous strip of insulation about 0·5 mm. thick whilst the whole winding is finally insulated to about 3 mm. with oiled linen and press-spahn.

The external cooling surface is given approximately by

$$\begin{aligned} & 1{\cdot}5\{2AB + 2E(A + B)\} \\ &= 1{\cdot}5\{2 \times 53{\cdot}5 \times 85{\cdot}4 + 2 \times 16{\cdot}4(53{\cdot}5 + 85{\cdot}4)\} \\ &= 20500 \text{ sq. cm.} \end{aligned}$$

The cooling surface of the air ducts can be taken as

$$85{\cdot}4 \times 11 \times 4 = 3800 \text{ sq. cm., say.}$$

There will be also additional ventilating space due to the two sizes of stampings which can be taken as

$$85{\cdot}4 \times 3 \times 8 = 2000, \text{ say (see Fig. 199).}$$

The total cooling surface is then

$$20500 + 3800 + 2000 = 26300 \text{ sq. cm.}$$

The watts dissipated at full load are of the order of 2·5 per cent. of 50000 = 1250 watts.

The watts per sq. cm. are approximately

$$\frac{1250}{26300} = 0{\cdot}0475.$$

If the transformer is oil immersed in a cast iron case with no mechanical circulation, this would correspond to a temperature rise of 50–55° C. (see Fig. 198). With an air-blast, however, this temperature rise could very easily be brought down to 40° C.

CHAPTER XVI

ALTERNATORS.—PRINCIPLES AND CONSTRUCTION

Simple Alternator.—An alternator is an alternating current dynamo and in its elementary form consists of a coil of wire rotating between the poles of a magnet, as shown in Fig. 2. The two ends of the coil do not need to be connected to a commutator and are, instead, connected to a pair of slip rings, on each of which a brush presses for the purpose of collecting the current. In an ideal case the induced E.M.F. will obey a sine law of the type

$$e = E_m \sin \theta.$$

An increase in the number of turns results in an increase of the induced voltage.

Frequency and Number of Poles.—The majority of actual alternators are multipolar machines, the number of poles employed being, in general, much greater than is the case in C.C. dynamos of the same size. The number of poles may, in fact, reach up to 100 in large alternators. Alternators are usually designed to generate at definite frequencies, and there is a rigid relationship between the speed, the number of poles, and the frequency. If there are p poles, the E.M.F. will go through $\frac{p}{2}$ cycles per revolution or $\frac{pn}{2 \times 60}$ cycles per second, where n equals the r.p.m. Thus

$$f = \frac{pn}{120}.$$

The r.p.m. at which alternators must run for various frequencies and numbers of poles are given in the following table :—

No. of Poles	2	4	6	8	10	12	16	20
Speed for $f =$ 25 ...	1500	750	500	375	300	250	187·5	150
,, ,, $f =$ 40 ...	2400	1200	800	600	480	400	300	240
,, ,, $f =$ 50 ...	3000	1500	1000	750	600	500	375	300
,, ,, $f =$ 60 ...	3600	1800	1200	900	720	600	450	360
,, ,, $f =$ 100 ...	6000	3000	2000	1500	1200	1000	750	600

Polyphase Alternators.—A simple two phase alternator may be obtained by adding a second coil similar to the first, but situated at right angles to it so that this angle is always maintained during rotation (see Fig. 79). In the case of multipolar machines, corresponding conductors in the two phases are always situated half a pole pitch distant from one another.

A simple three phase alternator is shown in Fig. 86, corresponding conductors being always situated two-thirds of a pole pitch distant from each other, this being independent of the total number of poles.

Rotating Field and Rotating Armature.—From a theoretical point of view it is immaterial whether the armature rotates between the poles or whether the poles rotate round the armature. In practice both methods are employed, the rotating armature usually being adopted for small machines and the rotating field for large ones. In the latter case the field system and the armature change positions, the armature being outside and presenting a hollow cylindrical surface to the field system. The latter now consists of a yoke supported from the shaft, with the poles projecting radially outwards and facing the armature. The stationary element is usually called the *stator* and the rotating element the *rotor*.

Modern practice favours the rotating field type as being the more economical design for large machines, but for small alternators the manufacturers usually adopt a rotating armature, largely on account of the fact that the demand is small and existing patterns of C.C. machines are available, thus enabling them to turn out a cheaper machine. If a large demand for such machines arose, it is an open question whether it would not be advisable to adopt the rotating field even for small alternators.

There are several inherent advantages of the rotating field type. A higher peripheral speed may be employed, since it is easier to make a sound mechanical job of the poles and pole windings than of the armature winding with its end connections, since, owing to the centrifugal force, the end connections of the armature must be braced securely in position. Also, since the armature is on the outside, there is more room for its winding, which is a help in the design, since the armature winding usually requires more space than the field winding, whilst in the case of high voltage machines it is preferable to have the H.T. winding stationary. There is a further advantage in connection with the slip rings. A three phase alternator with a rotating armature would require three slip rings, and a two phase machine four, whilst only two would be necessary in either case if a rotating field were adopted. Apart from the number of slip rings there is the question of their insulation and the liability to breakdown due to wear and tear, since higher insulation is probably necessary.

As opposed to these advantages the rotating armature machine presents the possibility of using stock patterns and a stock C.C.

armature winding with the commutator removed and slip rings substituted in its place.

Low Speed Alternators.—The usual form of modern low and medium speed alternators is the rotating field type, the magnet poles being bolted on to a yoke ring, which is supported by means of spider arms from the rotor hub. Since the majority of the weight of the rotor lies near its outer edge, its flywheel effect is so considerable that it avoids the necessity for any extra flywheel. This would otherwise be necessary, since a uniform angular velocity is desired for smooth running. In fact, this type of alternator is often referred to as the *flywheel type*.

Owing to the low speed, these alternators will usually have a large number of poles, this tending to make the diameter much larger for a given output than would be the case in a C.C. dynamo. The general tendency is for the machines to become large in diameter and small in length to a noteworthy extent. Even with the low speeds of revolution which are adopted, the peripheral speeds of the pole faces are kept up, due to the large diameters, the values of the peripheral speed ranging from 5,000 to 8,000 feet per minute.

This type of alternator is usually directly driven from a reciprocating engine and frequently has only one bearing. The end of the shaft on the engine side then terminates in a forged coupling which is bolted directly on to the engine crankshaft.

High Speed Turbo-Alternators.—The proportions of high speed alternators which are intended for direct coupling to steam turbines are much different from the low speed type already dealt with. Owing to the high speeds of rotation which are employed, the diameters are made very much less, resulting in a corresponding increase in the axial length. Even then the peripheral velocities of the rotating element reach from 12,000 to 20,000 feet per minute, and this necessitates a very sound mechanical rotor construction. Special types of rotating field systems are thus called into being, and these will be dealt with in detail later. The stator construction is not very different from that in low speed alternators except for the greatly changed ratio of diameter to length. The number of poles is also very much less, being usually either two, four or six.

Since the speed is very much higher, the bulk of a turbo-generator is very much less than that of a flywheel type alternator of the same output, and consequently the total surface available for dissipating heat is very much less. In order, therefore, to prevent an undue temperature rise, artificial cooling must be employed, and this usually takes the form of an air-blast whereby air is sucked or forced in through a hole in the bottom of the outer casing and makes its exit by a corresponding hole at the top. The outer casing which guides the air past the heated surfaces serves the additional purpose of deadening the noise of the machine, which would otherwise be

considerable, due to its high speed and the restricted air passages in its construction.

A separate class of turbo-alternators has been developed in conjunction with steam and water turbines having a vertical shaft. These machines may be designed for high speeds in connection with vertical steam turbines, or they may be used as low speed generators on hydro-electric stations where only a small fall is available.

Other Types of Alternator.—Various distinct types of alternator, other than the rotating field and rotating armature machines, have been introduced from time to time, but they have now practically disappeared from the market.

One such alternator is the Ferranti disc machine, which has a rotating armature consisting of a flat disc of copper strip suitably arranged so as to generate an alternating E.M.F. No iron is employed in the armature for magnetic purposes, the rotating element revolving between two sets of poles facing one another, these poles being alternately N. and S., those opposite being alternately S. and N.

Another type of A.C. generator is the inductor alternator. In this machine both armature and field windings are stationary, no rotating contacts being employed at all. The rotor consists of a heavy iron system with two rings of projecting pole pieces. These are magnetised by a central fixed exciting coil, so that all the poles of one ring have one polarity, whilst all the poles of the other ring have the opposite polarity. The E.M.F. in the armature coils is induced by the variation in the magnetic flux, due to the fact that the magnetic circuit is made alternately good and bad as the inductors move past the armature coils. The flux never cuts the armature coils in the reverse direction, simply varying between a maximum and a minimum value. This necessitates a large amount of flux, which, although it cannot be called leakage flux, is wholly waste as far as the generation of E.M.F. is concerned and results in a considerable increase in the iron weight of the machine. One characteristic of this class of alternator is the falling away of the voltage as the load comes on, which, although of advantage for parallel running, is undesirable as far as the voltage regulation is concerned.

Stator Construction.—The stator frame is merely an arrangement for holding the armature stampings and windings in position and does not fulfil any magnetic function. In large low speed alternators the size of the stator frame usually results in it being cast in two or three sections, these being bolted together as indicated in Fig. 200, which shows the general arrangement of rotor and stator, the frame of the latter being cut in two halves. In order to increase the ventilation a number of holes are cast in the frame, this being allowable since it does not interfere with the magnetic flux and does not unduly weaken the mechanical strength if done in modera-

tion. Fig. 201 is one example of stator construction, the diagram showing a cross section of the frame. This type of stator is suitable, with modifications, for medium and large size alternators. The stampings are held in position by being clamped between an end

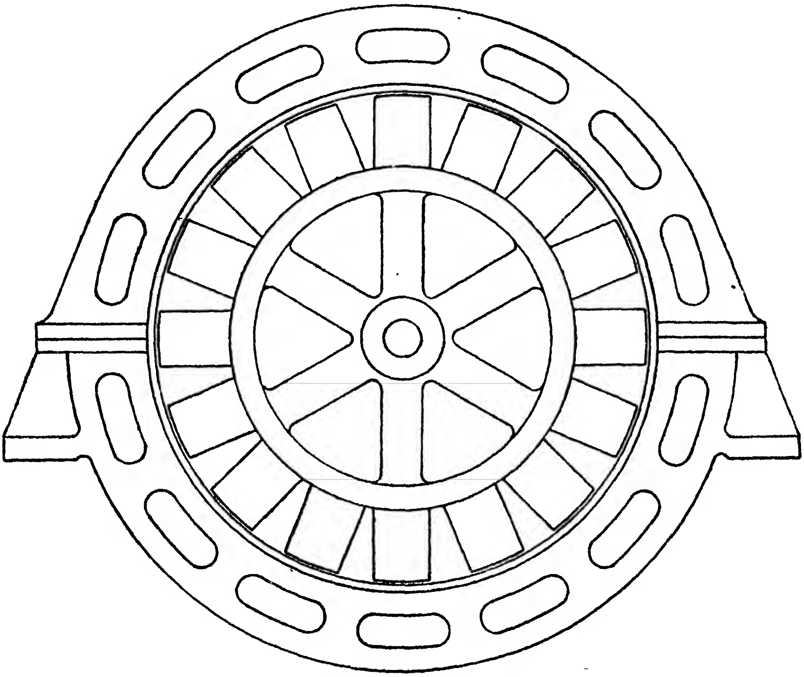


FIG. 200.—General Arrangement of Low Speed Alternator.

cheek cast on to the main frame on one side and a similar end cheek on the other, this one being cast separately and mounted in position after the stampings are arranged in place. The separate end cheek and the stampings are firmly held in position by means of

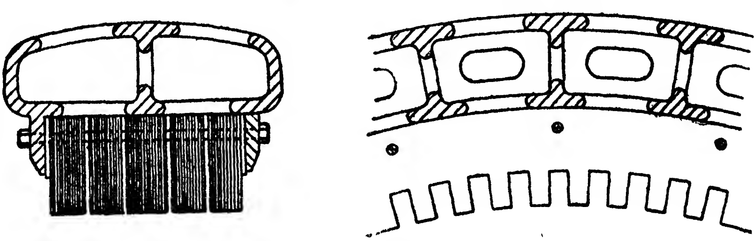


FIG. 201.—Stator Frame.

a number of bolts which pass right through from side to side. Radial ventilating ducts are provided in the stampings by the insertion of distance pieces, so that air can be thrown out from the air-gap by centrifugal force escaping by the vent holes in the outer case.

For smaller and narrower stators the central rib in the frame is sometimes omitted, as shown in Fig. 202.

Sometimes, instead of having an end cheek cast on to the frame,

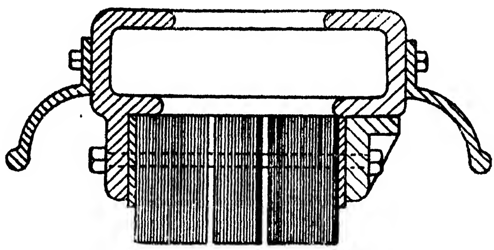


FIG. 202.—Small Stator Frame.

the stampings are gripped in position by means of two steel ring clamps bolted together as in the previous example (see Fig. 203).

Another method employing two fixed end cheeks is illustrated in Fig. 204, the whole stator frame being cast in two duplicate halves,

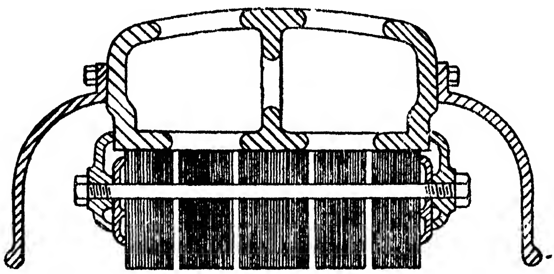


FIG. 203.—Stator Frame.

these being bolted together as shown in order to clamp the stampings in position.

The stators of turbo-alternators present very little real difference from those of low speed sets, although their external appearance is

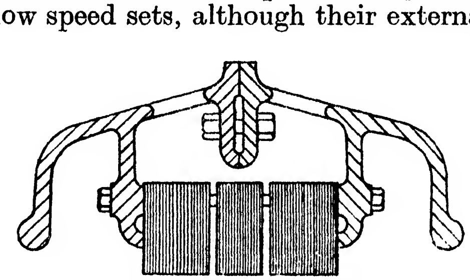


FIG. 204.—Split Stator Frame.

widely different. The great points of difference are the smaller diameter and the increased length. A special feature is, however, observed in the arrangements for clamping the end connections in

position owing to the large mechanical forces to which they are subjected when the machine is accidentally short-circuited or otherwise has a heavy current flowing momentarily through the armature winding. Practice has shown that these clamping devices must be made very strong. Of course, when a turbo-alternator is cooled by means of an external air-blast, this modifies considerably the mechanical design of the stator frame, the small vent holes round the outside being replaced by one large one at the top and a corresponding one underneath.

Cast iron end shields, built up in sections, are bolted on to both sides of the exterior of the stator frame for the purpose of providing a mechanical protection for the end connections, which frequently are at high potentials.

Staying of Stator Frames.—When the stator frame attains a very large diameter, precautions have to be taken to prevent any mechanical deformation due to its own weight, the tendency being for the frame to sink at the top and bottom in between the feet. This would cause the air-gap at the top to be decreased and that

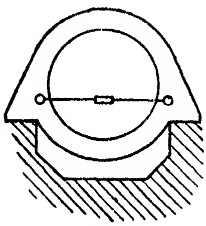


FIG. 205.

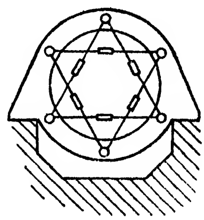


FIG. 206.

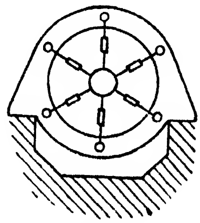


FIG. 207.

Systems of Tie-rods.

at the bottom to be increased, and to avoid this additional feet are sometimes cast on to the lower half of the stator frame, whilst in other cases systems of tie-rods are adopted. For example, a single tie-rod per side might be used, as shown in Fig. 205. This would pull the feet together and may be arranged to restore the air-gap at the top to normal, but for large, narrow stators it is not sufficient. Much more satisfactory stiffening of the stator is obtained by the system of tie-rods shown in Fig. 206, there being, in this instance, six tie-rods per side. Another method of producing the same result is obtained by mounting a pair of star frames on the stator, one on either side, as shown in Fig. 207, the radial tie-rods terminating in a ring concentric with, but free from, the shaft.

Stator Core.—In the rotating field type of alternator the stator stampings will, of course, consist of annular rings with the slots along the inside edge. For all except small machines these stampings will have to be built up in sections (see Fig. 208), being bolted and keyed on to the frame at the back. In order to nullify the effect of the joint as far as possible, the sections are staggered so that the

joints of adjacent stampings do not coincide. Radial ventilating ducts are provided at intervals by inserting spacing grids. These may consist of skeleton castings so arranged as not to impede the passage of the air, or they may be built up from two stampings

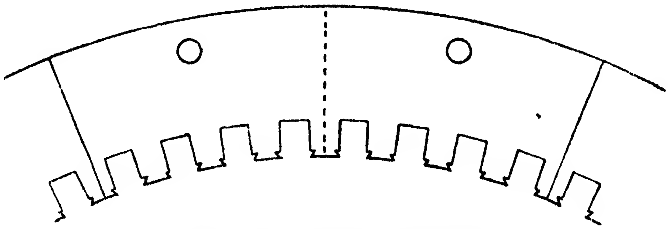


FIG. 208.—Armature Stampings.

thicker than the rest and separated by means of a number of distance pieces soldered on. One type of ventilating duct in common use is made by having two thickened stampings in which cuts are made as shown in Fig. 209 (*a*). The little tongues thus formed are bent

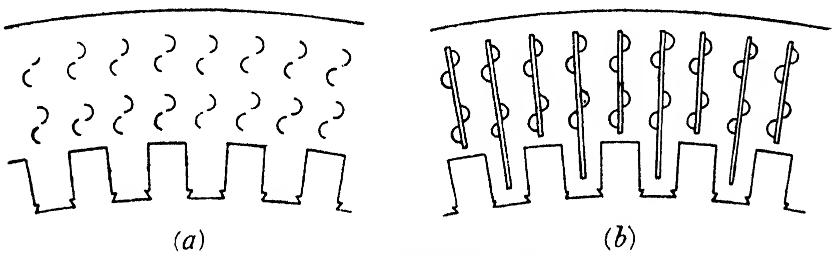


FIG. 209.—Ventilating Grids.

at right angles and form settings for a number of radial steel strips which serve as the distance pieces [see Fig. 209 (*b*)].

Slots.—Three distinct types of slots are used in A.C. machines, the first being the *open type* as used also in C.C. machines (see Fig. 210). These are frequently recessed near the mouth for the

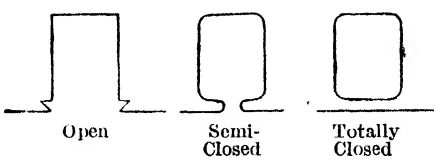


FIG. 210.—Types of Slots.

purpose of inserting a wooden strip designed to keep the armature winding from flying outwards under the action of centrifugal force when the armature is the rotating element. These wooden strips also take the place of binding wire on stator windings, since the use of binding wire on a concave surface is impossible. Open slots

enable former wound coils to be employed and are thus easier to wind than the other types. They also have the advantage of allowing the slots to be thoroughly impregnated with insulating varnish before the coils are wound. Their disadvantage is that the available area for the air-gap is less than is the case when the other types are employed and that they cause slight oscillations of the flux from side to side as the teeth move past the pole face. This is due to the great difference in permeability of the tooth and slot. The lines of force cling on to the retreating tooth as long as possible, and then snap across the slot to the next tooth coming along. This tends to produce harmonics in the E.M.F. wave form and necessitates laminated pole shoes in order to reduce the eddy currents which would otherwise be generated to a considerable extent.

The use of *totally closed slots* or *tunnels* removes these disadvantages, since a uniform iron surface is presented to the pole face, but other disadvantages are introduced. Due to the fact that the winding is totally surrounded by iron, the inductance of the coils is materially increased, and this has the effect of causing an increased drop in voltage as the load current is increased, thus making the regulation of the machine worse. To reduce this effect as much as possible, the bridge over the roof of the tunnel is made very thin in order to force up the flux density of the leakage lines in this neighbourhood and thus decrease the permeability. But if this bridge is made less than about $\frac{1}{32}$ inch the tunnels are apt to become broken open during the operation of stamping, this fact exercising a practical limit to the thickness of the roof. Another disadvantage of the tunnel type of slot is that the coils must be hand wound by threading them through the holes, thus increasing the labour of winding and the danger of abrasion during the process.

The *semi-closed slot* is a compromise between the wide open slot and the tunnel. The danger of bursting the narrow bridge is avoided and an air-gap is inserted in the path of the leakage lines of force. Moreover, nearly the whole of the surface of the armature is useful for flux bearing purposes, which tends to eliminate ripples from the wave form. The actual winding is easier to carry out than when totally closed slots are employed, as the conductors can be passed through the mouth of the slot, although more labour is involved than when open slots with former wound coils are adopted.

Rotors.—When the armature forms the rotating system, the mechanical construction is much the same as in C.C. machines, the stampings being mounted upon a cast iron spider which is driven from the shaft.

In the case of rotating field alternators, the yoke ring and spider form a flywheel upon which the poles are mounted. Up to a diameter of about eight feet the spider is generally cast in one piece, but for larger sizes it is usual to cast it in sections, which are bolted

together. To increase the mechanical strength, wrought iron rings are shrunk on to the hub and over projections near the rim as shown in Fig. 211. The rim has to be of sufficient cross section to

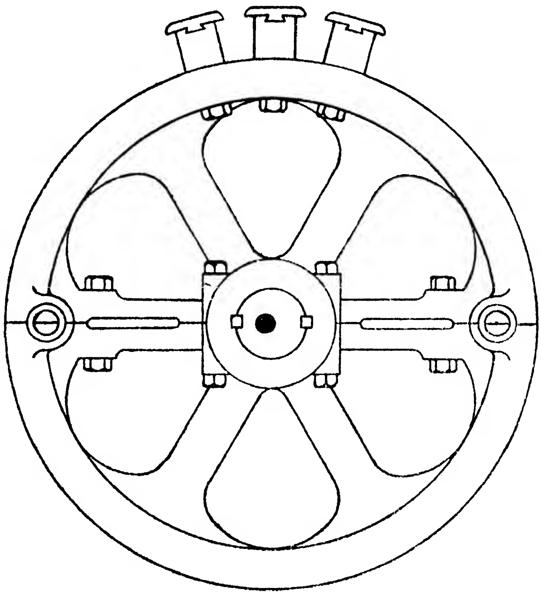


FIG. 211.—Split Rotor System.

carry half the flux per pole, and also to give the necessary flywheel effect, the calculation of which involves a knowledge of the engine which is to drive the alternator.

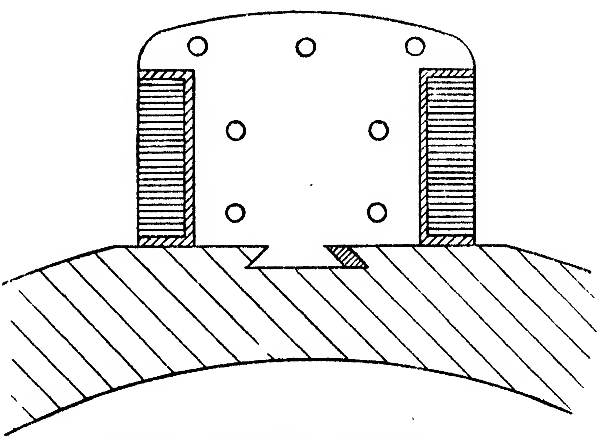


FIG. 212.—Laminated Pole Construction.

The poles may be fixed to the rim in several ways. A method in common use consists in screwing them on from the under side of the rim by means of set screws as shown in Fig. 211. When

laminated pole shoes are used with cast steel poles they may be attached by means of set screws driven in from the pole face or by dovetailing them on as shown in Fig. 211, lateral movement being prevented by means of keys. When the poles are laminated throughout they may be dovetailed on to the yoke ring in the same

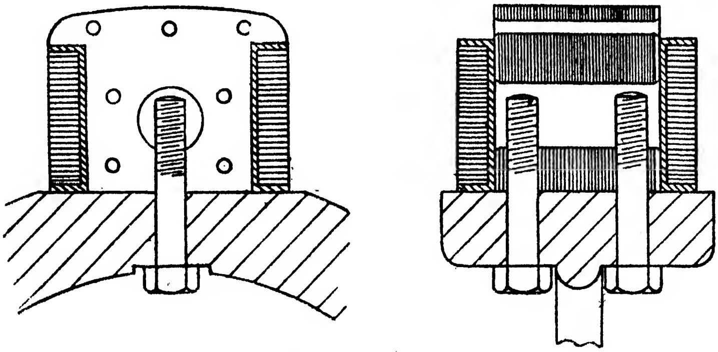


FIG. 213.—Laminated Pole Construction.

way (see Fig. 212). When ventilating ducts are employed in the poles, holes should be made through the yoke ring to meet them, and the ducts should further be in line with the stator air ducts so as to obtain a through passage for the air.

Sometimes laminated poles are attached to the yoke ring by means of set screws. In this case the laminations are pierced from side to side by a wrought iron bar so as to provide something solid for the set screws to grip (see Fig. 213).

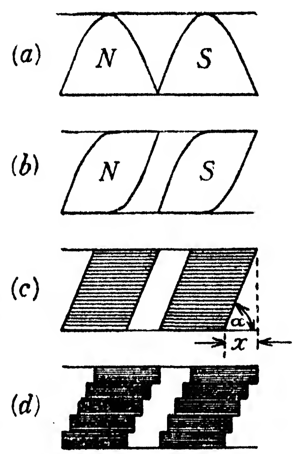


FIG. 214.—Shaped Pole Shoes to produce Sine Wave.

Shape of Pole Shoe.—If a rectangular pole shoe is employed with a uniform air-gap the resulting theoretical wave form obtained would be a rectangular one. Actually, due to fringing, the two vertical sides of the rectangle would become sloping ones, but notwithstanding this the wave form obtained in such a case is very far from being a sine wave. The E.M.F. induced in a particular conductor is proportional to its length in the magnetic field, the density of the magnetic field and the velocity of the conductor. The latter must be kept constant so that either the active length of the conductor or the density in the air-gap must be made to vary according to a sine law. The active length of the conductor is settled by the shape of the pole face. Fig. 214 (*a*) shows a development of the air-gap wherein the active length of the conductor is

proportional to $\sin \theta$. Apart from the mechanical difficulties presented by such a construction, there would be a tremendous amount of magnetic leakage across the adjacent tips of neighbouring poles. By turning the second part of each pole face round, however, as shown in Fig. 214 (*b*), the disadvantage of the magnetic leakage is obviated, and by the approximation shown in Fig. 214 (*c*) the manufacturing difficulties are done away with. If preferred, the same effect may be obtained by having bunches of stampings displaced with respect to one another, as shown in Fig. 214 (*d*).

The skewing of the pole shoes does not appear very noticeable in practice, as it is only necessary to do it to a small extent. Take the case of a 6-pole alternator with an air-gap diameter of 18 inches. With a ratio of pole arc to pole pitch of 0·65, the distance x in Fig. 214 (*c*) would be

$$\frac{\pi \times 18}{6}(1 - 0{\cdot}65) = 3{\cdot}3 \text{ inches.}$$

With an armature core length of 9 inches this means that the angle α is given by

$$\tan \alpha = \frac{9}{3{\cdot}3} = 2{\cdot}73,$$

whence $$\alpha = 70°.$$

Another method of obtaining a good approximation to a sine wave consists in having an air-gap of variable length, so that the magnetic density in the gap varies according to a sine law. The minimum air-gap occurs along the centre line of the pole and corresponds to the point of maximum induction and the peak of the wave. If this length is represented by l, the length of the gap at any other point displaced by $\theta°$ electrical from the centre line is given by

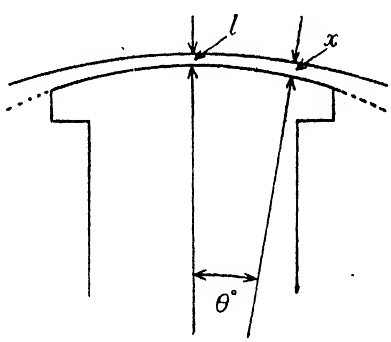


FIG. 215.—Variable Air-gap to produce Sine Wave.

$$x = \frac{l}{\cos \theta}.$$

Fig. 215 shows such a variable air-gap. When $\cos \theta$ gets small the air-gap becomes very long and the pole shoe is cut away altogether, the small amount of flux required being produced by fringing. Supposing a ratio of pole arc to pole pitch of 0·7 be employed, the length of the air-gap at the extreme end of the pole shoe is

$$\frac{l}{\cos(0{\cdot}7 \times 90°)} = \frac{l}{\cos 63°} = 2{\cdot}2\, l.$$

It is rarely worth while carrying the pole shoe beyond this.

A simple approximation to this form of pole shoe can be obtained by making the radius of curvature of the pole shoe equal to 0·7 times the radius of curvature of the stator core.

Field Coils.—In addition to the usual coil winding for the poles, a type of winding frequently adopted consists in having a single layer of strip wound on edge, the strip being bent to the right shape before assembling. The adjacent layers are insulated by a layer of paper or other insulation, the outside surface being coated with varnish. This construction is very sound mechanically, as there is no tendency for adjacent conductors to roll over one another, due to centrifugal force. Another advantage of this type of coil is its superior heat dissipating properties.

Slip Rings.—The slip rings necessary for conveying the field current to the rotor are generally made of gun metal, brass or cast iron. The usual type of construction consists of a cast iron spider carried from the shaft upon which the slip rings are bolted, insulating bushes and washers being employed as in the brush spindles of C.C. machines. A common type of construction is shown in Fig. 216, there being three lugs cast on to each ring for bolting on to the spider. Another lug is also cast on for the purpose of attaching the lead carrying the current to the field coils. The dimensions of the slip rings are usually settled from mechanical considerations, as these demand larger sections than would be required on purely electrical grounds.

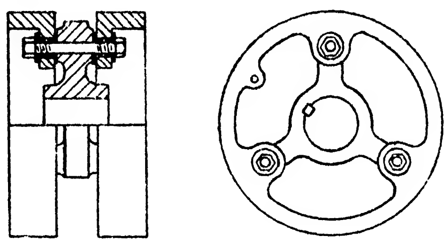


FIG. 216.—Slip Rings.

Carbon brushes are now employed in preference to copper gauze, as the wear on the slip ring is considerably less, current densities of from 30 to 40 amperes per square inch being adopted with ordinary types of brushes.

Rotors for Turbo-alternators.—Owing to the high speed at which turbines run, the number of poles is usually either two, four, or six, the diameter of the rotor being kept down in order to prevent the peripheral speed from exceeding the safe limit. This results in the appearance differing greatly from that of low-speed rotors and has led to the development of the *cylindrical* type of rotor, where the field windings are distributed in slots, in addition to what is known as the *salient* pole type, where the rotor has definite poles and windings as in the case of low-speed sets.

Salient Pole Type.—This type is frequently adopted for 4- and 6-pole machines and consists of a rotor body having a star-shaped cross section, forged to shape and bored out to receive the shaft as shown in Fig. 217. The pole winding consists of copper strip

wound either on edge or flat in sections. This winding is kept in position by means of the pole shoe, which is sometimes solid and sometimes laminated. The pole shoe is held securely in place by

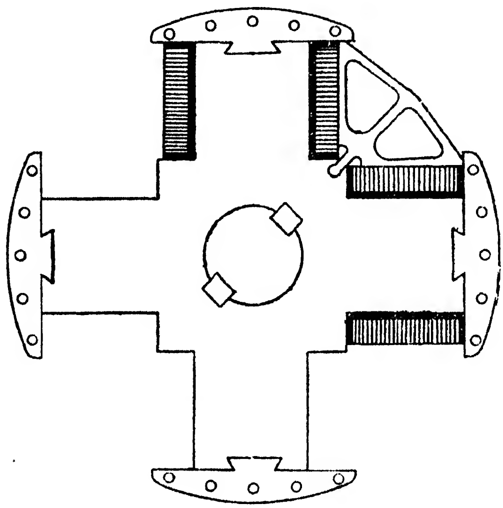


FIG. 217.—Salient Pole Type Rotor.

being dovetailed on to the pole. Owing to the large centrifugal force set up, a number of phosphor bronze clamps are provided, as shown in the diagram, for the purpose of preventing the winding from slipping. These clamps have a ball and socket action so as to allow of a little play when initially fixing them in position. Longitudinal vent spaces are bored at intervals along the poles to assist in keeping the machine cool.

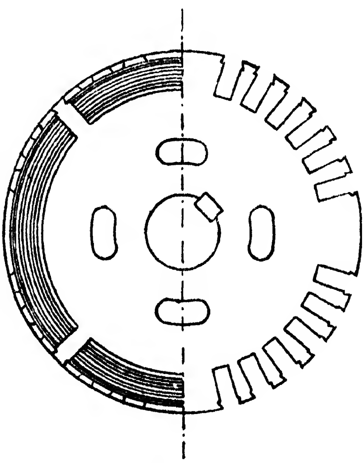


FIG. 218.—Cylindrical Type Rotor.

Cylindrical Type.—The cylindrical type of rotor consists of a number of mild steel laminations clamped together between two end plates, the laminations being slotted after the style of an ordinary armature, except that at certain places the slots are omitted, as shown in Fig. 218. This results in a uniform air-gap being formed all the way round. The slots receive the pole winding, which consists of former wound coils of copper strip laid in flat and suitably insulated. A phosphor bronze wedge is driven into the mouth of each slot so as to keep the winding in place. The

end connections are also covered by means of phosphor bronze end shields, which prevent them from flying outwards under the action of centrifugal force.

The cylindrical type of rotor has certain advantages over the salient pole type, inasmuch as a superior cooling surface is obtained, due to the distribution of the winding. Further, the mechanical balancing of the rotor will not be so difficult, and, due to the smooth surface exposed by the cylindrical type, noiseless running is more likely to be attained.

Since the winding of the cylindrical type rotor is distributed over several slots, the flux density in the air-gap will increase towards the centre of the pole, thus tending towards the generation of the ideal sine wave. For example, if the winding is distributed over three pairs of slots the theoretical wave form would be of the type shown in curve (*a*), Fig. 219, whilst if the same winding were concentrated in the outermost slots the theoretical wave form would be given by curve (*b*), Fig. 219. The latter arrangement generates a higher R.M.S. value of the voltage, but does so at the expense of the wave form.

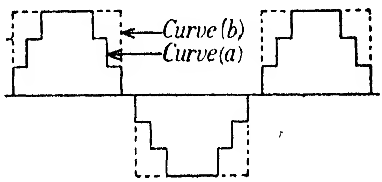


FIG. 219.—Theoretical Wave Form.

Armature Windings.—The ordinary type of winding used in C.C. machines is also suitable for alternator armatures, but in the larger

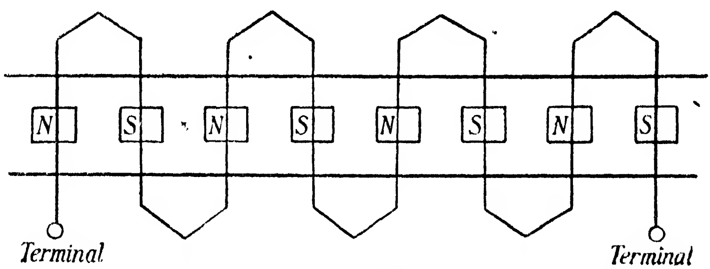


FIG. 220.—Single Phase Bar Winding. One Slot per Pole.

machines a somewhat different type is employed. The ordinary C.C. winding is called a *closed circuit* winding, because it closes on itself, whilst the special alternator windings about to be described are called *open circuit* windings, since there is no closed circuit in the armature itself.

One special feature in these windings is that there is an exact number of slots per pole, whilst in the case of polyphase alternators there is an exact number of slots per pole per phase.

The simplest example is perhaps the wave wound bar winding illustrated in Fig. 220, where the armature is shown in a development. The E.M.F.'s in all the conductors aid one another, there

being eight conductors in all (in an 8-pole alternator). This only utilises one slot per pole, and if more slots are to be used the winding

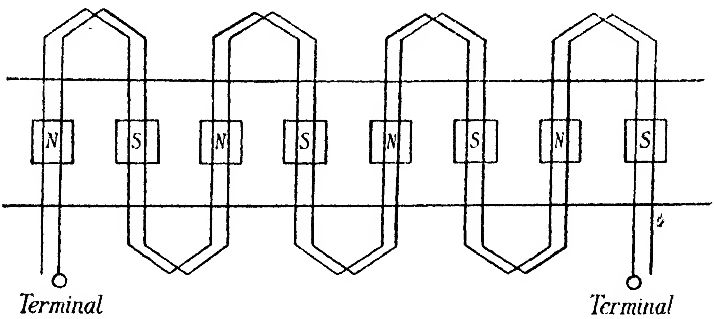


FIG. 221.—Single Phase Bar Winding. Two Slots per Pole.

would be as shown in Fig. 221, there being one unsymmetrical end connection for every time the winding goes right round the armature. In the same way, the winding can be extended to more slots per pole. It is not usual to fill up all the slots in a single phase alternator, since the last conductors add but little to the total voltage, as the phase of the conductors in the various slots differs considerably. For example, if there are six slots per pole, the phase difference between conductors in adjacent slots is 30°, and the vector diagram in Fig. 222 shows the various voltages obtained by using 1, 2, 3, 4, 5 and 6 conductors. The following table shows the numerical values of the voltage obtained in the different cases, and as each conductor adds to the impedance of the winding, it is usual in such cases to limit the number of slots used to about two-thirds of the total.

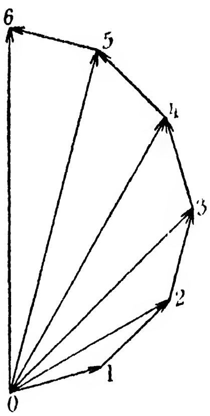


FIG. 222.—Vector Diagram showing Phase Difference of Conductors in Different Slots.

TABLE SHOWING EFFECT OF DISTRIBUTING WINDING.

No. of Slots used	Voltage.	Voltage added by last Conductor.
1	1·00	1·00
2	1·93	0·93
3	2·73	0·80
4	3·34	0·61
5	3·73	0·39
6	3·86	0·13

As an example of a single phase coil winding, the case of an armature having six slots per pole will be taken, four of them being wound. Fig. 223 shows what is known as an *ordinary* or *whole-*

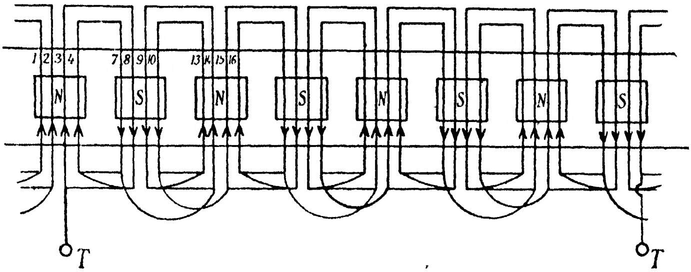


FIG. 223.—Ordinary or Whole-coiled Winding.

coiled winding, there being as many coils as there are poles. Each coil is taped up in one piece, the conductors in the different slots being separated, but the end connections being all taped up together, the individual coils looking like that shown in Fig. 224. Instead of the conductors in slots 9 and 10 being connected to conductors in slots 13 and 14, they might be brought back to the conductors in slots 1 and 2, which are in a similar position with respect to the field and are under a pole of the same polarity. Altering the connections in this manner throughout, the winding shown in Fig. 225 is obtained, this being called a *half-coiled* or *hemitropic* winding. There is now only one coil per pair of poles, but each coil is twice as large as in the previous case.

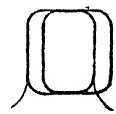


FIG. 224. Armature Coil.

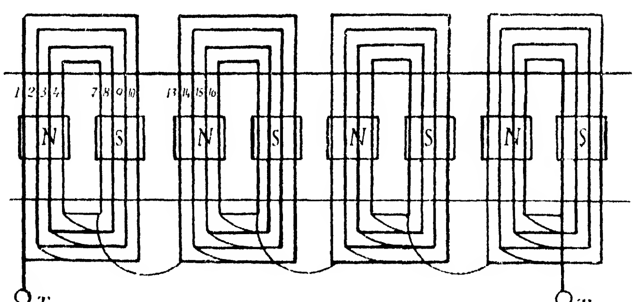


FIG. 225.—Hemitropic or Half-coiled Winding.

Two Phase Windings.—A two phase bar winding can be obtained from the single phase bar winding shown in Fig. 221 by adding an exactly similar winding midway between the conductors of the first. In this way Fig. 226 has been developed, there being four terminals instead of two. All the slots may now be used economically, whilst several slots per pole for each phase may be adopted.

A two phase coil winding may be evolved from Fig. 225 by placing a second winding in the vacant slots left by the first phase as shown in Fig. 227. In order to make the end connections of the

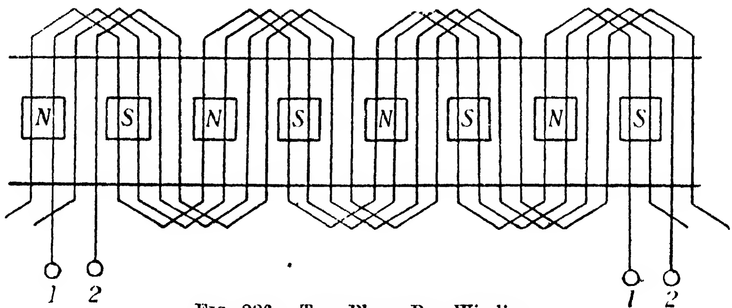


FIG. 226.—Two Phase Bar Winding.

two phases clear one another, those of one phase must be bent upwards as shown in Fig. 228, all the coils of the first phase having one shape of end connection and all those of the second phase having

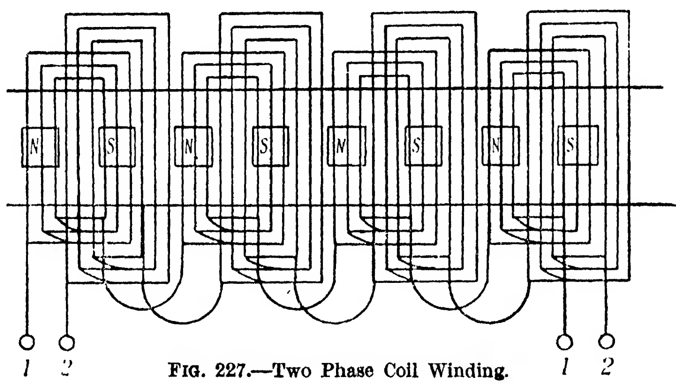


FIG. 227.—Two Phase Coil Winding.

the other shape. Such a winding is called a *two range* or a *two plane* winding.

Three Phase Windings.—A three phase bar winding can be evolved

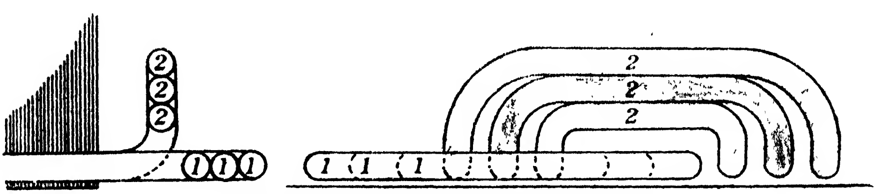


FIG. 228.—End Connections in Two Ranges.

from the single phase one shown in Fig. 221 by inserting two other windings, each displaced by two-thirds of a pole pitch as shown in Fig. 229, the three front ends being brought out to the terminals,

whilst the three rear ends are joined together, thus forming the star point. Since there must be a whole number of slots per pole per

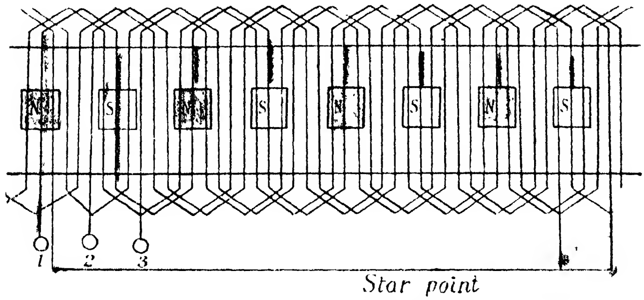


FIG. 229.—Three Phase Bar Winding.

phase, it follows that the total number of slots must be a multiple of three times the number of poles.

One type of three phase coil winding is obtained from that shown

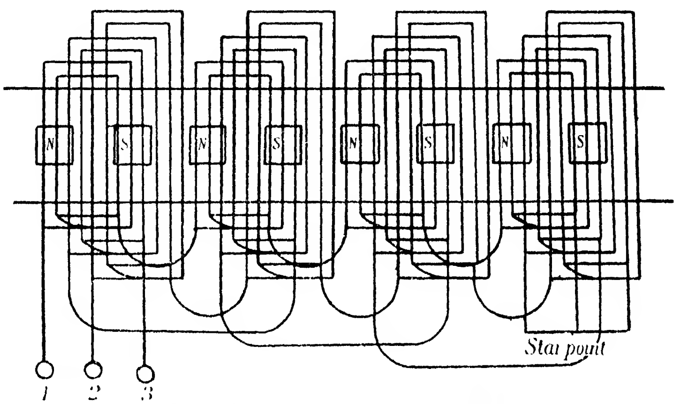


FIG. 230.—Three Phase Coil Winding.

in Fig. 225 by adding two other similar windings spaced two-thirds of a pole pitch apart as shown in Fig. 230. This type of winding requires three shapes of coil and results in the end connections

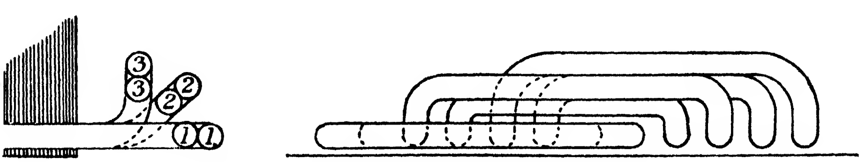


FIG. 231.—End Connections in Three Ranges.

occupying three planes, forming a three range winding as shown in Fig. 231. If the middle phase were connected in the same way as

the other two, it would result in the three phases being displaced by 60° with respect to one another instead of by 120°, and to remedy this the middle one is reversed, which has the effect of adding 180° to its phase, thus making it 240° out of phase with the first winding instead of 60°. An advantage of this type of winding is that it can be divided into sections and, in case of a breakdown, the faulty coil can be removed without seriously disarranging the remainder.

Another type of three phase winding is obtained by connecting

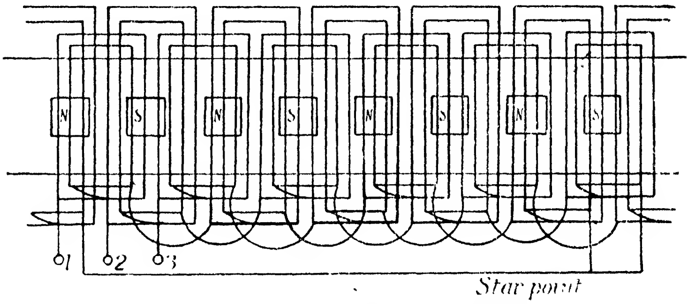


FIG. 232.—Three Phase Coil Winding.

the conductors of the middle phase (Fig. 230) under the second pole to those under the third pole instead of the first. This winding is shown in Fig. 232 and leads to only two shapes of end connection, coils of a particular phase being of each shape alternately. This has the advantage of equalising the impedances of the three phases should the lengths of the coils of different shape not be the same. The end connections in two ranges are shown in Fig. 233.

Breadth Factor.—In the majority of instances the armature

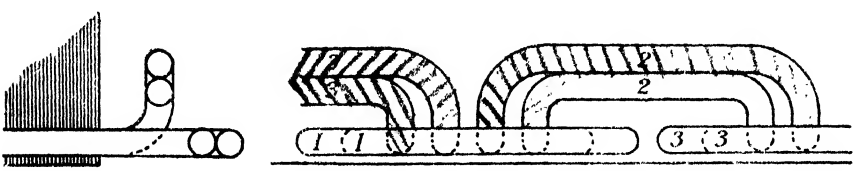


FIG. 233.—End Connections in Two Range Three Phase Winding.

winding is not concentrated in one slot per pole per phase. As the result of this, the conductors in neighbouring slots have E.M.F.'s induced in them which differ slightly from one another in phase. Since the total E.M.F. induced in the coil is due to the E.M.F.'s in the component conductors connected in series, this difference in phase causes a loss of voltage, since the individual E.M.F.'s must be added vectorially instead of arithmetically. For example, in a three phase alternator having three slots per pole per phase there are nine slots distributed over one pole pitch. The phase difference

between the E.M.F.'s induced in conductors in adjacent slots is therefore 20°. The vector diagram for this case is shown in Fig. 234, the resultant voltage being 2·88 instead of three times the voltage per slot. Distributing the winding in three slots thus causes a 4 per cent. loss in voltage, but is nevertheless preferable to having it concentrated in one slot of three times the size, as this would tend towards the production of irregularities in the wave form. For this and other reasons, the single slot winding has gone into disuse.

The ratio of the voltage actually obtained to the voltage which would be obtained if the winding were all concentrated in one slot

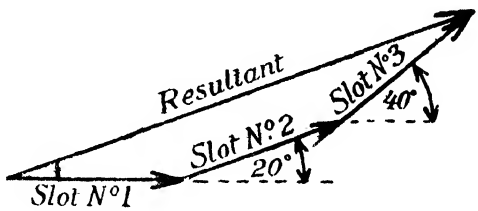


FIG. 234.—Effect of distributing Winding in Three Slots.

is called the *Breadth Factor*, and in the case shown in Fig. 234 its value is $\frac{2 \cdot 88}{3} = 0 \cdot 96$. The value of the breadth factor is unity if the winding is concentrated into one slot per pole per phase, but when it is distributed its value is always less than unity, decreasing as the number of slots is increased.

According to present practice, the number of slots per pole per phase is usually limited to five in the case of flywheel type alternators and six in the case of turbo-alternators. The following table gives the values of the breadth factor in a number of cases of single-, two- and three-phase machines.

TABLE OF BREADTH FACTORS.

Slots per Pole per Phase.	Single Phase, Half Slots Wound.	Single Phase, Two-thirds Slots Wound.	Two Phase, All Slots Wound.	Three Phase, All Slots Wound.
1	—	—	1·000	1·000
2	1·000	—	0·924	0·966
3	—	0·866	0·910	0·960
4	0·924	—	0·906	0·958
5	—	—	0·904	0·957
6	0·910	0·836	0·903	0·956

Effect of Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}}$.—In order to study the effect of this ratio on the induced voltage, the case of an alternator will be dis-

cussed where the pole pitch and the lines per pole are fixed. As a first approximation, a rectangular pole shoe and a uniform air-gap will be assumed, the resulting wave form being, of course, rectangular. A variation in the ratio $\frac{\text{pole arc}}{\text{pole pitch}}$ is now obtained by choosing different lengths of pole arc, getting different magnetic densities at the pole face. The area of the resulting flux curve is constant, since this is proportional to the product of the magnetic density

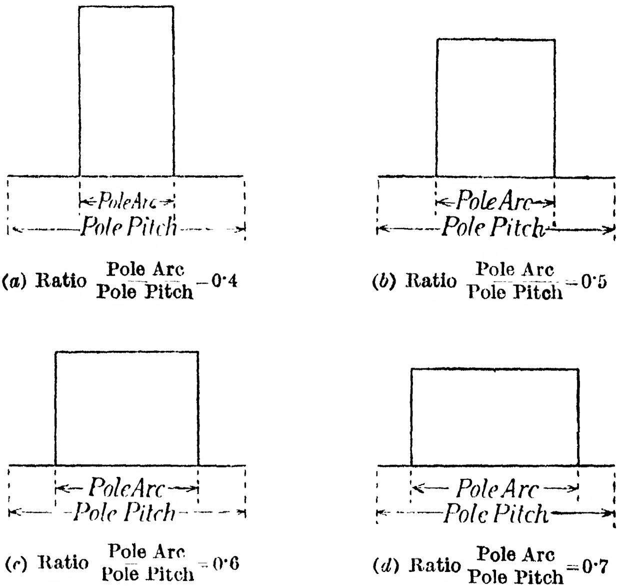


FIG. 235.—Effect of Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}}$ on Wave Form.

and the pole arc, and thus represents the total flux per pole, which is assumed constant. With a constant speed of rotation, the E.M.F. wave form is similar to the curve of flux distribution, and thus the area of the E.M.F. wave form is also constant. Working out the form factors and relative R.M.S. values of the E.M.F.'s for the various cases in Fig. 235, these are found to be different and they are tabulated in the following table.

Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}}$.	Form Factor.	Relative R.M.S. Voltage.	Relative Max. Voltage.
0·4	1·58	0·632	1·000
0·5	1·41	0·566	0·800
0·6	1·29	0·516	0·667
0·7	1·19	0·478	0·571

According to the above table, the ratio which gives a form factor closest to that of a sine wave is approximately 0·7, but if the pole face is shaped it will be found that the best value is somewhat lower than this.

Calculation of E.M.F.—Each conductor on the armature cuts Φp lines of force in every revolution of the armature, Φ being the number of lines of force per pole and p the number of poles. Since each turn consists of two active conductors together with the inactive end connections, the lines cut per turn per second are $2\Phi pn$, where n represents the revolutions per second. But the frequency f is equal to $\frac{pn}{2}$, so that the *average* E.M.F. induced per turn is

$$2\Phi pn \times 10^{-8} \text{ volts}$$
$$= 4\Phi f \times 10^{-8} \text{ volts.}$$

With a sinusoidal wave form the form factor $\left(= \frac{\text{R.M.S. value}}{\text{average value}}\right)$ is 1·11, and therefore the R.M.S. value of the induced E.M.F. per turn is

$$4{\cdot}44\,\Phi f \times 10^{-8} \text{ volts.}$$

For other shapes of E.M.F. wave the R.M.S. voltage per turn is given by

$$4k_1\Phi f \times 10^{-8} \text{ volts,}$$

where k_1 is the form factor.

If there are T turns in series per phase the voltage per winding becomes

$$E = 4k_1\Phi f T \times 10^{-8} \text{ volts.}$$

It has been shown, however, that distributing the winding in several slots per pole results in a slight loss of voltage, and so the breadth factor has to be taken into account, when

$$E = 4k_1k_2\Phi f T \times 10^{-8} \text{ volts,}$$

k_2 being the breadth factor.

The voltage obtained in this way is the open circuit voltage per winding, and, in the case of a three phase star connected alternator, the line voltage is obtained by multiplying by $\sqrt{3}$. To find the full load terminal voltage, the drop caused by the armature impedance and the effect of armature reaction must be taken into account, and this will be discussed later (see p. 258).

Clamps for End Connections.—The end connections of the armature coils consist of a number of conductors carrying currents flowing in the same direction, and this results in considerable mechanical stresses being set up on occasion, particularly in the case of turbo-alternators. In ordinary low-speed alternators, no special precau-

tions need be taken to deal with these stresses, but with turbo-alternators it is different. This is due to the longer pole pitches employed and to the greater ratio of short circuit current to full load current which is characteristic of the turbo sets. Experience has shown that very heavy clamping devices are necessary in these cases to prevent the coils being torn from their fastenings under the influence of an accidental short circuit. Fig. 236 shows a typical

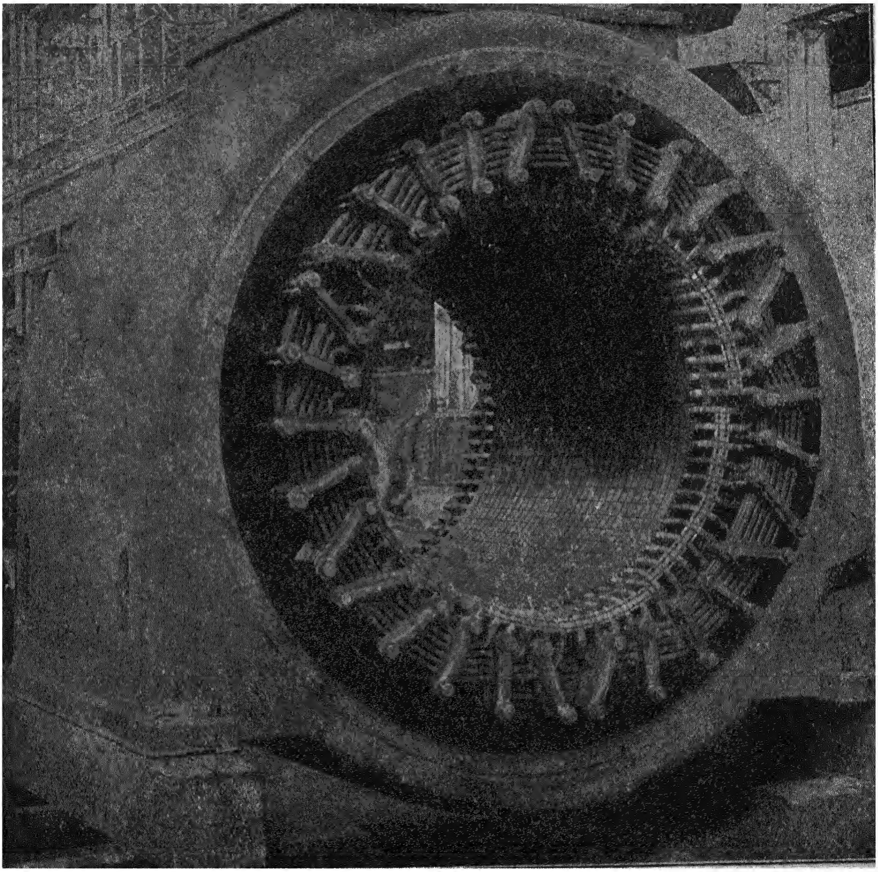


FIG. 236.—Turbo-alternator showing Clamping Arrangements.

example of the heavily clamped end connections of a turbo-alternator.

Damping Grids.—When a polyphase alternator delivers a current a rotating magnetic field is set up by the combined action of the armature currents, and if the speed of rotation of the rotor is not exactly uniform the poles will sometimes gain on this flux as it rotates and sometimes lag behind. The pole face can therefore

be imagined to be situated in a magnetic field which oscillates from side to side, and any closed circuit in the neighbourhood will have a current induced in it. The generation of such a current will tend to damp out the motion which produces it and thus tends towards smoother running. In order to effect this, special

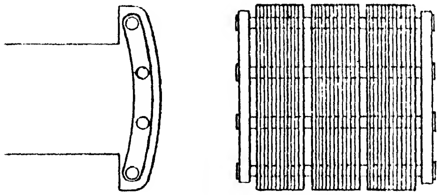


FIG. 237.—Damping Grid.

windings are sometimes wound in slots or holes in the pole shoe. These windings are known as *damping grids* or *amortisseurs*, and consist of a number of heavy copper rods, one in each hole, riveted at the ends to a common bar so as to form a short-circuited grid, as shown in Fig. 237.

Excitation and Exciters.—The ordinary types of alternator require to be excited by means of C.C. from some external source, and for this purpose it is usual to instal special C.C. dynamos called exciters in connection with large alternators. One system frequently adopted consists in having an exciter for every alternator, directly driven from the alternator shaft. An alternative method is to have one exciter of sufficient capacity to deal with the excitation of all the alternators in question, the exciter being steam-driven. In this case the exciter can also be made to supply additional power for the purpose of lighting the generating station and for other auxiliary services. Sometimes also there will be a pair of C.C. bus bars convenient, in which case the necessary exciting current can be obtained from the mains.

When each alternator is fitted with its own exciter, the regulation of the exciting current can be effected by means of a resistance in the exciter field, which deals with a comparatively small current, but when a common exciter serves a number of alternators the field regulation must be effected by resistances in the exciter main armature circuit. The field regulating resistances must therefore be capable of carrying larger currents, thus resulting in an increase in size. On the other hand, the efficiency of the single exciter is higher and the initial cost lower than when individual exciters are employed.

Compounded Alternators.—When an alternator is made to deliver a current its terminal voltage drops by an amount which depends upon the magnitude and phase of the current. If the load changes slowly, this can be corrected by hand regulation, but when the load

is of a rapidly fluctuating character this is not sufficient. In order to make the alternator self-regulating, various methods have been devised depending upon either the action of some form of automatic regulator or else some form of compounding in the alternator itself. The latter frequently takes the form of boosting up the excitation by means of a rectified current obtained from the armature, a special form of commutator being attached to the alternator shaft.

CHAPTER XVII

ALTERNATORS.—PERFORMANCE AND TESTING

Magnetisation Curve.—The graphical relationship which exists between the exciting current and the terminal voltage of the armature is called the magnetisation curve, and can be obtained experimentally by taking various values of the exciting current and observing the corresponding armature voltages, the machine being on open circuit. Usually it will be necessary to insert a considerable resistance in series with the field winding in addition to the ordinary shunt regulator, in order to bring down the exciting current so as to obtain points on the lower portion of the curve. The resulting graph follows the general shape of the *B—H* curves. Another magnetisation curve can be obtained by making the alternator give out full load current at unity power factor and obtaining the relation between the exciting current and the terminal voltage as before. Since the armature current is to be kept constant, it is necessary to have a load, the resistance of which can be varied between wide limits. A water resistance will be frequently of use in this connection both for single and polyphase alternators. When the machine is a three-phaser, the three legs of the load resistance are usually connected in star, but when a water resistance is employed a mesh arrangement is adopted, there being three parallel plates immersed in the liquid. If the necessary inductances are available, the test can be repeated with a power factor of less than unity. Fig. 238 shows the nature of the curves which are obtained in this test.

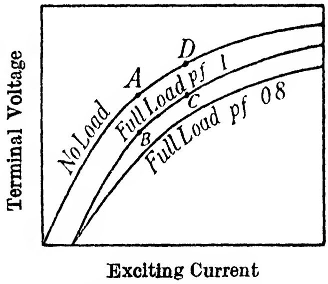


FIG. 238.—Magnetisation Curves.

Load Characteristic.—The load characteristic of an alternator is obtained by determining the relationship between the terminal voltage and the load current, the exciting current and speed being kept constant. For the purposes of test, an alternator is frequently driven by a C.C. shunt motor and as the load comes on the speed slightly falls. This must be compensated for by shunt regulation in the motor field circuit.

As the armature current is increased the terminal volts drop, due to a number of causes. The resistance and reactance of the armature winding absorb some of the volts which are generated, whilst the armature reaction generally results in an actual decrease of the volts generated, owing to its weakening effect on the magnetic field. The magnitude of the latter effect depends to a very large extent upon the angle of lag or lead of the armature current and results in a strengthening of the magnetic field when a leading current is delivered. In fact, if the angle of lead be sufficient, the increase in the generated volts is such that, taking into consideration the phase of the volts absorbed by the armature impedance, the terminal voltage on load is actually higher than the no-load E.M.F. Fig. 239 shows a typical series of load characteristics obtained at different power factors, and clearly shows the effect of lagging and leading currents. When the alternator is a polyphase machine,

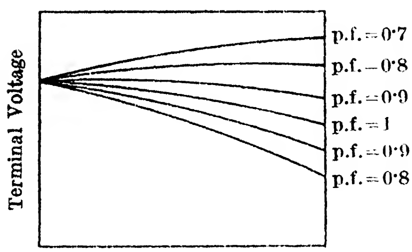


FIG. 239.
Load Characteristics.

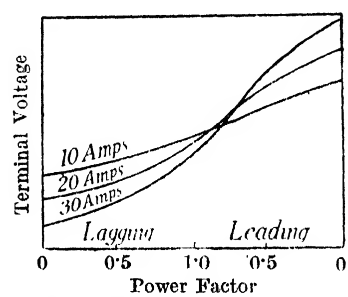


FIG. 240.—Effect of Power Factor on Terminal Voltage.

care must be taken to obtain a balanced load, both as regards power factor and current.

When performing the test at any particular power factor, it will be found convenient to insert a power factor indicator, as otherwise great difficulty will be experienced in maintaining it constant. An alternative method is to take a series of readings with a constant current but varying power factors, obtained by altering the relative amounts of resistance and reactance in the circuit. A number of such curves must be taken, each one corresponding to a different current (see Fig. 240). To obtain the load characteristic for a particular power factor, draw a vertical line through the value chosen and read off the voltages corresponding to the various currents from the curve. In this way, one point is obtained from each of the curves in Fig. 240, but this can be repeated for as many power factors as desired.

Regulation.—When an alternator is subjected to a varying load, the voltage at the terminals of the armature varies to a certain extent, and the amount of this variation determines the *regulation* of the machine. The numerical value of the regulation is usually

given as the percentage fall in voltage when full load is switched on, or, alternatively, the percentage rise in voltage when full load is switched off, the excitation being adjusted initially to give normal voltage in each case. The latter method of expressing the regulation gives a lower figure than the former and finds favour, therefore, with the manufacturers. The reason for this difference can be seen by referring to Fig. 238. Assume the machine to be on open circuit and giving a voltage represented by the point A on the no-load curve. On switching on full load at unity power factor the voltage drops to that represented by the point B and the drop in voltage is given by the vertical distance between A and B. Now let the excitation be increased until the original voltage is reached, full load being maintained all the time. In this way point C is reached. On throwing off the load, the voltage rises to a value given by point D, and the vertical distance between C and D measures the rise in voltage. Owing to the greater saturation due to the larger exciting current, this rise is less than the fall obtained when the load is switched on. If this be repeated with a load having a power factor of less than unity, the current being a lagging one, the percentage fall or rise of the voltage would be correspondingly greater.

The numerical values of the regulation of modern alternators will be usually about 6 per cent. for unity power factor and about 20 per cent. when operating on a power factor of 0·8.

If it is not possible or desirable to measure the regulation by direct experiment, it can be obtained indirectly by other means which do not necessitate the alternator being fully loaded. For this purpose it is necessary to perform two tests, viz., the open circuit test to obtain the magnetisation curve (already described) and a short circuit test to obtain the impedance of the alternator armature. The latter test will now be dealt with.

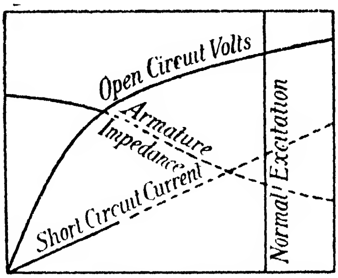


FIG. 241.—Method of obtaining Armature Impedance.

Short Circuit Test.—In this test the armature is directly short-circuited through an ammeter capable of carrying at least the full load current of the machine, whilst additional resistances must be inserted in the field circuit to limit the exciting current so as not to burn out the armature. The relation between the armature current on short circuit and the exciting current is then obtained over as large a range as possible, the speed being kept constant. As will be explained later, small variations of the speed do not affect the armature current to any appreciable extent, so that there is no necessity to have close speed regulation. The resulting curve should be a straight

line, as shown in Fig. 241, and, assuming the linear relationship still to hold good, it may be produced, this portion being shown dotted. In this way the probable short circuit current can be obtained corresponding to normal excitation. At this same point the open circuit voltage can be read off the magnetisation curve, and, assuming the same voltage to be generated when on short circuit, the impedance of the armature can be calculated (see p. 257). In fact, the ratio of the open circuit voltage to the short circuit current at each value of the excitation gives the impedance of the armature, but since the two curves differ in shape the ratio will not be constant. The third curve in Fig. 241 shows the values of the armature impedance calculated in this manner.

If the resistance of the winding be measured by means of a continuous current, the reactance can be calculated and the impedance triangle determined.

In the majority of cases the resistance will be small compared with the reactance, so that if the speed varies, the frequency and the reactance ($= 2\pi fL$) will vary accordingly. Thus the impedance is practically proportional to the speed, and since the induced voltage is also proportional to the speed, the short circuit current will remain practically constant over a wide range.

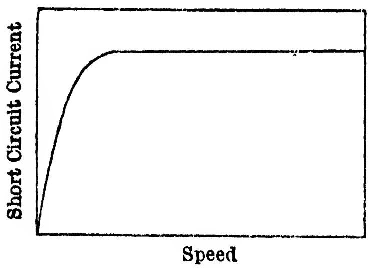


FIG. 242.—Effect of Speed on Short Circuit Current.

As an example, consider an armature having a resistance of 0·1 ohm and a reactance of 0·8 apparent ohm at a frequency of 50. The impedance is $Z = \sqrt{0{\cdot}1^2 + 0{\cdot}8^2} = 0{\cdot}806$ apparent ohm, and if the induced voltage is 100 the short circuit current is $\frac{100}{0{\cdot}806} = 124$ amperes. On halving the speed, the frequency drops to 25 and the impedance becomes $Z_1 = \sqrt{0{\cdot}1^2 + 0{\cdot}4^2} = 0{\cdot}412$ apparent ohm. The voltage is now 50 and the short circuit current becomes $\frac{50}{0{\cdot}412} = 121$ amperes, a drop of only 3 per cent. for a speed reduction of 50 per cent. In fact, quite a respectable current may be obtained by turning the armature round by hand. Fig. 242 shows a typical example of the variation of the short circuit current with the speed.

Polyphase alternators behave in the same way as single phase ones in this respect, all the terminals being short-circuited through the necessary ammeters.

Armature Reaction.—When the armature is supplying current there are two distinct sources of M.M.F. acting upon it, viz., that

due to the main field system and that due to the armature current itself. The former is approximately constant, but the latter is not so in a single phase case, since it alternates with time and rotates in space. Fig. 243 represents successive instants of time in a single phase alternator having one conductor per pole, the current being supposed to be in phase with the E.M.F. Assuming a sinusoidal space distribution of the main flux and also that the curve of armature M.M.F. follows a sine law with respect to time, the maximum induced E.M.F. will occur when the conductors are passing the centre line of the poles. The armature current will be also a maximum at this instant, and the curve of armature M.M.F. will now have its maximum height. Points above the line are taken to represent M.M.F.'s aiding the main N pole. Successive instants are shown at intervals of 30° electrical, the armature M.M.F. dying down and reversing synchronously with the current. Now consider a point situated on the centre line of the N pole. There is the constant main M.M.F. acting, but in addition to this there is an alternating M.M.F. due to the armature alternately weakening and strengthening it. The average flux is unaffected, but it pulsates from instant to instant, now less than and now greater than its mean value. Next consider a point situated at the right-hand edge of the N pole (the trailing pole tip). This will be subjected to similar fluctuations of M.M.F., but the strengthening will be much more than the weakening. The opposite will be the case for a point situated at the leading pole tip of the N pole where there is more weakening than strengthening. The same thing occurs under the S pole, with the result that there is no resultant strengthening or weakening, but only distortion. What one side of a pole gains the other side loses, so that it corresponds to a pulsating swing of the flux from side to side of the poles.

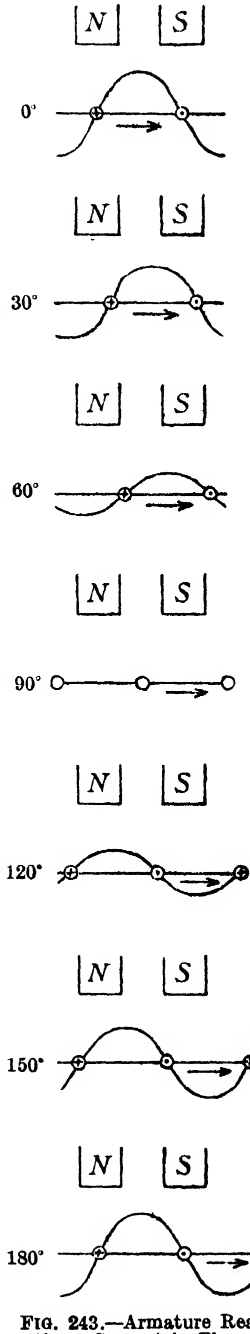


FIG. 243.—Armature Reaction. Current in Phase.

Next consider the same case when the current lags behind the induced E.M.F. by 90°. These conditions are represented in

Fig. 244 and can be traced out in the same way as in the previous example. Taking a point lying under the centre line of a pole, it is found that there is always a weakening effect, although variable in magnitude. The trailing pole tip will be subjected to an action which is mainly weakening, although a slight strengthening occurs about 120° from the start. The leading pole tip is also subjected to a general weakening effect with a slight strengthening about 60° from the start. The net result is a perpetual weakening with a slight periodical swaying of the resultant flux from side to side.

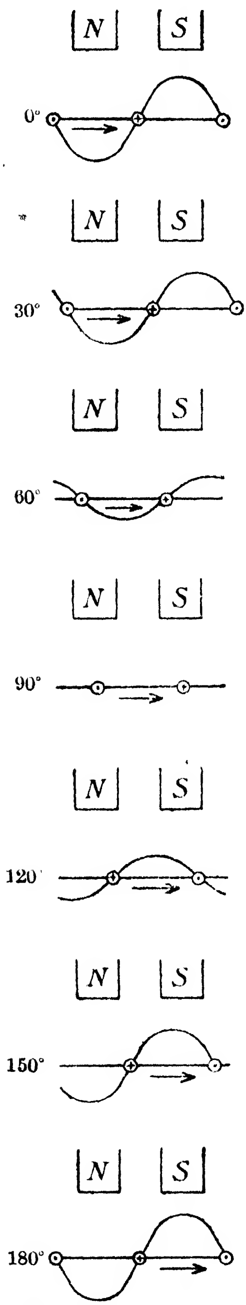


FIG. 244.—Armature Reaction. Current lagging 90°.

When the current lags by an angle less than 90° there is both weakening and distortion, the weakening getting greater and the distortion less as the angle of lag increases.

When a leading current is considered, the current arrives at its maximum value before the conductor gets to the centre line of the pole, and the weakening effect is turned into a strengthening one, again accompanied by distortion, the strengthening increasing and the distortion decreasing as the angle of lead is increased.

It is thus seen that when an alternator is delivering a lagging current the magnetic flux is decreased in magnitude so that the induced voltage is actually less than on no-load, whilst the phase relationship of the current causes an increased loss of the voltage which is generated, thus accounting for the increased rate at which the voltage falls away when the machine is supplying an inductive load.

Conversely, when an alternator is supplying a leading current the flux is increased, thus causing an increase in the voltage generated, whilst the phase relationship existing between the current and voltage reduces the drop in voltage and may even make it a rise if the current leads by a sufficient angle. The load characteristic of the alternator, therefore, rises with increase of load under these conditions.

In the case of a polyphase alternator operating under a balanced load, the resultant armature M.M.F. of the various phases is constant and remains fixed with respect to the main field system so that distortion is produced

when the power factor is unity, this being combined with a weakening effect when the current is lagging and a strengthening effect when the current is leading.

Synchronous Impedance.—The method of determining the impedance of the armature described on p. 254 assumes that the voltage generated when short circuited is the same as when on open circuit, and this assumption is not justified, for in addition to the armature possessing resistance and reactance there is a true armature reaction as well. Armature reaction is due to the magnetising action of the armature upon the main field. Armature reactance is due to an E.M.F. being induced in the armature conductors, due to the current carried by it. The first effect, by weakening the field, reduces the voltage actually generated; the second uses up some of the generated volts before the terminals are reached. Really, the main induced E.M.F. and the E.M.F. of self-induction do not combine as they do not exist separately, but the M.M.F.'s which produce the respective fluxes exist and produce a resultant flux. It is convenient, however, to regard the two E.M.F.'s as separate from a mathematical standpoint.

When the current in the armature is a lagging one the effect of armature reaction is to weaken the main field besides distorting it, whilst when the armature current leads the weakening action becomes a strengthening one. Again, when the armature current lags armature reactance sets up an E.M.F. which is in partial opposition to the induced E.M.F., causing a drop in voltage, whilst when the armature current leads the reactance introduces an E.M.F. having a component which increases the induced E.M.F. From this point of view the armature reaction and reactance produce similar effects and may be combined in what is called the *synchronous reactance.* This is not a true reactance, but can be considered as such for a variety of purposes. When combined with the resistance in the usual way, a quantity called the *synchronous impedance* is obtained, which thus takes into consideration the armature reaction and the armature reactance in addition to its resistance.

The synchronous reactance is usually not constant, but pulsates periodically on account of the varying reluctance of the armature magnetic circuit, which also varies between maximum and minimum limits synchronously. A very approximate value of the synchronous reactance can be obtained experimentally by passing an alternating current of the correct frequency through the armature whilst stationary and measuring the impedance for different positions of the armature with respect to its field system. If the resistance be also measured, the synchronous reactance can be calculated for each position. This experiment may be carried out with the main field either on or off, this making a considerable difference to the result, the average value with normal field excitation coming out lower than when unexcited, due to the higher values of the flux density

obtained and the consequent drop in the permeability. F245 shows the way in which the synchronous reactance varies a particular small 4-pole alternator.

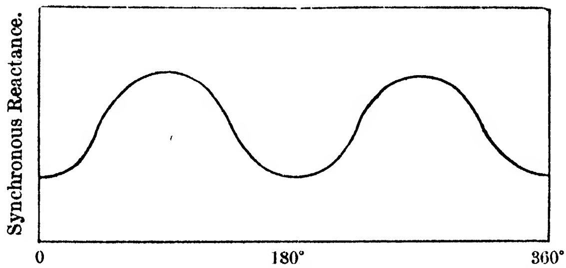


FIG. 245.—Variation of Synchronous Reactance with Position.

Predetermination of Regulation by Behn-Eschenburg's E.M.F. Method.—In this method the open circuit magnetisation curve and the short circuit characteristic are necessary, so that the synchronous

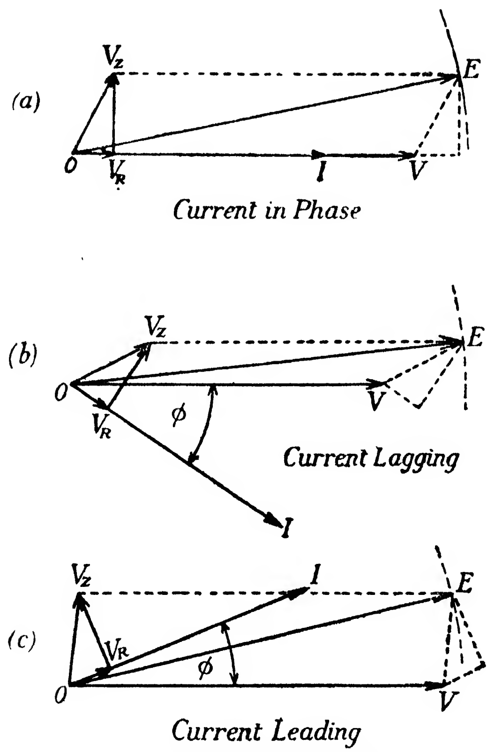


FIG. 246.—Behn-Eschenburg's E.M.F. Diagram.

impedance may be obtained as explained above. The impedance triangle is then determined and the regulation can be obtained by drawing a vector diagram as shown in Fig. 246. In the first case

the full load current is assumed, the power factor of the external circuit being unity. By multiplying each side of the impedance triangle by the current, a voltage triangle is obtained representing the voltages absorbed by the armature resistance and synchronous reactance. Taking the phase of the current as a starting point, this voltage loss triangle can be erected in position. The total generated voltage will consist of the terminal voltage together with the impedance voltage added at its proper phase angle, so that the tip of the induced voltage vector OE will lie along a horizontal line drawn through V_Z. The magnitude of OE is obtained from the open circuit magnetisation curve, and the vector can be drawn by taking O as centre and with OE as radius, striking an arc so as to cut the horizontal line through V_Z. A line VE is then drawn equal and parallel to OV_Z, when OV will represent the terminal voltage.

When a lagging current is dealt with [see Fig. 246 (*b*)], the phase of OV is taken as the starting point and OI is drawn lagging behind it at the correct angle ϕ and the same construction repeated, remembering that the voltage overcoming the resistance of the armature is in phase with the current.

Fig. 246 (*c*) shows the vector diagram for a leading current.

These vector diagrams can be repeated for other values of the load current by making the size of the impedance voltage triangle proportional to the current.

In this way, points on the load characteristic can be calculated for all values of the load current and for any power factor.

In practice this method gives values for the voltage drop which are too high and has been named, in consequence, the *pessimistic method.*

Predetermination of Regulation by Rothert's Ampere-Turn Method.—This method involves the vector addition of M.M.F.'s instead of E.M.F.'s, but since these are proportional to ampere-turns the vector diagram is usually drawn to a scale of ampere-turns instead of in units of M.M.F. The previous method assumed no armature reaction, including it in the synchronous reactance, whilst this method assumes no reactance by including its effects in those of the armature reaction. Thus, instead of allowing for a voltage drop, this portion of the voltage is assumed never to be generated. As before, the open circuit and short circuit characteristics are required, and since on short circuit there is no terminal voltage, the whole of the field ampere-turns are supposed to be balanced by those of the armature which are in direct opposition. This neglects the ohmic resistance of the armature, which will not involve any serious error, as by far the greater portion of the drop of voltage on load is due to the armature reaction and reactance. The ampere-turns set up by the armature when carrying full load current can, therefore, be obtained from the short circuit curve by multiplying the field excitation

by the number of turns per pole. When delivering full load current at unity power factor, the same armature ampere-turns will be set up, but since the armature reaction is now purely a cross-magnetising or distorting effect, these ampere-turns will be in quadrature with those producing the terminal voltage. The vector diagram is shown in Fig. 247. The first diagram represents the case where the current is in phase with the voltage. Since *OE* represents the induced voltage, the armature current and the armature ampere-turns have the same phase and are represented to scale by *AB*. The field excitation for which the diagram is drawn, when multiplied

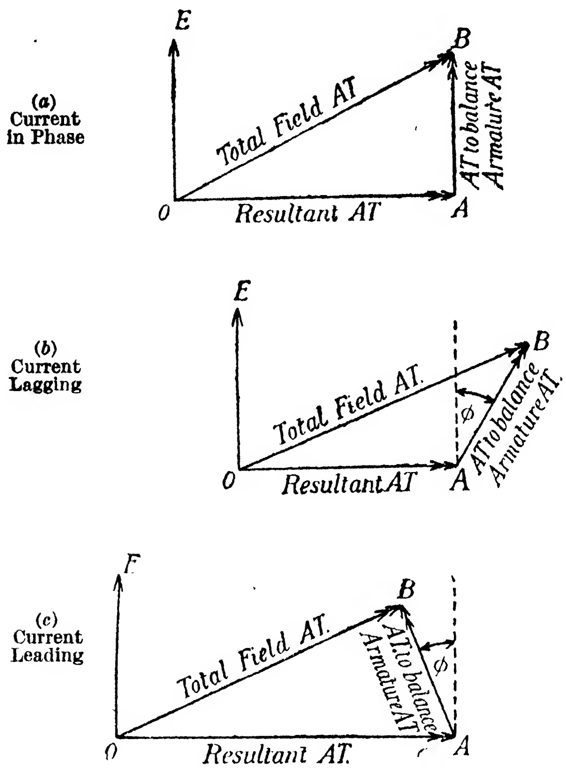


FIG. 247.—Rothert's Ampere-turn Diagram.

by the turns per pole, gives the magnitude of *OB*, and the point *O* can be definitely fixed by drawing a horizontal line through *A* and by striking an arc having the point *B* as centre and *BO* as radius. The triangle *OAB* is now determined, and the vector difference of *OB* and *AB* gives *OA* the resultant ampere-turns. On referring to the open circuit characteristic, the voltage corresponding to these ampere-turns can be read off, and this gives the terminal voltage neglecting the drop due to the armature resistance, which is usually justifiable.

If the armature current lags by an angle ϕ, it makes the armature ampere-turns lag by a corresponding amount and the vector diagram then takes the form shown in Fig. 247 (*b*). AB and OB have the same magnitude as before and consequently OA is reduced, resulting in a further fall in the terminal voltage. In a similar way, Fig. 247 (*c*) refers to a leading current, the line AB being inclined to the left instead of to the right.

This method gives values of the voltage drop which are too small, particularly when working above the knee of the magnetisation curve, and for this reason it has been termed the *optimistic method*, in contrast with that of Behn-Eschenburg (pessimistic method).

Sudden Short Circuits.—When an alternator is running under normal conditions the strength of the field is several times as great as when it is running under a steady short circuit with the same excitation, this being due to the direct weakening action of the armature reaction. When a short circuit takes place suddenly, therefore, the field has to decrease in magnitude to a very considerable extent. This weakening of the field does not occur instantaneously, but takes a certain time to accomplish, because every change in the flux causes a change in the number of lines of force linked with the field coils. As soon as the lines of force begin to die down, therefore, a voltage is induced in the field coils, giving rise to a current opposing the fall of the flux. Eddy currents are also induced in the iron itself, particularly in any solid portions, and this also helps to retard the fall of the flux, which thus takes a definite time to accomplish. On suddenly short circuiting the armature, a large current will flow before the armature reaction has had time to make its effects felt, and the first few current waves after the short circuit will indicate an abnormally high current, quickly dropping down until the steady short circuit current has been reached. But in the meantime large mechanical forces have been set up which may have torn the end connections from their fastenings; hence the necessity for adequate clamping arrangements.

Cyclical Variation in Exciting Current.—It has been shown that the flux linked with each pole fluctuates slightly from instant to instant, and this induces an alternating E.M.F. in the exciting coil which will produce a small alternating current superposed upon the main continuous exciting current. The frequency of this current will be double that of the armature current, as can be seen from Fig. 244, for the flux of any particular pole goes through its whole cycle of events whilst the armature coils are advancing through one pole pitch, during which time the armature current goes through half a cycle. If a third harmonic be present in the main current wave, a fourth harmonic will be produced in the field circuit. In general, only even harmonics will be generated in the field windings, and their magnitude will depend upon the shape of the armature

current wave form and upon the angle of lag. The presence of these fluctuations can easily be shown by allowing an alternator to come to rest with its load still connected. Just before it reaches standstill the frequency is very low, and the field ammeter will indicate the beats in the exciting current. Under actual working conditions, their presence can be detected by means of an oscillograph or by a Joubert's contact, the instantaneous voltage drop over a known resistance placed in series with the field being measured.

Parallel Running.—In practice, alternators are run in parallel and not in series, in common with C.C. practice. In fact two separately driven alternators connected in series will always automatically adjust themselves so as to do a minimum of work. This results in them setting themselves in direct phase opposition, so that the resultant voltage is zero. This is not true, of course, if the two alternators are mechanically coupled together, but that is not a practical case.

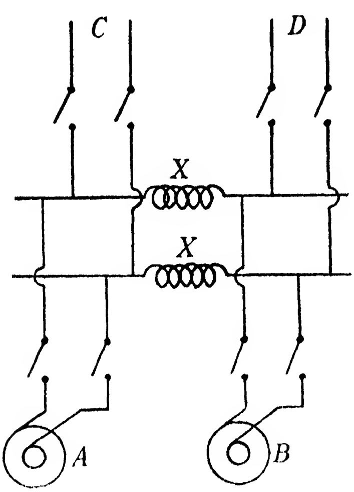
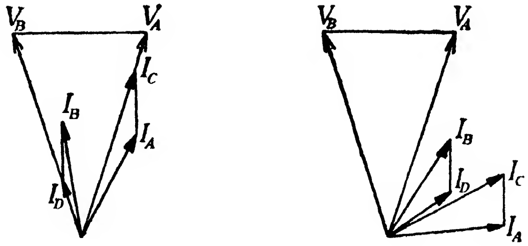


FIG. 248.—Reactances in Bus Bars.

In a generating station a number of alternators feed into a set of common bus bars from which the various feeder circuits are taken, and in a number of cases the short circuit current is limited in the case of a breakdown by the insertion of reactances in series, either in the alternator or in the feeder circuits. These reactances are sometimes placed in between sections of the bus bars, the effect being to confine the trouble to the particular section in which the fault occurs. This results in the various generators being operated with a certain angular displacement in their voltages. For example, consider the two alternators, A and B, in Fig. 248, connected to two feeder circuits, C and D, the bus bars being divided into two sections connected through the reactances XX. Assuming the power factor to be unity, the load on C greater than that on D, and the total load shared equally between the two generators, it follows that a certain amount of current must flow through the two reactances. Neglecting the losses in these, this current is in quadrature with the voltage across the terminals of the reactances. The vector diagram is shown in Fig. 249 (a), which represents the conditions for unity power factor. V_A and V_B represent the voltages of the two alternators and $V_A V_B$ the voltage across the reactance. I_C and I_D represent the currents in the two feeder circuits. The current flowing through the reactance is determined by the magnitude of

$V_A V_B$, and hence the phase displacement of the two alternators depends upon the difference in the loads on the two feeders. By subtracting $I_A I_C$ (at right angles to $V_A V_B$) in the one case and adding $I_B I_D$ in the other, these two currents being the same, the currents



(a) Unity Power Factor

(b) Power Factor Less than Unity Current Lagging

FIG. 249.—Effect of Reactance in Bus Bars.

delivered by the two alternators, I_A and I_B, are obtained. When the current is a lagging one, the conditions are represented in Fig. 249 (b), the current $I_A I_C = I_B I_D$ flowing through the reactance being added to one alternator current and subtracted from the other to obtain the two feeder currents.

Cyclical Variation.—When an alternator is driven by a reciprocating steam engine its speed is not absolutely constant throughout a revolution, due to the varying torque developed by the engine. It follows, therefore, that if such an alternator were run in parallel with an ideal one running at a perfectly constant speed, it will sometimes be in advance of and sometimes behind its correct position with respect to its partner. The two voltages will set up a resultant at all positions where there is a relative phase displacement which produces a torque tending to pull the faulty machine into its correct position. If this angle of phase displacement is allowed to exceed certain limits, however, it interferes with the smooth running of the machine, and practice has shown that the maximum displacement should be limited to 3 electrical degrees either way. To obtain the corresponding mechanical angle, this figure must be divided by the number of pairs of poles. Obviously a three-crank engine is superior to a single-crank engine for smooth running. A low frequency is also an advantage in this respect, as this reduces the number of poles for the same speed and allows a larger mechanical displacement so that a smaller flywheel may be employed.

Synchronising.—The operation of paralleling two alternators is known as *synchronising*, and certain conditions must be fulfilled before this can be effected. The incoming machine must have its voltage and frequency adjusted to that of the bus bars, and, in

addition, the phase of the two voltages must be the same for correct synchronising. The instruments or apparatus for determining when these conditions are fulfilled are called *synchronisers* or *synchroscopes*.

The simplest method of synchronising is by means of two lamps connected across the ends of the double pole paralleling switch, as shown in Fig. 250. If the conditions for synchronising are fulfilled there will be no voltage across the lamps and the switch may be closed. It is very rarely that the lamps are completely black for any length of time, and the speed of the incoming machine must be adjusted as closely as possible so that the lamps light up and die down at a very slow rate. The alternator may then be switched in at the middle of the period of darkness, which must be judged by the speed at which the light is varying. When the incoming machine is 180° out of phase, there will be double the voltage of one machine acting across the two lamps in series, so that each lamp must be capable of standing the full terminal voltage of one alternator.

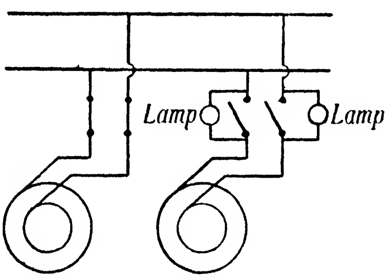


FIG. 250.—Synchronising with Lamps dark.

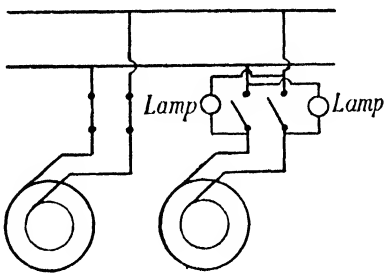


FIG. 251.—Synchronising with Lamps bright.

An alternative system of connections is shown in Fig. 251, where the lamp connections are crossed. For synchronising there must be no volts across the switch, and as there are full volts between the two poles of the switch, it follows that the lamps will be lit with their maximum brilliance at the correct moment for synchronising. This method has an advantage over the previous one inasmuch as the lamps are much more sensitive to changes of voltage at their maximum candle power than when they are quite black, and a sharper definition of the exact point of synchronism is thus obtained. Both methods are, however, employed.

When the voltage of the alternator is such that lamps cannot be used directly, small potential transformers may be installed, the synchronising lamps being connected across the secondaries. The primaries may be connected straight across the switch or they may be cross-connected as with low tension alternators. These two transformers may be replaced by a single three limb transformer, but it is not admissible to use a single two limbed transformer with

two primaries, since at certain times the ampere-turns of these two windings would directly oppose one another. This would practically destroy the flux in the iron path and would render the two primaries approximately non-inductive except for the leakage flux, and the current taken by the primaries under these conditions would cause a burn-out. When a three limbed transformer is employed the connections are those shown in Fig. 252, the secondary being wound for a suitable lamp voltage. At one instant the two primaries aid one another to drive the flux round the outer magnetic circuit, there being no flux in the central limb. When one primary gains half a period on the other, the two oppose each other, but unite to force the flux through the central limb. To change over from the lamps dark to the lamps bright method of synchronising, the connections to one primary must be reversed.

A pair of special synchronising bus bars are often provided on switchboards where there are a number of alternators running in parallel. There is only one set of synchronising gear, and this is

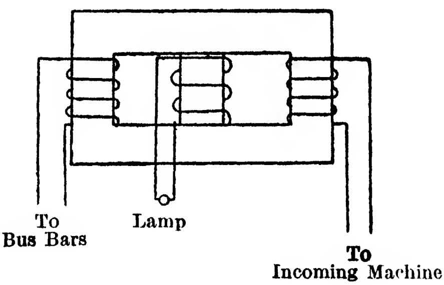


FIG. 252.—Synchronising Transformer.

connected between these auxiliary bus bars and the main bus bars. The incoming machine is plugged on to the synchronising bus bars, one voltmeter indicating the main bus bar voltage, and a second that of the incoming machine. The one set of synchronising gear can thus be used for any of the alternators as desired.

For polyphase machines it is only necessary to synchronise one phase, as all the other phases will then be in synchronism as well. When connecting up an alternator in the first place, however, it is necessary that they are *phased up* correctly, *i.e.* the phases must be connected in their correct order of 1, 2, 3 and not 1, 3, 2. In the latter case it is, of course, impossible ever to synchronise the machines. Any two leads of the new alternator must then be interchanged to reverse the sequence.

A commonly adopted method of synchronising three phase alternators, introduced by Siemens and Halske, consists in having three lamps connected as shown in Fig. 253 (*a*). The order in which the lamps light up shows whether the incoming machine is

running too fast or too slow. The lamps are not connected symmetrically, as they would then light up and die down simultaneously, but L_1 is connected between A and D, L_2 is connected between B and F (not B and E), and L_3 is connected between C and E (not C and F). Fig. 253 (b) shows the corresponding vector diagram. If the frequencies of the two machines are slightly different, the two stars will have a relative movement with respect to each other. Assuming that the incoming machine DEF is rotating a little fast, the star DEF will have a slow rotation in a counter-clockwise direction with respect to the star ABC. The voltage AD on lamp L_1 will then be increasing from zero, the voltage CE on lamp L_3 will be increasing and near its maximum, and the voltage BF on lamp L_2 will be decreasing, having passed through its maximum. The lamps will then light up one after the other in the order 2, 3, 1, 2, 3, 1,

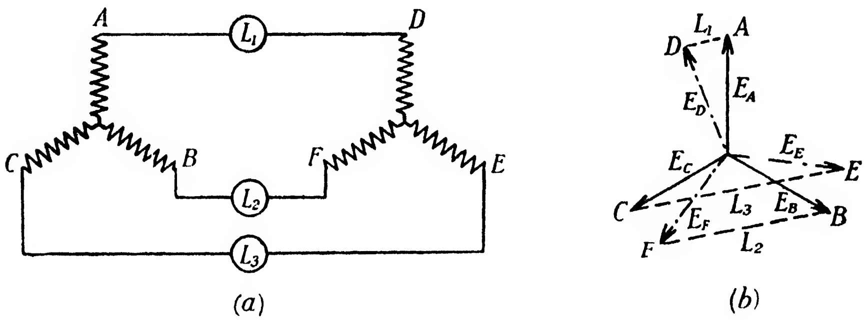


FIG. 253.—Siemens and Halske Synchroniser.

etc. Next, suppose the machine DEF is running a trifle slow. The star DEF in the vector diagram will now rotate slowly in a clockwise direction with respect to the star ABC. The voltage AD on lamp L_1 will be decreasing, having passed through its maximum some time previously. The voltage CE on lamp L_3 will be decreasing, having just passed through its maximum, whilst the voltage BF on lamp L_2 will be increasing up to its maximum value. The lamps will therefore light up in the order 1, 3, 2, 1, 3, 2, etc., which is the reverse order to that in the previous case. It is common practice to mount the three lamps at the angles of a triangle, and the apparent direction of rotation of the light indicates whether the machine is running too fast or too slow. The actual synchronising is done when the lamp L_1 is in the middle of its dark stage.

Synchroscopes.—Synchronising by means of lamps is not very exact, as a considerable amount of judgment is called for in the operator, and in large machines even a small angle of phase displacement causes a certain amount of shock to the machines. On this account a number of more complicated *synchroscopes* have been devised, of which two types will be described.

If an ordinary dynamometer wattmeter has both its fixed and

moving windings made of fine wire and each is supplied with an alternating voltage, the deflection will depend upon the phase difference of these two voltages. When they are in quadrature the deflection will be zero, and if one advances in phase with respect to the other it will cause a continued deflection of the pointer until they are in phase, after which the pointer will slowly retrace its path. It will thus appear to oscillate from side to side, assuming a central zero. If a condenser is placed in series with one coil, the deflection will be zero when the two voltages are in phase, and this is what is required, and to compensate for the resistance in the condenser circuit a little reactance is placed in the other to establish exact quadrature. The switch is closed when the pointer is stationary and on the zero. In order to indicate whether the incoming machine is running too fast or too slow, an ordinary synchronising lamp is arranged to illuminate the dial of the instrument. This lamp is bright when the pointer is swinging one way and dark on the return swing, which is consequently not seen.

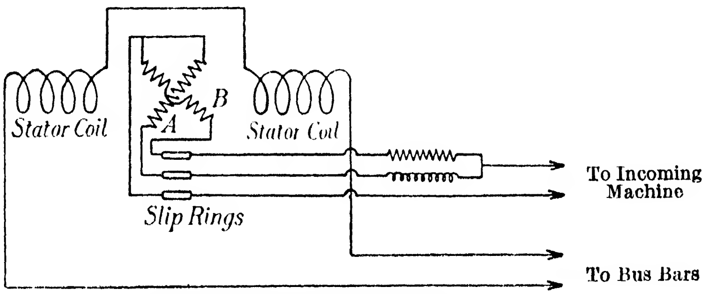


FIG. 254.—Rotary Synchroscope.

When viewed from a little distance the pointer has the appearance of rotating in a constant direction. In this way "too fast" is indicated by an apparent rotation in one direction and "too slow" by an apparent rotation in the other, since the lamp now lights up the opposite swing of the pointer.

A type of rotary synchroscope often employed is very similar in construction to the power factor indicator shown in Fig. 147. In reality it consists of a small motor, the field of which is provided by the bus bar volts, whilst the rotor currents are supplied by the incoming machine. The rotor is wound with two coils at right angles, the currents in which differ in phase by approximately 90°. This is obtained by connecting a resistance in series with one and a reactance in series with the other, as shown in Fig. 254. Only three slip rings are required, since a common return is used for both coils. A pointer is attached to the rotor and serves to indicate the correct time for synchronising. The E.M.F.'s of the bus bars and of the incoming machine will now be in phase, so that the currents in the stator and in coil A on the rotor will be in phase

since they both lag by 90° behind their respective E.M.F.'s. Coil *A* will set itself so that the path of the resultant flux is as short as possible. There will be no torque acting on coil *B*, since its current is in quadrature with the stator current. The pointer now rests on the zero of the scale. If the two alternators are running at the same speed but with a constant phase difference, the stator current will produce a torque in both the rotor coils, so that the rotor will take up a new position of equilibrium, the deflection of the pointer indicating the difference in phase of the bus bar volts and those of the incoming machine. If one machine is gaining on the other, the angle of phase difference is continually increasing, and this results in a continually increasing deflection. In other words, the pointer rotates with a speed proportional to the difference in the two frequencies. If the fast machine is now made to go slower than its companion the direction of rotation is reversed,

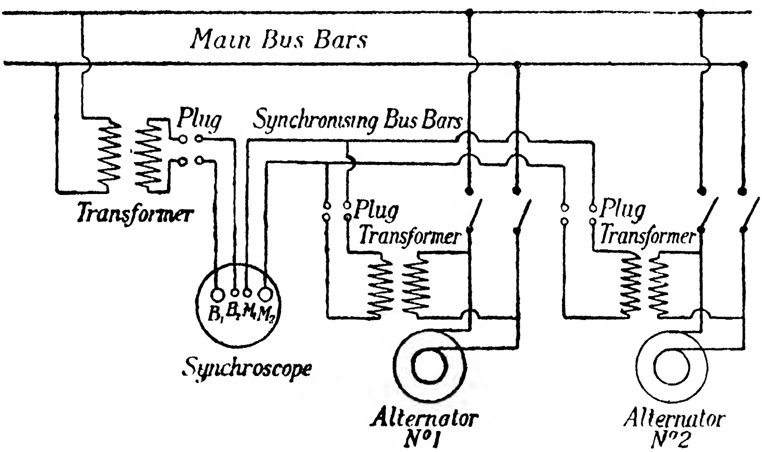


FIG. 255.—Synchroscope Connections.

and the correct time for synchronising is when the pointer is stationary and vertical.

The actual instrument is made with four poles, and a double-ended pointer is used, either end being considered, since a gain of a complete cycle is now indicated by half a revolution.

For use in engine rooms where the synchroscope is viewed from a distance, a signalling arrangement is provided whereby a red or a green light is shown, depending upon whether the speed is too high or too low. This is obtained by a toothed disc on the rotor spindle, which engages with one of two pawls according to the direction of rotation. These operate a vertical arm which falls over to one side or the other and thereby interposes a red or a green glass in front of the lamp.

A diagram showing how the alternators and synchroscope are connected to the synchronising bus bars is shown in Fig. 255.

Earthed Neutral.—Three phase star-connected generators are frequently earthed at their star points in order to prevent the volts to earth rising to any undue value. With a balanced load and sinusoidal wave forms, this earth circuit will not carry any current, but this is not true with an unbalanced load. It was shown on p. 130 that the third harmonic, and all the other harmonics which are a multiple of 3, neutralise one another in a three phase star winding because they are exactly in phase opposition in two adjacent phases. But considering the local circuit formed by joining the two star points together, the outer ends of the phases

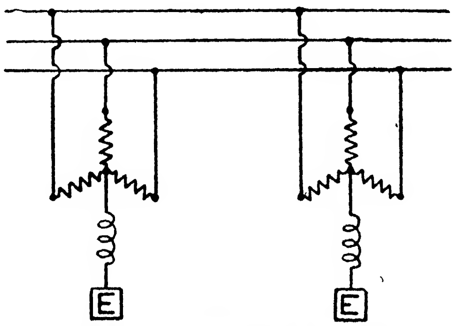


FIG. 256.—Earthed Neutrals.

being paralleled on to the same bus bars, it is seen that, whilst the fundamental voltages are in opposition, the third harmonics aid one another to produce a current which is relatively large owing to the fact that the conditions are the equivalent of a short circuit. The ninth and fifteenth harmonics act in a similar way, but in practice the triple frequency currents are the only ones which need be considered. To limit this current, reactances may be inserted in the earth lead as shown in Fig. 256. These choking coils provide a triple impedance to the third harmonic currents, the value of which automatically drops to one-third when dealing with an ordinary out-of-balance current.

Efficiency Test.—The most convenient method of testing an alternator for efficiency is by means of a motor the efficiency of which is known for all the required loads. The alternator can then be loaded under various conditions and the output and input measured, the latter being equal to the output of the motor. The efficiency can thus be obtained directly.

Measurement of Losses.—If it is desired to measure the various losses separately, a motor having a capacity of about one-tenth that of the alternator will be found to give the best results. The alternator can then be run unexcited at its correct speed and the watts output of the motor determined from its input. This gives the friction and windage loss. The alternator is now normally excited and the increase in the watts output gives the normal iron

loss at no-load. In the same way, the iron loss at different excitations can be obtained.

In order to determine the armature copper losses, it is not sufficient to measure the resistance on C.C., as, owing to the skin effect and eddy currents, this will give values which are too low. A better way is to short circuit the armature and excite the fields sufficiently to cause full load current to flow through the armature. The friction and the iron loss for this excitation must be determined by the auxiliary motor as described above, and then the increase in the power to drive the alternator when the armature is short circuited.

To measure the excitation loss, all that is necessary is to know the exciting voltage and exciting current, these being measured on a C.C. circuit.

The efficiency for any output can now be calculated by adding all the losses to the output to get the input. The power factor of the load should be specified, as this affects the armature copper loss. Strictly speaking, the other losses are also slightly affected, but not to such an extent as to make it worth while allowing for it.

Hopkinson Test.—This method of determining the efficiency is convenient when two similar alternators are available for the purpose. One acts as a generator and drives the other as a motor, the balance of the power being supplied mechanically by means of a third machine the efficiency of which at various loads must be known. In order to make a circulating current flow between the two alternators, their excitations are made different, and, in addition, the two alternators are rigidly coupled, so that they are out of phase to a certain extent. An angle of phase displacement of about 25° gives good results.

The output of the alternator can be measured by wattmeters in the usual way, and the total losses supplied by the third machine can be divided equally between the two test machines, and in this way the individual efficiency of each alternator can be calculated.

Retardation Test.—The retardation method of testing, which is particularly applicable in the case of flywheel type alternators, consists in measuring the rate at which an alternator slows up under different conditions when the driving power is removed. The rate at which the speed is decreasing is a measure of the rate at which kinetic energy is given out, this being used up to overcome the losses at that particular instant. The instantaneous power (in watts) given out by the rotating system when slowing down is

$$0{\cdot}011 \times 10^{-7} \times Mn \times \text{instantaneous rate of decrease of speed in (r.p.m.) per second,}$$

where n is the speed in revolutions per minute and M is the moment

of inertia in C.G.S. units.[1] The test, therefore, demands a knowledge of the moment of inertia of the rotor and the various instantaneous values of the speed and the slope of the speed-time curve. In order to carry out the test, the alternator is run up to speed, preferably by means of a belt drive. The belt is then thrown off and the alternator allowed to come to rest with nothing but friction and windage to cause it to slow up. If the alternator is motor-driven through a direct coupling, the power is switched off the motor and the combined set allowed to come to rest. The losses occasioned by the motor are then calculated by carrying out a similar test on the motor alone and deducted from the total to get the alternator losses. Whilst the alternator is coming to rest, speed measurements are taken from instant to instant, and these are plotted as shown in Fig. 257. The test is then repeated with the alternator fields excited, the machines now coming to rest in a shorter time since there is the added drag due to the iron losses.

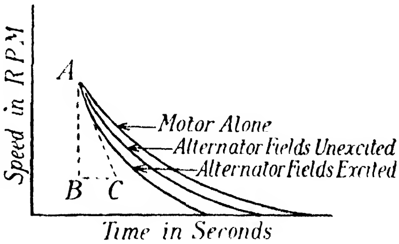


FIG. 257.—Retardation Curves.

At any particular speed, the slope of the speed-time curve can be determined by drawing a tangent to the curve at that speed and measuring the drop in speed corresponding to one second, *i.e.* $\frac{AB}{BC}$ to the correct scales (Fig. 257). It now remains to determine the moment of inertia. This can be done by ordinary mechanical methods, but this is a cumbersome task and can be avoided in the following way. The total power to drive the alternator light at one particular speed is measured by noting the input, and the moment of inertia is determined from the equation

$P = 0{\cdot}011 \times 10^{-7} \times Mn \times$ instantaneous rate of decrease of speed,

P being the watts required to drive the set at n revolutions per minute, the slope being obtained from the retardation curve. This equation can now be re-written

$$P = kn \times \text{instantaneous rate of decrease of speed},$$

where the constant k can be evaluated as shown.

[1] The kinetic energy E at any angular velocity ω is $\frac{1}{2}M\omega^2$. Then

$$P = \frac{dE}{dt} = \frac{d(\frac{1}{2}M\omega^2)}{dt} = M\omega\frac{d\omega}{dt} = M\frac{2\pi n}{60} \times \frac{2\pi}{60}\frac{dn}{dt}$$

$$= \frac{\pi^2}{900}Mn\frac{dn}{dt} = 0{\cdot}011\,Mn\frac{dn}{dt}\ \text{C.G.S. units}$$

$$= 0{\cdot}011 \times 10^{-7}\,Mn\frac{dn}{dt}\ \text{watts.}$$

The losses can now be calculated at various speeds, and curves can be drawn, as shown in Fig. 258, from which the friction and the iron loss at any speed can be determined separately by means of the vertical distance between the various curves.

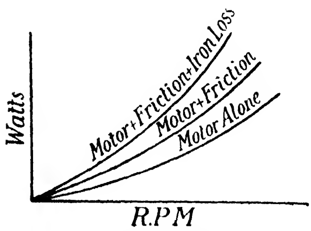


FIG. 258.—Separation of Losses by Retardation Test.

Heating Tests.—Before an alternator is put into actual operation it is desirable to test its temperature rise when working under full load. Since the temperature rise does not attain a maximum value until many hours have elapsed, this test involves a considerable expenditure of energy which, apart from the cost, is difficult to get rid of. The Hopkinson test avoids the latter difficulty, but it is seldom that two similar alternators are available for test. It is therefore desirable to set up artificial conditions which will imitate the real heating effects, and one method of doing this is by reversing half the poles on the rotor, which method will now be described briefly.

Reversed Poles Test.—The field windings are divided into two halves, one of which is reversed so that the E.M.F.'s induced in the corresponding halves of the armature winding are in opposition. On short circuiting the armature, therefore, no current will flow, since the E.M.F.'s are balanced. But by increasing the excitation in one half of the field winding or by decreasing the excitation in the other half the equilibrium of the armature E.M.F.'s is destroyed and a current will flow the magnitude of which can be regulated by adjusting the excitations. Full load current can be obtained in this way without the expenditure of the full amount of power, since one half of the machine is acting as a generator whilst the other half is acting as a motor. It is, in fact, a kind of Hopkinson test on the two halves of a single machine. The grave disadvantage of this test is the large unbalanced mechanical stresses which are set up and which cause considerable vibration. This can be mitigated to a certain extent by reversing alternate pairs of poles instead of reversing one half completely. Thus in a 12-pole alternator poles Nos. 3, 4, 7, 8, 11 and 12 would be reversed. It is necessary that the alternator should have an even number of pairs of poles, as otherwise there would be one pair left over and these two poles would have to be cut out.

CHAPTER XVIII

ALTERNATORS.—DESIGN

Speed and Number of Poles.—When designing an alternator the output, voltage and frequency must be specified, the speed depending to a large extent upon the output and the type of prime mover adopted. In general, the larger the output the slower the speed, since the diameter goes up and the peripheral velocity must be kept within limits. These speeds are now to a large extent standardised and range from about 200 r.p.m. in the case of a 100 k.V.A. set down to about 75 r.p.m. in the largest sizes when driven by slow speed reciprocating engines, these speeds being about trebled when high speed engines are employed. For turbine-driven sets, the available speeds are very limited owing to the fact that either 2, 4 or 6 poles are almost invariably employed. In any case, only certain definite speeds are available as they must fulfil the equation

$$\text{number of poles} = \frac{120f}{\text{r.p.m.}}.$$

The table on p. 225 shows these speeds for various numbers of poles and frequencies.

Efficiency.—In order to compete with rival machines similar efficiencies must be obtained, and Fig. 259 shows the average efficiencies which ought to be reached by modern alternators.

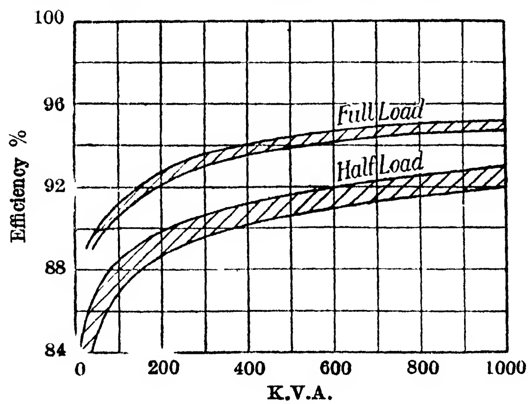


FIG. 259.—Alternator Efficiencies.

Output Coefficient.—It has been found that an empirical expression can be obtained connecting the output, speed and the volume swept out by the rotating element. This formula may be written

$$\frac{\text{k.V.A.}}{\text{r.p.m.}} = k \times D^2L,$$

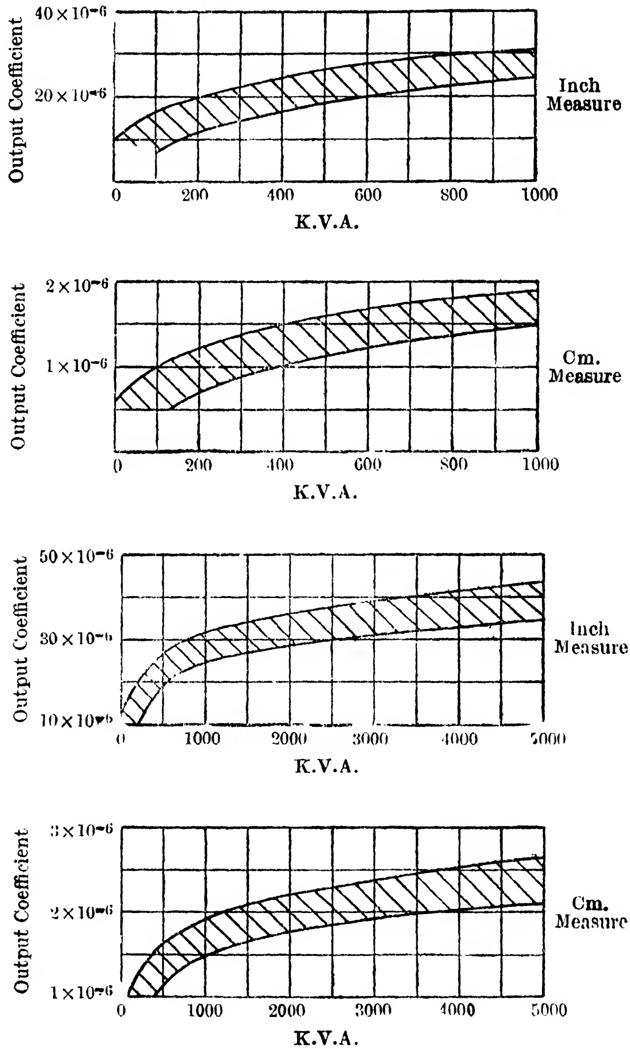


FIG. 260.—Output Coefficients.

where D is the air-gap diameter, L is the gross length of the armature core and k is a constant known as the *output coefficient*. Assuming the output of a given carcase to be proportional to the speed, k represents $\frac{4}{\pi}$ times the output per unit volume at one revolution

per minute, and practice has shown that this constant varies with the size of the machine. Fig. 260 shows the average values of the output coefficient for modern polyphase alternators, but it must be remembered that these figures need not be rigidly adhered to. For turbo-alternators about 75 per cent. of these values should be taken, and they should also be multiplied by about 0·85 in the case of single phase machines.

Air-gap Diameter.—When the value of D^2L is known there are a number of various values of D and L which can be inserted. For example, a 500 k.V.A. polyphase alternator running at 300 r.p.m.

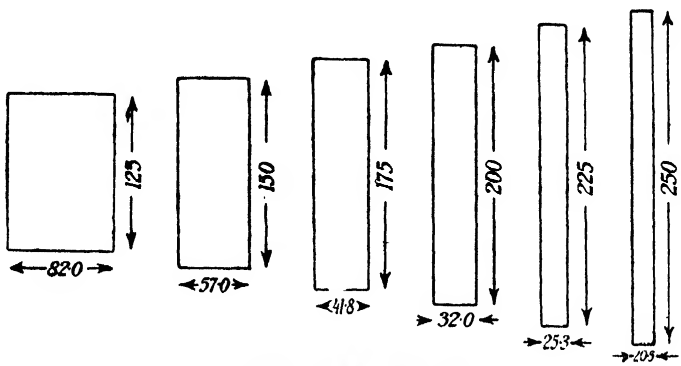


FIG. 261.—Shapes of Rotor.

has an output co-efficient $1·3 \times 10^{-6}$ (cm. measure). The value of D^2L is, therefore,

$$D^2L = \frac{500}{300} \times \frac{10^6}{1·3}$$

$$= 1·28 \times 10^6.$$

The following values of D and L are possible :—

D	125	150	175	200	225	250
L	82·0	57·0	41·8	32·0	25·3	20·5

The diagrams in Fig. 261 show the shapes of the section of the rotating element in the different cases.

An upper limit is reached when the peripheral speed exceeds, say, 8000 feet per minute or 4000 cm. per second, and a lower limit is reached when the poles become too crowded together. Assuming a minimum diameter of pole of 8 cm. and allowing 5 cm. per side for winding and clearance, it is seen that the minimum pole pitch is about 18 cm. But the pole pitch is equal to

$\frac{\pi D}{p}$, and therefore D (in cm.) should not be less than $\frac{18p}{\pi}$ or $5{\cdot}75p$. In the example shown in Fig. 261, the upper and lower limits of air-gap diameter are $\frac{4000 \times 60}{300 \times \pi} = 255$ cm. and $5{\cdot}75 \times 20 = 115$ cm. respectively. Assuming a normal ratio of $\frac{\text{pole arc}}{\text{pole pitch}}$ of 0·65, the shapes of the pole shoes in the six cases shown in Fig. 261 work out as shown in the following table and are represented in Fig. 262.

Air-gap Diameter $= D$	125	150	175	200	225	250
Pole Arc $= 0{\cdot}65 \frac{\pi D}{p} = a$	12·7	15·3	17·8	20·4	23·0	25·5
Area of Pole Shoe $= a \times L$	12·7 × 82·0 = 1040	15·3 × 57·0 = 873	17·8 × 41·8 = 745	20·4 × 32·0 = 653	23·0 × 25·3 = 582	25·5 × 20·5 = 523

It is preferable to choose such a diameter as will enable an approximately square pole to be adopted, as this reduces the amount of copper required by the field system. The usual ratios of armature length to pole arc for 25- and 50-cycle machines vary from 1 to 1·2 and 1·7 to 2·0 respectively.

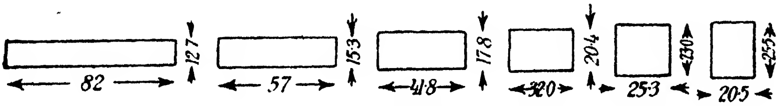


FIG. 262.—Shapes of Pole Shoe.

The magnitude of the output coefficient is fixed by the maximum flux density in the air-gap and the permissible number of ampere-conductors per inch diameter on the armature. The voltage induced per phase is given by

$$E = 4k_1k_2\Phi f T \times 10^{-8} \text{ volts},$$

where k_1 is the form factor, k_2 is the breadth factor, Φ is the flux per pole, f is the frequency, and T the turns per phase. Putting $k_1 = 1{\cdot}11$ and $k_2 = 0{\cdot}96$, the induced voltage becomes

$$E = 4 \times 1{\cdot}11 \times 0{\cdot}96\, \Phi f T \times 10^{-8} \text{ volts}$$
$$= 4{\cdot}26\, \Phi f T \times 10^{-8} \text{ volts}.$$

Choosing a suitable maximum flux density in the air-gap, B, the total useful flux per pole is given by

$$\Phi = \frac{2}{\pi} B \times \text{area of pole shoe}$$
$$= \frac{2}{\pi} B \times 0{\cdot}65 \frac{\pi D}{p} L$$
$$= 1{\cdot}3 \frac{BDL}{p} = \frac{n}{120f} \times 1{\cdot}3BDL,$$

where the ratio $\dfrac{\text{pole arc}}{\text{pole pitch}}$ is taken as 0·65.

The expression for the induced voltage then becomes

$$E = 4{\cdot}26 \times \frac{n}{120f} \times 1{\cdot}3\,BDLfT \times 10^{-8} \text{ volts.}$$

The total output in k.V.A. is

$$\text{k.V.A.} = mIE$$
$$= mI \times 4{\cdot}26 \times \frac{n}{120f} \times 1{\cdot}3BDLfT \times 10^{-11},$$

where m is the number of phases.

But $\text{k.V.A.} = nkD^2L.$

Therefore

$$nkD^2L = mI \times 4{\cdot}26 \times \frac{n}{120f} \times 1{\cdot}3BDLfT \times 10^{-11},$$

$$kD = mI \times \frac{4{\cdot}26 \times 1{\cdot}3}{120} BT \times 10^{-11},$$

$$k = 4{\cdot}61B \frac{mIT}{D} \times 10^{-13}.$$

Again, if A represents the ampere-conductors per inch diameter, $AD = 2mIT$ and $k = 2{\cdot}3BA \times 10^{-13}.$

If B and A are fixed the magnitude of the output coefficient is fixed, and hence it is desirable to work with as high a flux density and as large a number of ampere-conductors per inch as possible.

The following tables show the average values adopted in practice for the ampere-conductors per cm. and per inch diameter for polyphase machines and also the usual magnetic densities employed.

Output in k.V.A.	Ampere-conductors per inch Diameter.			
	Flywheel Type Alternators.		Turbo-alternators.	
	$f=25$.	$f=50$.	$f=25$.	$f=50$.
100— 250	1200	960	—	—
250— 500	1520	1200	—	—
500—1000	1700	1430	1520	1350
1000—2000	1850	1600	1680	1520
2000—5000	2000	1770	1830	1680

Output in k.V.A.	Ampere-conductors per cm. Diameter.			
	Flywheel Type Alternators.		Turbo-alternators.	
	$f=25$.	$f=50$	$f=25$.	$f=50$.
100—250	480	380	—	—
250—500	600	480	—	—
500—1000	670	560	600	540
1000—2000	740	630	660	600
2000—5000	800	700	720	660

Output in k.V.A.	Air-gap Flux Densities.			
	Flywheel Type Alternators.		Turbo-alternators.	
	Lines per cm.^{2}	Lines per in.^{2}	Lines per cm.^{2}	Lines per in.^{2}
0—500	5500—7000	35000—45000	4500—5500	30000—35000
500—1000	7000—8000	45000—52000	5500—6500	35000—42000
1000—2000	8000—8500	52000—55000	6500—7000	42000—45000
2000—5000	8500—9000	55000—58000	7000—7500	45000—48000

The above figures are for 50-cycle machines and may be increased by 10 per cent. to 15 per cent. for $f = 25$. Also in the case of high voltage alternators they should be reduced by about 5 per cent. to 10 per cent.

Length of Stator Core.—The length of the stator core is definitely fixed when once the values of the output coefficient and the air-gap diameter have been decided upon, since

$$L = \frac{\text{k.V.A.}}{nkD^2} \text{ or } \frac{D^2L}{D^2}.$$

Ventilating Ducts.—The usual practice with respect to flywheel type alternators is to have a number of radial ventilating ducts in the stator about $\frac{3}{8}$ in. to $\frac{5}{8}$ in. in width and spaced about 3 in. apart. In the case of turbo-alternators an increased ventilation is necessary, since the overall dimensions are much smaller for a given output. This means that each cubic inch of material must dissipate more heat, since the efficiency and the total losses are approximately the same in each case. The distance between the ventilating ducts is therefore reduced to about 2 inches.

Number of Slots.—These vary from two to five per pole per phase in the ordinary rotating field alternators. The larger the number of slots the nearer does the wave form approach the ideal sine wave, but the copper space factor of the slot goes down at the same time,

particularly with high voltage machines. In the latter case, therefore, the tendency is to use two and three slots per pole per phase, whilst with low voltage machines four and five are frequently employed. With turbo-alternators, the pole pitch is usually considerably greater, and this leads to larger numbers such as five and six slots per pole per phase. The total number of slots is then obtained by multiplying by the number of poles and the number of phases.

Armature Winding.—The number of armature conductors is settled by E.M.F. considerations, but this number must be divisible by the total number of slots. The induced voltage per winding is first calculated, this being equal to the line voltage in a single and two phase case and $\frac{1}{\sqrt{3}}$ times the line voltage in the case of a three phase star-connected alternator. The total turns in series per phase is then given by the equation

$$E = 4k_1k_2\Phi fT \times 10^{-8} \text{ volts (see p. 247)}$$
$$= 4{\cdot}26\Phi fT \times 10^{-8} \text{ volts}$$

as a first approximation. The approximate value of the useful flux per pole can be obtained from a knowledge of the dimensions of the pole shoe and by assuming a suitable flux density taken from the table on p. 278. The number of turns required can then be calculated, and the nearest possible number is adopted, taking into consideration the number of slots. The flux is then adjusted to suit the number of conductors. The winding itself may be either coil or bar wound, the particular type being chosen from amongst those explained on p. 239 or others.

Size of Armature Conductors.—The section of the conductor to be employed is obtained from a knowledge of what is a suitable current density to adopt. This varies with the total current carried by the conductor, and values are shown in the curve in Fig. 263. Solid wire of circular cross section should be used up to sections of about 0·25 (cm.)^2, but for larger sizes the conductor becomes difficult to bend and two or more conductors are connected in parallel. For the larger sizes of conductor rectangular strip is often employed, as this brings about a higher copper space factor in the slot.

Size of Slots.—When the number of conductors per slot and the cross-sectional area of the conductors are settled, the space that they will occupy, when covered with the necessary insulation, can be drawn out and the slot dimensions fixed. Since the width of slot plus tooth is fixed, the dimensions of the slot determine the dimensions of the tooth, and this should not be too narrow. A suitable maximum magnetic density in the teeth is 19,000 lines per sq. cm. or 120,000 lines per sq. in.

Armature Losses.—In order to predetermine the heating of the

armature, it is necessary to obtain an estimate of the losses incurred by it. These will consist of copper and iron losses, in addition to which there is a frictional loss produced and dissipated in the bearings when the armature is the rotating element.

The copper loss can be predetermined from a knowledge of the number and dimensions of the conductors. The mean length of a turn must be estimated, and the number of turns in series per phase being known, the total length of conductor per phase can be obtained. Then, knowing the cross-sectional area, the resistance per phase can be calculated, taking care to allow for the rise of temperature. At a temperature of about 50° C. the specific resistance of copper is approximately 2 microhms per cm. cube or 0·8 microhm per inch cube. From the current per phase and the number of phases the total watts lost due to ohmic resistance can be calculated. The figures obtained in this way will be frequently

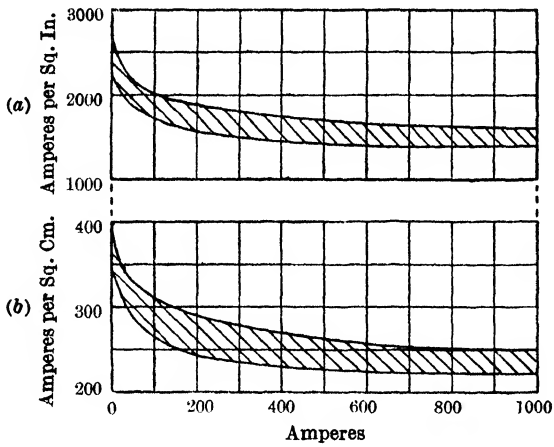


FIG. 263.—Current Densities.

too low, as the current is not always uniformly distributed over the conductors, and this always results in an increase in the watts lost in a conductor.

The iron loss, comprising the hysteresis and eddy current losses, is dependent upon the flux density in the core, and tests have been conducted by several experimenters in order to determine the combined iron loss at various flux densities and frequencies. It is found that for all practical purposes the losses can be taken as being proportional to the frequency, although this is not true theoretically. The curves in Fig. 264 show the watts lost per unit volume at unit frequency for stationary armatures. These figures require to be multiplied by the frequency in question.

Cooling Surface.—In determining the cooling surface of the armature, the inner and outer cylindrical surfaces of the stampings

may be considered together with the two ends of the iron core and one side of each ventilating duct.

Estimated Temperature Rise.—The estimated temperature rise of the armature depends upon the watts which have to be dissipated per square inch of the cooling surface. With the usual ventilating ducts, a temperature rise of approximately 40° C. will be obtained when the watts per square inch are 1–1½ in the case of rotating armature alternators and ¾–1 in the case of rotating field alternators, the cooling surface being estimated in the above manner. Here also the temperature rise may be taken as being proportional to the watts per square inch of cooling surface, but these preliminary calculations are by no means accurate and are subject to a number of disturbing conditions.

Flux per Pole.—The no-load useful flux per pole can be calculated from the E.M.F. formula, since the number of conductors is now settled. The form factor, k_1, and the breadth factor, k_2, can

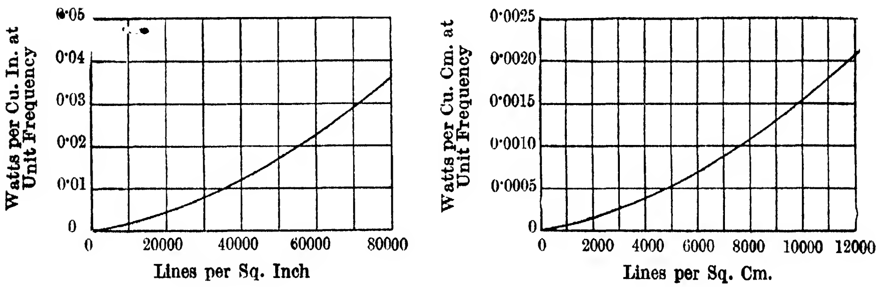


FIG. 264.—Core Losses.

also be estimated fairly accurately, and the useful flux per pole on no-load is given by

$$\Phi = \frac{E \times 10^8}{4k_1k_2fT},$$

f being the frequency, T the turns in series per phase and E the no-load induced E.M.F. per phase. For the same terminal voltage the induced E.M.F. on full load must be greater owing to the loss of voltage occasioned by the synchronous impedance of the armature. The resistance may be calculated from a knowledge of the dimensions of the winding and an estimate may also be made for the synchronous reactance, but the calculations are rather involved. As a rough approximation, the latter may be taken as 15 per cent. or 20 per cent. of the no-load voltage. The full load useful flux per pole is thus obtained, to which must be added the waste leakage flux, which will ordinarily be of the order of 10 per cent. to 35 per cent. of the useful flux. The figures in the following table refer to the leakage factors of rotating field alternators and may be taken as average practice.

k.V.A. Output ...	2·5	5	10	25	50	100	200	300	500	1000	2000
Leakage Factor = λ ...	1·4	1·35	1·30	1·28	1·25	1·22	1·20	1·18	1·15	1·12	1·10

Air-gap.—The radial length of the air-gap in modern alternators is fairly constant for a given size of machine, increasing as the bore of the stator goes up. Fig. 265 shows the usual values of the air-gap length in the case of modern rotating field alternators for slow and medium speeds. The lengths given are those at the centre of the air-gap, where a variable length is employed for the purpose of improving the wave form. Considerably longer air-gaps are employed in the case of turbo-alternators.

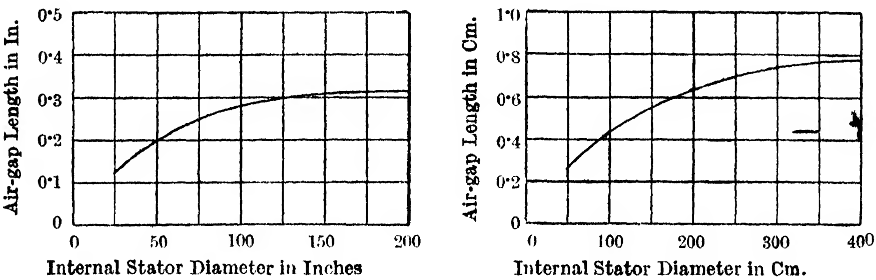


FIG. 265.—Air-gap Length.

Ampere-turns per Pole.—Before the ampere-turns per pole can be calculated, it is necessary to know the flux densities in the various parts of the magnetic circuit, and also the lengths of the lines of force in the different materials. The various magnetic sections are settled by choosing suitable flux densities, and for this purpose reference may be made to the following table :—

	Lines per Sq. In.	Lines per Sq. Cm.
Armature cores ...	45000— 65000	7000—10000
Armature teeth ..	65000— 90000	10000—14000
Magnet poles ...	90000—100000	14000—15500
Magnet yokes ...	70000— 80000	11000—12500

The flux in the armature core and teeth is alternating and thus sets up an iron loss. At higher frequencies, therefore, a lower density is adopted, this accounting for the considerable range shown in the table, where the higher figures correspond to a frequency of 25 and the lower figures to a frequency of 50.

The magnetic sections being chosen and the various flux densities calculated, the corresponding values of H must be obtained from a

B—H curve of that particular material. For this purpose the curves in Fig. 56 may be used. From the formula

$$H = \frac{4\pi}{10} \times \text{Ampere-turns per cm.}$$

the ampere-turns required for the various parts can be obtained. Adding all these ampere-turns together, the total no-load ampere-turns required per pole can be evaluated. It is usual to consider the poles and yoke as carrying all the leakage flux in addition to the useful flux, whilst the armature only carries the useful flux.

The above calculation should now be repeated for full load at the minimum specified power factor. The difference between the two numbers of ampere-turns fixes the size of the field regulator required, a suitable margin being allowed.

Field Winding.—A tentative length of winding space and an approximate depth of the winding must first be decided upon. If wire wound, the length of the mean turn can be estimated from the known dimensions. Then

$$\begin{aligned}\text{Volts per Coil} &= \text{Exciting Current} \times \text{Resistance of Coil (Hot)}\\ &= \text{Exciting Current} \times \text{Turns} \times \text{Resistance of Mean Turn (Hot)}\\ &= \text{Ampere-turns} \times \text{Resistance of Mean Turn (Hot)}\end{aligned}$$

and $$\text{Resistance of Mean Turn (Hot)} = \frac{\text{Volts per Coil}}{\text{Ampere-turns}}.$$

From this the resistance per yard can be obtained, and a workshop rule to get the cold resistance is to multiply by $\frac{6}{7}$. The nearest wire to this in the wire tables is chosen and its insulated diameter noted. Allowing 5 per cent. for loss of winding space due to imperfect winding, the number of turns per layer and the number of layers can be calculated. The total number of turns per pole and the resistance of the winding are next evaluated, giving the current taken by the coil. The ampere-turns thus obtained should agree substantially with the number aimed at in the first instance. For a temperature rise of about 40° C. the watts per square inch of cooling surface should be about 1–1½ for rotating fields and about ¾–1 for stationary fields, as in the case of armatures. Further, the watts wasted in excitation should range from about 4 per cent. for small alternators to about 1½ per cent. for very large alternators. To adjust the field winding for ampere-turns or for watts lost extra layers may be put on or taken off. Taking off turns increases the watts lost and increases the ampere-turns slightly, since the turns which are removed have more than the average resistance per turn, and therefore the current is increased by a larger percentage than the turns are decreased. Similarly, putting more turns on reduces the

watts lost and the ampere-turns at the same time. This does not apply to a winding consisting of a single layer of strip on edge, as all the turns now have the same length. The depth of winding having been fixed in this case, a suitable cross section is chosen so as to give the required ohms per turn from which the number of turns and the exciting current follow.

Estimated Open Circuit Magnetisation Curve.—The ampere-turns for one value of the voltage and flux have already been worked out, and the corresponding exciting current can also be evaluated, since the field turns per pole are known. This gives one point on the curve, and a number of other points can be obtained in a similar manner. A value of the voltage is chosen and the corresponding flux calculated. The ampere-turns required to drive this flux through the various parts of the magnetic circuit are worked out, from which the exciting current is evaluated as above.

Estimated Short Circuit Characteristic.—If the synchronous reactance is known, the short circuit characteristic can be obtained straight away, as shown on p. 253, but otherwise a more approximate method must be adopted. The ampere-turns of the field may be considered as being in direct opposition to the ampere-turns of the armature, diverting the resultant field into leakage paths. For the ordinary type of alternators, the ratio of the field ampere-turns per pole to the armature ampere-turns per pole per phase will be fairly constant, and the following approximate relationship may be stated :

Field ampere-turns per pole =

$$\text{Armature ampere-turns per pole per phase} \times \text{Constant},$$

and this constant may be taken as 2·5–3 for three phase and 1·3–1·6 for single- and two-phase alternators. The short circuit current is then given by

$$\frac{\text{Field ampere-turns per pole}}{\text{Armature turns per pole per phase} \times \text{Constant}}.$$

Since the excitation corresponding to these field ampere-turns is known, one point on the short circuit characteristic can be plotted and the graph taken as a straight line passing through the origin. The short circuit current should be about three or four times the full load current for normal excitation in the case of slow and medium speed sets and about twice full load current in the case of turbo-alternators.

Predetermination of Regulation.—Since a predetermination has been made above of both the open circuit magnetisation curve and the short circuit characteristic, the estimated regulation for a particular alternator design can be calculated in the way explained on pp. 258–259.

Example of Design.—As an example of a design, the main dimen-

sions will be worked out for a three phase rotating field alternator having an output of 200 k.V.A., 2,200 volts, 50 cycles per second, and 375 r.p.m.

The number of poles is $\frac{120 \times 50}{375} = 16$.

The alternator being star connected, the volts per winding will be $\frac{2200}{\sqrt{3}} = 1270$ volts.

The full load current will be $\frac{200000}{\sqrt{3} \times 2200} = 52{\cdot}5$ amperes.

An output coefficient of $0{\cdot}9 \times 10^{-6}$ will be chosen [see Fig. 260 (*b*)], and the value of D^2L becomes

$$D^2L = \frac{\text{k.V.A.}}{\text{r.p.m.}} \times \frac{1}{k}$$

$$= \frac{200}{375} \times \frac{10^6}{0{\cdot}9} = 593000.$$

The diameter is limited by the peripheral speed, which should not exceed 4,000 cm. per second. Then

$$\frac{\pi D \times 375}{60} = 4000$$

and $$D = \frac{4000 \times 60}{\pi \times 375} = 204 \text{ cm. (maximum).}$$

The minimum diameter settled by the crowding of the poles is equal to $5{\cdot}75 \times 16 = 92$ cm.

Several corresponding values of D and L are given in the following table, together with the pole arcs assuming a ratio of $\frac{\text{pole arc}}{\text{pole pitch}} = 0{\cdot}65$.

D	100	125	140	150	175	200
L	59·3	38·0	30·2	26·4	19·4	14·8
Pole Arc	12·8	16·0	17·9	19·1	22·3	25·5

Taking a ratio of armature length to pole arc about 1·7 to 2·0 the diameter of 140 cm. is suggested, which accounts for its inclusion in the table. This gives a ratio of

$$\frac{\text{Armature Length}}{\text{Pole Arc}} = \frac{30{\cdot}2}{17{\cdot}9} = 1{\cdot}69.$$

An air-gap diameter of 140 cm. and a gross core length of 30 cm. will be adopted.

There will be three ventilating ducts, each 1 cm. wide, leaving four batches of stampings 7, 6·5, 6·5 and 7 cm. in width respectively.

With three slots per pole per phase the total slots number $3 \times 16 \times 3 = 144$, having a slot pitch of $\frac{\pi \times 140}{144} = 3{\cdot}05$ cm.

Before the number of armature turns can be settled a tentative value of the useful flux per pole must be obtained. Referring to the table on p. 278, a density of 6,000 lines per sq. cm. may be assumed, giving a useful flux per pole of $6000 \times 17{\cdot}9 \times 30 \times 0{\cdot}9 = 2{\cdot}9 \times 10^6$ lines, assuming the iron is 90 per cent. of the pole face.

The turns in series per phase are then given by

$$E = 4{\cdot}26 \Phi f T \times 10^{-8} \text{ (see p. 279)}$$

and $$T = \frac{2200}{\sqrt{3}} \times \frac{10^8}{4{\cdot}26 \times 2{\cdot}9 \times 10^6 \times 50} = 206.$$

The slots per phase are 48, and with 8 conductors per slot this gives 192 turns per phase. The flux per pole will, therefore, need to be increased somewhat.

As a check, the ampere-conductors per cm. diameter will now be worked out. The total armature conductors are $144 \times 8 = 1152$ and the full load current is 52·5 amperes. The ampere-conductors per cm. diameter are, therefore

$$\frac{1152 \times 52{\cdot}5}{140} = 432.$$

Comparing this with the table on p. 278, it is seen to be about right.

To determine the size of conductor reference is made to Fig. 263 (*b*) to see what is a suitable current density. This will be taken at 300 amperes per sq. cm., giving a conductor having an approximate cross-sectional area of $\frac{52{\cdot}5}{300} = 0{\cdot}175$ sq. cm. Using round wire, this corresponds to a diameter of 0·472 cm. The nearest wire to this is No. 6 S.W.G. having a diameter of 0·488 cm. When insulated with d.c.c. the diameter becomes 0·52 cm. Allowing 0·25 cm. of press-spahn per side for the slot lining, the width of the slot becomes $0{\cdot}25 + 0{\cdot}52 + 0{\cdot}52 + 0{\cdot}25 = 1{\cdot}54$, say 1·60 cm., and as the slot pitch is 3·05 cm., it leaves a minimum thickness of tooth of $3{\cdot}05 - 1{\cdot}60 = 1{\cdot}45$ cm. The depth of the slot is $2 \times 0{\cdot}25 + 4 \times 0{\cdot}52 = 2{\cdot}58$, say 2·70 cm., with a bridge 0·3 cm. thick and an opening at the mouth of the slot 0·3 cm. wide (see Fig. 266).

The average flux per tooth is approximately

$$\frac{2{\cdot}9 \times 10^6}{9} = 0{\cdot}32 \times 10^6 \text{ lines.}$$

The maximum flux per tooth is

$$\frac{\pi}{2} \times 0{\cdot}32 \times 10^6 = 0{\cdot}5 \times 10^6 \text{ lines.}$$

(This figure will be adjusted later, as the correct flux has not yet been worked out.)

The minimum cross-sectional area of a tooth is

$$1{\cdot}45 \times (30 - 3) \times 0{\cdot}9 = 35{\cdot}2 \text{ sq. cm.,}$$

and the maximum flux density is $\frac{0{\cdot}5 \times 10^6}{35{\cdot}2} = 14200$, which is rather low.

To get the density in the iron behind the teeth, the flux per magnetic circuit in the armature is

$$\frac{2{\cdot}9 \times 10^6}{2} = 1{\cdot}45 \times 10^6 \text{ lines.}$$

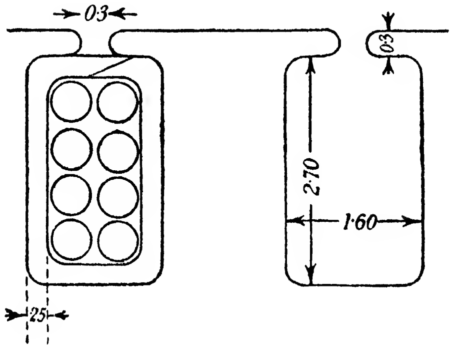


FIG. 266.—Slot Dimensions.

The diameter at the tooth roots is $140 + 2 \times 2{\cdot}7 = 145{\cdot}4$ cm. If a depth of 8·5 cm. of iron behind the teeth be chosen, the flux density behind the teeth becomes

$$\frac{1{\cdot}45 \times 10^6}{8{\cdot}5 \times 27 \times 0{\cdot}9} = 7000 \text{ approximately,}$$

which is satisfactory.

To calculate the iron loss refer to Fig. 264. Then

Watts per c.c. at unity frequency $= 0{\cdot}00087$,

Watts per c.c. at 50 frequency $= 50 \times 0{\cdot}00087 = 0{\cdot}0435$,

$$\text{Volume of iron} = \left\{\frac{\pi}{4}(162{\cdot}4^2 - 145{\cdot}4^2) + 144 \times 2{\cdot}7 \times 1{\cdot}45\right\} \times 27 \times 0{\cdot}9 = 113000 \text{ c.c.}$$

Total armature iron loss $= 0{\cdot}0435 \times 113000 = 4900$ watts. This figure cannot be guaranteed as to its accuracy.

The copper loss is calculated as follows :—

Pole pitch $= \dfrac{\pi \times 140}{16} = 27{\cdot}5$ cm.

Length of end connection assumed = 30 cm.

Embedded length of conductor = 30 cm.

Length of turn $= 2 \times 30 + 2 \times 30 = 120$ cm.

Length per circuit $= 120 \times 192 = 23040$ cm.

Cross-sectional area $= \dfrac{\pi}{4} \times 0{\cdot}488^2 = 0{\cdot}187$ sq. cm.

Resistance per winding (hot) $= \dfrac{2 \times 10^{-6} \times 23040}{0{\cdot}187}$

$= 0{\cdot}247$, say $0{\cdot}25$ ohm.

Total armature I^2R loss $= 3 \times 52{\cdot}5^2 \times 0{\cdot}25$

$= 2070$, say 2100 watts.

Total armature loss $= 2100 + 4900 = 7000$ watts.

The cooling surface is determined as follows :—

Inner cylindrical surface $= \pi \times 140 \times 30 = 13200$ sq. cm.

Outer cylindrical surface $= \pi \times 162{\cdot}4 \times 30 = 15300$ sq. cm.

Area of ends of iron core $= 2 \times \dfrac{\pi}{4}(162{\cdot}4^2 - 140^2)$

$= 10600$ sq. cm.

Total cooling surface $= 13200 + 15300 + 10600$

$= 39100$ sq. cm.

Armature watts per sq. cm. $= \dfrac{7000}{39100} = 0{\cdot}179$,

or per sq. in. $= 1{\cdot}15$.

This is rather high, and the ventilating ducts should be looked to in order to prevent an undue temperature rise.

The calculations for the field system are as follows :—

No-load useful flux per pole

$$= \frac{2200}{\sqrt{3}} \times \frac{10^8}{4k_1k_2fT}$$

$$= \frac{2200}{\sqrt{3}} \times \frac{10^8}{4 \times 1{\cdot}11 \times 0{\cdot}96 \times 50 \times 192}$$

$$= 3{\cdot}11 \times 10^6.$$

Leakage coefficient λ (from p. 282) $= 1{\cdot}20$.

Total no-load flux per pole $= 1{\cdot}20 \times 3{\cdot}11 \times 10^6$

$$= 3{\cdot}73 \times 10^6 \text{ lines.}$$

Resistance drop per phase at full load $= 52{\cdot}5 \times 0{\cdot}25$

$$= 13 \text{ volts.}$$

Reactance ,, ,, ,, ,, $= 0{\cdot}20 \times 1270$ (say)

$$= 254 \text{ volts.}$$

The E.M.F. required to be generated on full load at a power factor of 0·8 is obtained from Fig. 267 and is 1,440 volts.

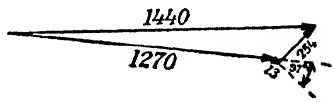


FIG. 267.—Vector Diagram for Full Load Voltage.

The full load flux per pole is, therefore,

$$3{\cdot}73 \times 10^6 \times \frac{1440}{1270}$$

$$= 4{\cdot}23 \times 10^6 \text{ total lines}$$

or $$3{\cdot}52 \times 10^6 \text{ useful lines.}$$

A length of air-gap (see Fig. 265) of 0·6 cm. will be chosen. The approximate air-gap flux density on full load is

$$\frac{3{\cdot}52 \times 10^6}{17{\cdot}9 \times \frac{30 + 27}{2} \times 0{\cdot}9} = 7670.$$

The field ampere-turns per pole required for the air-gap are

$$\frac{10}{4\pi} \times 7670 \times 0{\cdot}6 = 3680.$$

Density in teeth

$$= \frac{3{\cdot}52 \times 10^6}{9 \times 1{\cdot}45 \times 27 \times 0{\cdot}9} = 11100.$$

$$H = 5 \text{ (from Fig. 57).}$$

Ampere-turns for teeth $= \dfrac{10}{4\pi} \times 5 \times 2{\cdot}7 = 11.$

Density in armature core

$$= \frac{3{\cdot}52 \times 10^6}{2 \times 8{\cdot}5 \times 27 \times 0{\cdot}9} = 8500.$$

$$H = 4 \text{ (from Fig. 56).}$$

Length of path $= 20$ cm. (say).

Ampere-turns for armature core $= \dfrac{10}{4\pi} \times 4 \times 20 = 64.$

Section of pole $= 15 \times 30$ with semicircular ends

$$= \frac{\pi}{4} \times 15^2 + 15 \times 15$$

$$= 402 \text{ sq. cm.}$$

Density in pole $= \dfrac{4{\cdot}23 \times 10^6}{402} = 10500.$

$$H = 9.$$

Assume a length of 8 cm.

Ampere-turns for pole $= \dfrac{10}{4\pi} \times 9 \times 8 = 58.$

Assume section of yoke $= 180$ sq. cm.

Density in yoke $= \dfrac{4{\cdot}23 \times 10^6}{180 \times 2} = 11800.$

$$H = 11{\cdot}5.$$

Length of path $= \dfrac{\pi \times 110 \text{ (say)}}{16 \times 2} = 11$ cm. (say).

Ampere-turns for yoke $= \dfrac{10}{4\pi} \times 11{\cdot}5 \times 11 = 101.$

Total ampere-turns per pole $= 3680 + 11 + 64 + 58 + 101.$
$= 3914.$

Assume a c.c. supply of 480 volts.

Volts per coil $= \dfrac{480}{16} = 30$ volts.

Resistance of mean turn (hot) $= \dfrac{30}{3914} = 0{\cdot}00765$ ohm.

" " " (cold) $= \dfrac{6}{7} \times 0{\cdot}00765$

$= 0{\cdot}00655$ ohm.

Estimated length of mean turn with 3 cm. depth of winding

$$= \pi \times 18 + 2 \times 15 = 86{\cdot}5 \text{ cm.}$$

Ohm per metre $= 0{\cdot}0076.$

The nearest wire to this is No. 16 S.W.G., having a resistance of 0·00818 ohm per metre and a diameter of 0·163 cm. (bare) and 0·195 cm. (d.c.c.).

Turns per layer	$= \frac{8}{0{\cdot}195} \times 0{\cdot}95 = 39.$
Number of layers	$= \frac{3}{0{\cdot}195} \times 0{\cdot}95 = 14.$
Turns per coil	$= 39 \times 14 = 546.$
Length of coil	$= 546 \times 0{\cdot}865$
	$= 473$ metres.
Resistance of coil (hot)	$= 473 \times 0{\cdot}00818 \times \frac{7}{6}$
	$= 4{\cdot}5$ ohms.
Current	$= \frac{30}{4{\cdot}5} = 6{\cdot}67$ amperes.
Ampere-turns	$= 6{\cdot}67 \times 546$
	$= 3650.$
Watts per coil	$= 30 \times 6{\cdot}67 = 200.$
Total watts lost in excitation	$= 16 \times 200 = 3200$ watts.
Cooling surface per pole	$= (\pi \times 21 + 2 \times 15) \times 8$
	$= 770$ sq. cm.
Watts per sq cm.	$= \frac{200}{770} = 0{\cdot}26,$
or per. sq. in.	$= 1{\cdot}68.$

This calculation has, however, neglected the ends of the coil which will bring down the watts per sq. cm. somewhat.

Allowing about 2,000 watts for the frictional loss the full load efficiency works out at

$$\frac{200000}{200000 + 2100 + 4900 + 3200 + 2000} \times 100$$
$$= 94{\cdot}2 \text{ per cent.}$$

The question of the E.M.F. wave form has not been gone into, but by suitably skewing the pole shoe this could be made approximately sinusoidal.

CHAPTER XIX

HIGH TENSION

Standard Voltages.—The voltages commonly employed in electric supply systems are now more or less standardised, although various non-standard voltages are often met with, particularly in old schemes. The various voltages are classified by the Board of Trade into four groups, there being a number of regulations in force in this country dealing with each of the several groups. These four groups of voltages are called low, medium, high and extra high pressure respectively, and their ranges are as follows :—

Low pressure	Not exceeding 250 volts.
Medium pressure	Above 250 and not exceeding 650 volts.
High pressure	Above 650 and not exceeding 3,000 volts.
Extra high pressure	Exceeding 3,000 volts.

High pressure and extra high pressure systems are also frequently called High Tension (H.T.) and Extra High Tension (E.H.T.) respectively.

For lighting purposes 110 and 220 volts are the standards, but 200, 230, 240 and even 250 volts are also met with. For mining work 250 volts are used, this being the limit of the low pressure supply. For tramways and railways 500–650 volts is the usual pressure except where H.T. is used. The standard H.T. voltage for a single phase A.C. supply is 2,200 volts, whilst for three phase 6,600, 11,000 and 22,000 volts are now standard practice. Extremely high voltages such as 55,000 and 110,000 are also occasionally met with, but on account of the relatively few examples in existence these can hardly be said as yet to be standardised by practice.

Commercial Frequencies.—The standard frequencies are 50 for lighting and 25 for traction, the latter being undesirable for lighting on account of the flickering produced in the illumination, whilst the various types of traction motors employed work better on low frequencies. In fact, even lower frequencies such as 15 have been adopted on occasion for traction schemes. In systems erected

before the standardisation took place, a number of peculiar frequencies are still met with, the most common being probably 40 and 60, and these are perpetuated because extensions have to be made at the original frequencies.

Insulation Resistance.—The insulation resistance is the joint resistance of all the leakage paths in parallel between the two insulated conductors concerned, and is determined by measuring the leakage current and calculating it from Ohm's law. In the case of two insulated cables, an increase in the length is really an increase in the leakage surface, and results in a decrease in the insulation resistance which is inversely proportional to the length. It is usually specified in megohms per mile and varies to a considerable extent from day to day, since it is largely dependent upon the amount of moisture present, which in turn depends upon the humidity of the atmosphere, even when the insulation is impregnated with non-hygroscopic compounds. The effect of the latter causes a considerable time lag, however, since the impregnated insulation does not immediately take up the moisture from the atmosphere.

Dielectric Strength.—The dielectric strength of an insulator is a totally different thing from its insulation resistance and is measured by the voltage which each centimetre thickness of the insulation will stand, and a high insulation resistance does not at all imply a high dielectric strength. The higher the voltage the greater the strain on the insulation, and thus the maximum value of the voltage must be considered and not the R.M.S. value, since it is possible, by having two different wave forms, to strain the insulation differently, although the R.M.S. value may be the same in the two cases. When a specimen of insulation breaks down it becomes punctured, and on a subsequent application of voltage this puncture acts as a conducting path, with the result that the insulation is permanently ruined. Further, it has been found by experiment that straining a specimen of insulation close up to its breakdown point is bad for it, since it has a weakening effect. By applying a voltage slightly lower than that which will cause breakdown for a considerable time, a reduction in the maximum voltage which the insulation will stand is brought about, thus necessitating a considerable factor of safety. In fact, time curves can be obtained for solid insulators showing the relation between the applied voltage and the time taken to break down the insulation. These curves are of the type shown in Fig. 268. This time effect is also present in the case of air insulation, but is so irregular as to prevent such a curve being obtained. Another point in connection with the breakdown voltage is that it is not by

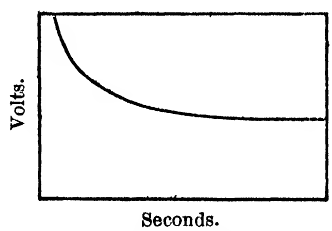


FIG. 268.—Time Curve of an Insulator.

any means proportional to the thickness of the insulation, increasing more slowly than the thickness. Also two layers of material will give a different value from one layer of double the thickness, whilst different specimens of the same material, even from the same batch, will give surprisingly different results.

H.T. Switchboards.—In view of the great personal danger which would be incurred in operating H.T. switchgear directly, special precautions have to be taken, resulting in the evolution of switchboards totally different from those in use on L.T. circuits. The H.T. apparatus is all placed in chambers which are normally inaccessible when the gear is " live " and is operated from a distance by what is known as *remote control.* The switchboard itself, as known in L.T. work, is done away with and the apparatus is mounted upon the wall, usually in separate cells or cubicles corresponding to the panels of a L.T. switchboard. Two types of remote control are employed, viz., mechanical and electrical. In the mechanical remote control the switches are operated by long rods and levers, which project through a protecting partition in front of the switchgear. In some cases the H.T. switchgear itself is placed on the ground level, whilst the operating is done from a gallery situated above, in which case the switch levers come up through the floor. All ammeters and voltmeters are operated through transformers, so that only L.T. apparatus is dealt with by the switchboard attendant.

Electrical remote control consists in having the switches operated by means of electromagnets or small motors, the controlling gear being supplied from an auxiliary source at a low pressure.

Examples of pneumatically controlled switchgear are also in existence, but they suffer from the disadvantage of complication in common with electrical remote control.

Sectionalised Bus Bars.—In large generating stations it is advisable to divide the bus bars into sections in order to minimise the possibility of a shut-down. It may also be desired to run the different sections separately on occasion. Duplicate bus bars are employed, each one frequently being built in the form of a closed ring so that it may be supplied from either end. A complete set of synchronising gear is required for each section, and in addition another set is necessary for the purpose of indicating whether the various sections are in phase before paralleling them. A modern development consists in inserting reactance coils between the sections in order to limit the effects of a possible short circuit. These do not cause any loss of power and, further, do not interfere with the regulation, merely limiting the amount of power transferred from one section to another.

H.T. Fuses.—The great difficulty with H.T. fuses lies in effectually extinguishing the arc which is formed when they operate. For this purpose very long fuses are employed, often encased in porcelain

tubes which sometimes reach a length of several feet. These porcelain tubes are apt to become broken by the violence of the explosion on a sudden short circuit, and to reduce the chance of this they are sometimes lined with plaster of Paris, which serves as a buffer.

Oil immersed fuses (such as the Ferranti oil fuse) are also employed, the fuse itself being kept in tension by means of springs, so that when it melts the broken ends are drawn rapidly apart and dragged under the oil, where the arc is effectually extinguished.

Relays.—Relays are largely used on H.T. circuits for the purpose of operating circuit breakers and have superseded fuses to a large extent on account of their greater reliability. Relays may be divided broadly into the following groups :—

1. Maximum or Overload Relays.
2. Minimum Relays.
3. No-load or No-voltage Relays.
4. Reverse Current Relays.
5. Reverse Power Relays.
6. Differential Relays.

A simple form of the solenoid type of overload relay is shown in Fig. 269. The main current is passed through a solenoid which attracts an iron plunger the movement of which is retarded by means of a dashpot, producing what is known as a *time lag*. In the majority of cases instantaneous action is not desired, the amount of time lag required being dependent upon the duties which the relay fulfils. When the plunger is lifted two switch contacts are closed, allowing an auxiliary battery to send a current through the trip coil of the circuit breaker, which causes it to operate. The induction or Ferrari's principle is also used, the action being the same as in the induction type ammeter and voltmeter except that instead of a pointer a cord runs over a pulley and lifts or lowers a movable switch contact which closes and opens the trip coil circuit.

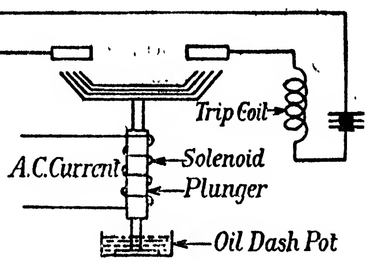


FIG. 269.—Overload Relay.

Minimum current and no-voltage relays are constructed on the same principle as the above, only the switch contacts are closed when the plunger sinks to a certain level.

An example of a reverse current relay for A.C. work is shown in Fig. 270, which illustrates the Andrews' discriminating relay. The relay contacts are carried on an iron core which is magnetised by two windings in parallel. As these two are magnetically in opposition, there is no resultant pull in normal conditions. The two free

ends of the coils on the plunger are connected to a similar differential winding on a choking coil, the whole arrangement being connected across the bus bars through a potential transformer. The choking coil is also magnetised by another winding obtained from a current transformer in series with the main line. If the main current is normal the effect of this additional winding is to weaken the top

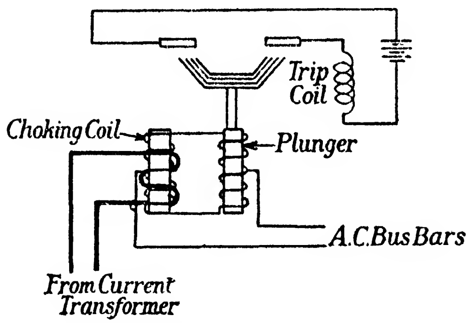


FIG. 270.—Andrews' Discriminating Relay.

half of the differential winding and to strengthen the lower half, thus holding the plunger down. On the other hand, if a reversal of the current occurs, such as is obtained when an alternator takes a motoring current from the bus bars, the upper half of the differential coil is strengthened and the plunger is raised, thus closing the trip coil contacts.

The wattmeter principle is frequently adopted for reverse power relays, since the deflection of a wattmeter pointer reverses when the direction of transference of power is reversed. In the relay the pointer is replaced by a disc which rotates, thereby raising or lowering the trip coil switch contact. A diagram of connections of such a relay is given in Fig. 271. The relay, *R*, has a series and shunt coil fed from a current transformer, *C.T.*, and a potential transformer, *P.T.*, respectively. The latter circuit is protected by a fuse, *F*. When the relay contacts are closed the battery, *B*, sends a current through the trip coil, *T.*, so as to operate the circuit breaker and thus isolate the generator, *G*.

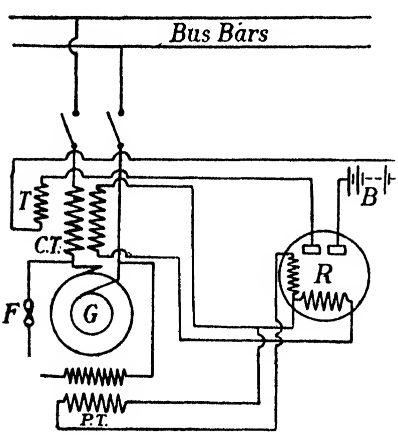


FIG. 271.—Reverse Power Relay Connections.

Merz-Price Gear.—The Merz-Price protective gear has for its object the isolation of a faulty feeder when a breakdown occurs, the relays not operating in the case of a surge or a temporary over-

load. The feeder to be protected has a small current transformer, *C.T.*, placed in series with it at each end, the two secondaries being joined in series with one another, but in opposition, and in series with two relays, *R*, as shown in Fig. 272. This necessitates a small auxiliary cable joining the two relays, which are situated at opposite ends of the feeder. Since the two current transformers are similar and in opposition, they will balance one another for all loads, and the relays will not open the circuit breakers even on an overload. But if a fault develops on the feeder, the current flowing out at the

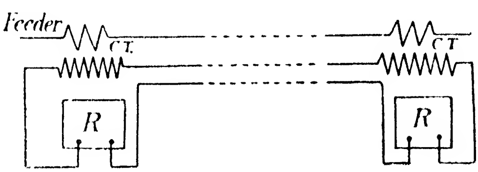


FIG. 272.—Arrangement of Merz-Price Gear.

far end is no longer equal to the current flowing in at the near end, and the two secondaries will no longer balance each other. The resulting current in the relay circuit now operates the circuit breaker and isolates the faulty feeder. In the case of a feeder linking up two sub-stations, the current may flow either way in normal circumstances, and thus a reverse current relay would not be admissible, whilst this apparatus would protect the circuit.

In the case of a three phase system, a three core pilot cable is laid with each feeder to provide the relay circuits.

Instruments.—For voltmeters in use on H.T. circuits the usual practice is to operate them through potential transformers [see Fig. 175 (*a*)], the scales of the instruments being marked so as to indicate the line voltage. Each voltmeter must be calibrated in conjunction with its own particular potential transformer.

In a similar way, ammeters are run from the secondaries of current transformers [see Fig. 175 (*b*)], so that the instruments themselves are not in electrical contact with the H.T. system.

In the case of wattmeters and watt-hour meters, the pressure coils are treated like voltmeters, being run through potential transformers, and the current coils are treated like ammeters, being run through current transformers.

Line Choking Coils.—Choking coils of a particular design and carrying the full line current are employed at the generator end of a transmission line for the purpose of protecting the apparatus behind it from the effects of a lightning discharge the frequency of which is supposed to be of the order of millions of cycles per second. These choking coils usually consist of two flat copper spirals, mounted one on each side of a marble slab and connected in parallel with one another, but in series with the main line. At the normal

frequencies of supply these are practically non-inductive, but with the extremely high frequencies obtained in the case of a lightning discharge thèy produce a very strong choking effect, so that practically none of the induced current flows through them, but is dissipated by the lightning arresters instead.

Earthing.—The advisability of earthing A.C. systems has given rise to much discussion. In the case of single phase railway systems an earthed return is used, the line wire being insulated. In polyphase transmission schemes it is the neutral point which is earthed, except in those cases where the whole system is left insulated. With perfectly balanced loads the neutral point automatically assumes the earth potential, but these conditions are by no means always fulfilled. The potential of the neutral point then tends to assume a value other than that of the earth, and this is prevented by putting the two points in electrical contact. This tends to maintain equality in the capacity currents to earth in the different phases and prevents the consequent unbalancing of the voltages. It also tends to maintain equality in the maximum stress in the insulation to earth, the magnitude of which would be increased if the potential of the neutral point were allowed to alter.

On the other hand, a single earth fault on the system is sufficient to cause an interruption of the supply, which is not the case if the whole system is insulated throughout in normal circumstances. In order to limit the short circuit current resulting from such a fault, it is usual to insert a resistance or a choking coil in the earth connection from the neutral point.

CHAPTER XXI

SYNCHRONOUS MOTORS

Alternator used as a Motor.—If two alternators are run in parallel and the driving force of one is suddenly cut off, the machine will continue to run as a motor and will take the power necessary to drive it from the other machine, which will be loaded to a certain extent on this account. The supply of C.C. to the field system must be maintained throughout. For an alternator to act as a motor it must, therefore, be supplied with A.C. for the armature and C.C. for the field system, and further, as will be seen later, the machine must be brought up to the speed of synchronism before the motoring action takes place.

In order to examine the action in detail, take the case of the elementary two pole single phase synchronous motor shown in Fig. 285. This machine is supposed to be exactly the same as the corresponding alternator, and may have a stationary field and a rotating armature, or *vice versa*, the current being led into the rotating element by means of slip rings. Consider the conductor which arrives at A at the moment when the current is zero. The instantaneous value of the torque due to this conductor is also zero, since it is proportional to the product of the field strength and the armature current. The field strength is assumed to be constant throughout. A moment later the conductor has arrived at B and the armature has rotated through an angle, θ. But the current has also advanced in phase by $\theta°$ and is supposed to be flowing away from the observer. This produces a torque in a clockwise direction and causes the armature to revolve. When the conductor reaches C the current has reached its maximum value, and by the time the conductor reaches D the current has died down to zero. Throughout the whole of this half-revolution, which has taken place whilst the current has advanced through half

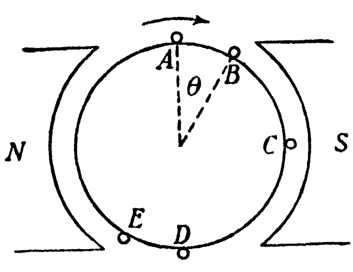


Fig. 285.—Action of Synchronous Motor.

a cycle, the torque has been in the same direction. A little later the conductor arrives at E, but the current has now started to grow in the *reverse* direction. However, since this reverse current is cutting the magnetic field in the reverse direction, the torque still tends to produce rotation in a clockwise direction. By the time the armature has completed one whole revolution the current has advanced through one whole period. This is the essential condition for the continuance of rotation; the armature must rotate synchronously with the current, and hence the machine is called a synchronous motor. The currents in the other armature conductors produce torques which aid one another, for during the first half-period of the current they are cutting the field in one direction, and during the second half-period, when the current reverses, they are cutting the field in the other direction.

The same principle operates in the case of two and three phase synchronous motors, these being much superior in their performance to the single phase machines, their great advantage lying in the fact that they are much less likely to drop out of step when subjected to overloads.

Conditions for Running.—Such a motor as is described above is not self-starting, and it is necessary first to run up the machine to synchronous speed by some external means and then to synchronise it with the supply, the field having first been excited. In other words, the machine must be run up and synchronised exactly like an alternator, and the same conditions apply. The machine will then continue to run and take a motoring current from the supply as long as it keeps in step. By this is meant keeping in synchronism. If the armature does not rotate synchronously with the variations of the current, there come certain points in the revolution where a reverse torque is produced, and this tends to pull the motor up. The retardation of the armature causes it to fall still further out of step, with the result that the motor quickly comes to rest. Since the armature is still supplied with current, the circuit should be opened immediately, to prevent damage, since the motor at standstill has only the impedance of its armature to limit the current.

Speed.—The speed of a two pole synchronous motor has already been shown to be one revolution per cycle, and this corresponds to a speed of $60f$ revolutions per minute, where f is the frequency. If the motor is a multipolar machine having p poles, the armature conductors will advance past one pole pair every cycle, or, in other words, the armature will rotate through $\frac{2}{p}$ ths of a revolution. The revolutions per minute will then be equal to

$$\frac{2}{p} \times 60f = \frac{120f}{p}.$$

The speed of a polyphase synchronous motor is the same as that of the corresponding single phase machine with the same number of poles.

The only way to vary the speed is to vary the frequency, and this is not a practical method. Synchronous motors are, therefore, essentially constant speed machines.

No-load Conditions.—When a synchronous motor is first synchronised with an alternator, the two machines momentarily act as generators in parallel across the same bus bars, their E.M.F.'s acting in the same direction. Considering the local circuit formed by the two armatures, these may be regarded as being in series with each other with their two E.M.F.'s always opposing, as shown in Fig. 286. The armature of the motor may, therefore, be considered as setting up a back E.M.F. equal to and opposite in phase to the applied voltage. The resultant voltage in this circuit is zero, and so no current is supplied to the motor armature. Since the latter receives no power in an electrical form from the supply, it immediately commences to slow up when the mechanical driving power is removed. But as soon as the motor armature falls behind the position where it should be if it maintained an absolutely synchronous motion, the back E.M.F. and the applied E.M.F. no longer neutralise each other, for, notwithstanding the fact that they are equal, no two voltages can completely neutralise one another unless they are in phase opposition. The applied and the back E.M.F. now produce a resultant voltage which causes a current to flow through the motor armature and supplies it with a certain amount of power. If this power is sufficient to maintain the rotation, the motor continues to rotate synchronously, but always lagging by a constant small angle. If the power supplied to the motor in this manner is not sufficient to overcome the losses at this speed, the armature is retarded and lags behind by a greater angle. The effect of this is to increase the resultant voltage and the armature current, and the power supplied to the motor is thereby increased. This action goes on until the motor lags by such an angle as to produce a resultant voltage which will cause the necessary amount of power to be transferred to the motor.

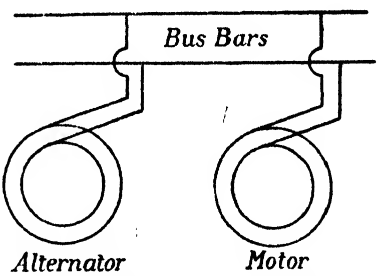


FIG. 286.—Alternator driving Synchronous Motor.

The action is similar in a way to that which takes place in a C.C. shunt motor where the back E.M.F. is exactly in opposition to the applied E.M.F., only in the A.C. case the back E.M.F. lags by rather more than 180° behind the applied E.M.F. instead of being in exact phase opposition.

Torque.—In a simple two pole single phase synchronous motor the axis of the armature field may be considered as lying along the line joining the two conductors which are connected to the slip rings, just as in a C.C. motor the armature field lies along the line joining the brushes. This field rotates with the armature, but also varies according to a sine law with time, and it reacts with the main field produced by the exciting current. Since the latter field is constant, the torque is proportional to that component of the armature field which is at right angles to the main field. The magnitude of the armature field after the armature has moved through $\theta°$ from the zero position may be represented by $\Phi \sin \theta$, where Φ is the maximum value. At the same instant, the vertical component of this (see Fig. 287) is $\sin \theta$ times its actual value, and thus the instantaneous torque is proportional to $\Phi \sin^2 \theta$. This expression shows that the torque is of a pulsating character, although it never reverses in direction.

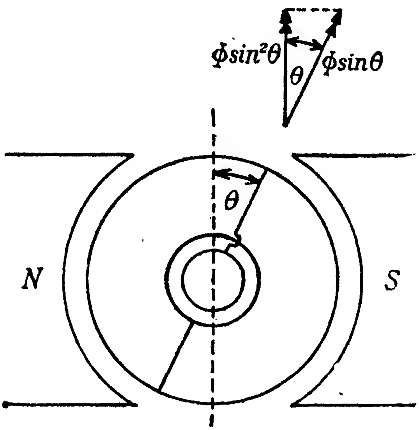


FIG. 287.—Hypothetical Armature Flux.

In the case of a three phase synchronous motor, the armature field may be regarded as consisting of three components spaced 120° apart in space and differing by 120° in phase. The useful part of each component is obtained in the same way as before. The instantaneous magnitudes of the three hypothetical fluxes are $\Phi \sin \theta$, $\Phi \sin (\theta + 120°)$, and $\Phi \sin (\theta + 240°)$, as shown in Fig. 288. The vertical components are $\Phi \sin^2 \theta$, $\Phi \sin^2 (\theta + 120°)$, and $\Phi \sin^2 (\theta + 240°)$. The resultant vertical field is obtained by adding these together, and is equal to

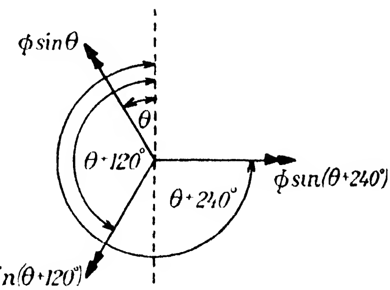


FIG. 288.—Hypothetical Armature Fluxes in Three Phase Motor.

$$\Phi \sin^2 \theta + \Phi \sin^2 (\theta + 120°) + \Phi \sin^2 (\theta + 240°)$$

$$= \Phi \{\tfrac{1}{2} - \tfrac{1}{2} \cos 2\theta + \tfrac{1}{2} - \tfrac{1}{2} \cos 2(\theta + 120°) + \tfrac{1}{2} - \tfrac{1}{2} \cos 2 (\theta + 240°)\}$$

$$= \Phi \{\tfrac{3}{2} - \tfrac{1}{2} \cos 2\theta - \tfrac{1}{2} \cos (2\theta + 240°) - \tfrac{1}{2} \cos (2\theta + 120°)\}$$

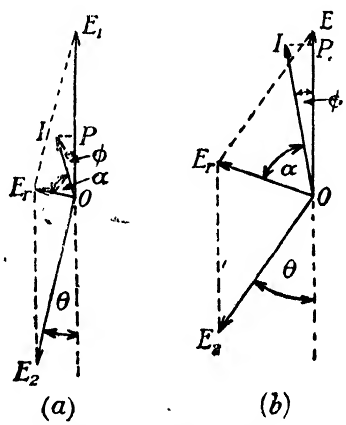
$$= \Phi \{\tfrac{3}{2} - \tfrac{1}{2} \cos 2\theta - \tfrac{1}{2} \cos 2\theta \cos 240° + \tfrac{1}{2} \sin 2\theta \sin 240° - \tfrac{1}{2} \cos 2\theta \cos 120° + \tfrac{1}{2} \sin 2\theta \sin 120°\}$$

$$= \Phi \left\{\tfrac{3}{2} - \tfrac{1}{2} \cos 2\theta + \tfrac{1}{4} \cos 2\theta - \frac{\sqrt{3}}{4} \sin 2\theta + \tfrac{1}{4} \cos 2\theta + \frac{\sqrt{3}}{4} \sin 2\theta\right\}$$

$$= \tfrac{3}{2}\,\Phi.$$

In other words, the magnitude of the torque is independent of the position of the armature, and the same statement holds true for a two phase machine. The fact that the torque is constant in magnitude in a polyphase machine whilst it is of a rapidly pulsating character in a single phase motor, dropping to zero twice per period, indicates the reason why a single phase synchronous motor is so much more liable to drop out of step when a heavy overload or a sudden change of load occurs.

Effect of Load.—When a load is put upon the shaft of the motor, the first tendency is to retard the rotation, but as soon as the armature commences to drop behind, it causes the back E.M.F. to lag by a rather larger angle than before. It will be remembered that the angle of phase difference between the applied and the back voltage is rather more than 180°, the applied voltage leading. But an increase in the angle of lag of the back voltage causes an increase in the resultant voltage acting on the circuit, and this in turn causes an increased current to flow, with the result that the motor takes more power from the supply. The motor armature then drops behind until its position, relatively to that of the driving alternator, is such as to produce a resultant voltage capable of causing sufficient current to flow to deal with the load. The armature will then continue to rotate synchronously with the driving alternator. If the motor should drop too far behind, the power which it will take from the supply will be greater than is necessary, and so the armature will be accelerated until it is in its correct relative position. Some motors are subject to this overshooting the mark, and when a motor is constantly retarded and accelerated in this manner the effect is called *hunting* or *phase-swinging*.



(a) (b)

FIG. 289.—Vector Diagrams of Synchronous Motor.

(a) On No-load. (b) On Load.

Vector Diagram.—The study of the vector diagram of the synchronous motor is the best means of understanding what is really going on inside the motor. For the sake of simplicity, the case of a single phase machine will be considered, but the same principles apply in the case of polyphase machines. In Fig. 289 (*a*) OE_1 represents the applied voltage and OE_2 the back voltage. In the first instance these two voltages will be taken as equal, representing the conditions which occur when the machine is perfectly synchronised, OE_2 lagging behind OE_1 by an angle $180° + \theta$. Combining these two voltages together, the resultant voltage OE_r is obtained. This voltage leads the applied voltage by rather less than 90°. For the purposes of this vector diagram, the armature reactance

and armature reaction will be combined together under the term synchronous reactance as in the case of alternators (see p. 257). The effect of armature reaction need not then be allowed for separately, and the resultant voltage, OE_r, can be considered as acting on a circuit having a definite resistance and synchronous reactance. As these two quantities are approximately constant, the angle of lag of the current behind the resultant voltage will be constant. If the synchronous impedance of the armature be known, the current vector, OI, can be plotted, lagging behind OE_r by a fixed angle, α, equal to $\tan^{-1}\frac{X}{R}$, the magnitude of the current being given by $\frac{E_r}{\sqrt{R^2+X^2}}$. The angle of lag or lead, ϕ, of this current with

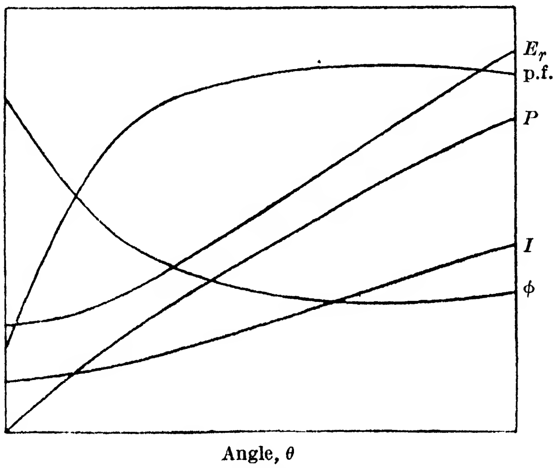


FIG. 290.—Effect of θ.

respect to the applied voltage determines the power factor under which the motor is operating, the current leading in the example shown in the figure. Since the applied voltage, E_1, is supposed to be constant, the power taken by the motor is represented to scale by OP, which is the projection of OI on OE_1. After subtracting the losses incurred by the motor, the output is obtained.

An increase of load results in an increase in the angle θ, as shown in Fig. 289 (*b*). This swings the vector OE_r round a little and increases its magnitude. The magnitude of the current is now also increased, since the synchronous impedance remains unaltered, and the power taken by the motor ($E_1 I \cos\phi$) goes up as well. By choosing a number of values for θ the corresponding values of E_r, I and $\cos\phi$ can be obtained, and these are plotted in Fig. 290 for a particular example, where the applied E.M.F. is 100 volts, the back E.M.F. is 75 volts, $R = 0{\cdot}04$ and $X = 0{\cdot}20$. These curves are re-plotted in Fig. 291 on a base of power input instead of θ.

Effect of Excitation.—The direct effect of varying the excitation is to vary the back E.M.F., and this reacts upon the behaviour of the machine to an enormous extent. In a C.C. shunt motor a variation in the exciting current produces a variation in the speed, but in the synchronous motor this is not possible, and so a variation in the current is produced instead. But as the power supplied to the motor depends upon the load and not upon the exciting current (except in so far as the losses are varied), the variation in the current must be accompanied by a variation in the power factor. The effect can be studied best by a reference to the vector diagram. For this purpose Fig. 289 (*b*) has been reproduced in Fig. 292 (*a*). Now suppose that the exciting current has been decreased so as to reduce the back E.M.F. to 75 per cent. of its former value. The value of

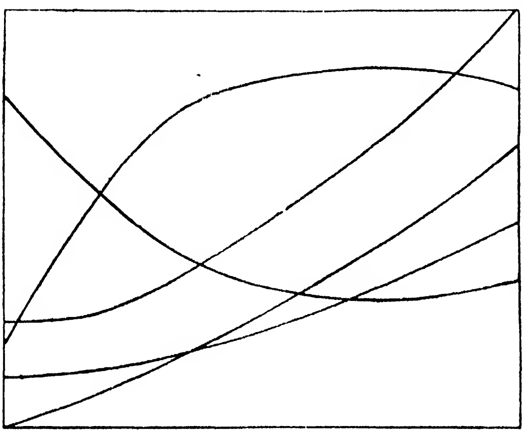


FIG. 291.—Power Curves.

the angle θ will not change appreciably, as this depends largely upon the load. The resultant voltage, E_r, will be brought nearer in phase to E_1, and the current, lagging by the fixed angle α behind E_r, will be retarded in phase, as shown in Fig. 292 (*b*). A further reduction of the back E.M.F. to 50 per cent. of its original value accentuates this effect and causes the current to lag still further behind its former position [see Fig. 292 (*c*)]. On the other hand, an increase in the exciting current raising the back E.M.F. to 125 per cent. of its original value advances the phase of E_r and I, thus tending to reduce the lag or increase the lead of the current [see Fig. 292 (*d*)].

It thus appears that the phase of the current with respect to the applied voltage may be adjusted at pleasure by the simple expedient of varying the excitation, and the motor may be made to operate on any power factor with either a leading or a lagging current. There is, of course, a limit to this reduction of the power factor, due

to the dangerous heating which is set up on account of the excessive current, or the motor may drop out of step. Synchronous motors have, however, been known to remain running even though the excitation has been wholly interrupted, the machines running on their residual magnetism.

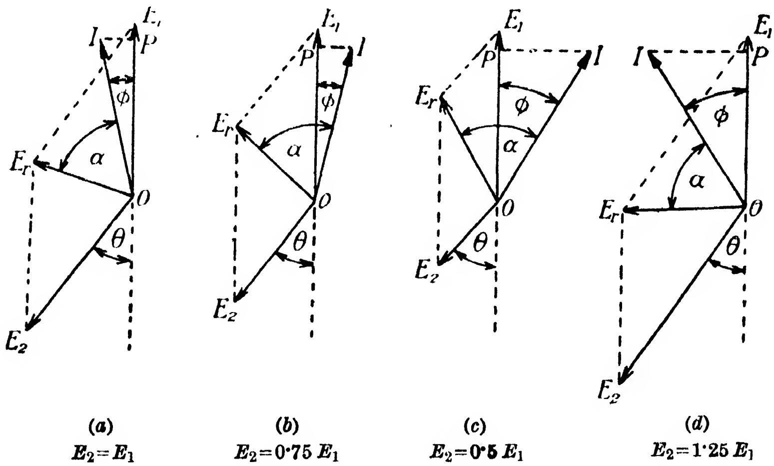


FIG. 292.—Effect of Excitation.

V-curves.—If a series of vector diagrams is drawn with various back E.M.F.'s like those in Fig. 292, a number of currents can be obtained all relating to the same power output. Each current will correspond to a particular back E.M.F., which in turn will correspond to a particular excitation. The latter can be determined

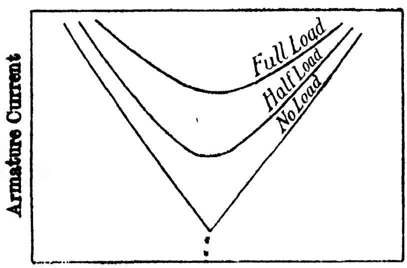


FIG. 293.—V-curves.

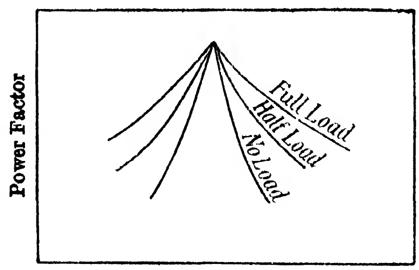


FIG. 294.—Power Factor Curves.

from the magnetisation curve of the machine. By varying the excitation, a number of corresponding pairs of armature current and exciting current can be obtained. It is found that on plotting these figures the resulting curve takes the form of a V, as shown in Fig. 293, and is known as a V-curve. When the excitation is small, the back E.M.F. is low, giving rise to a resultant voltage leading the applied voltage by a relatively small angle. This causes the

current to lag behind the applied voltage by a considerable angle, and since the power factor is small the current is relatively large. As the excitation is increased, the back voltage is also increased, thus swinging the resultant voltage vector round and advancing it in phase. The current is also advanced in phase, its magnitude decreasing since the power factor is increased. When the current becomes in phase with the applied voltage it reaches a minimum value, after which it commences to increase again. The excitation corresponding to this minimum current is called the normal exciting current for that particular load. The effect of over-exciting a synchronous motor is to cause the current to lead the applied voltage due to the lengthening of the back E.M.F. vector. As the excitation is still further increased the armature current also increases and the power factor falls. On no-load, the point on the V-curve is sharply accentuated, but if the machine is loaded the tendency is to round off the point, this effect being more marked at the higher loads.

Power Factor.—From the preceding paragraph it is seen that the armature current varies between wide limits for the same power output, and this causes the power factor to vary widely in accordance with it. The curves of power factor corresponding to the armature currents represented in Fig. 293 are shown in Fig. 294, where it is seen that they look like inverted current curves. Again, the no-load power factor curve is fairly sharp at the apex, whilst the others are less sharp, an increase in load tending to flatten out the curve. An examination of these power factor curves shows that particularly on no-load the armature current and the power factor are very susceptible to changes of excitation. The warming up of the field coils is quite sufficient to cause a material change in the armature current.

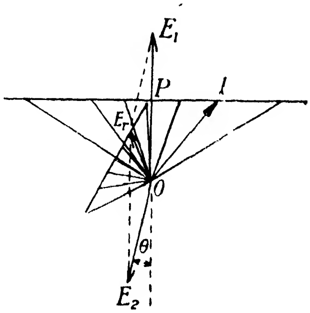


FIG. 295.—Locus Vector Diagram of Synchronous Motor.

Example of V-curve.—Instead of assuming the angle θ to be constant for a given load, the power input may be assumed constant instead. In this case, since the applied voltage is fixed, the power component of the current remains unaltered, even although the excitation varies. In Fig. 295, where OE_1 represents the applied voltage and OI the armature current, a line drawn through I at right angles to OE_1 is the locus of the current vector. A line drawn from O to any point in this line represents a current the power component of which is OP. Taking any random value of OI, the resultant voltage producing it can be obtained by multiplying by the synchronous impedance. This voltage leads the current by an angle, α, equal to $\tan^{-1}\frac{X}{R}$. Then,

knowing the value of this resultant voltage and the applied voltage, the back E.M.F. generated is obtained by subtraction. The excitation corresponding to this back E.M.F. is then read off the magnetisation curve. Working backwards in this way, a series of vector diagrams can be constructed without making any assumption as to the magnitude of the angle θ. If a series of values of I are taken in this way, a series of values of E_r and E_2 can be obtained and the locus drawn in for each, this being done in Fig. 295. The whole diagram can then be repeated, if desired, for another load. Strictly speaking, the locus of the current for a given output is not a straight line, since the losses are increased when the current goes up, so that the input is not quite constant. This can be allowed for by making the current locus bend upwards a little on each side.

Experimental Determination of V-curves.—The actual experimental determination of one of these curves is carried out by maintaining a constant load throughout a single series of observations and by varying the excitation through as wide limits as the machine will permit without overheating or falling out of step. The applied voltage should be maintained constant throughout, and this will often be a source of trouble, because, if the driving alternator is comparable in output with the motor, the large current at the very low power factor obtained will necessitate a certain amount of field regulation in order to maintain the voltage constant.

A series of such curves can be obtained from the one motor, each one corresponding to a particular load.

Ampere-turn Diagrams.—In the previous paragraphs a constant synchronous reactance was assumed, the effects of both armature reactance and armature reaction being included in it. An alternative method of treatment is to assume a constant reaction, including in its effects those of the true armature reactance. The armature is thus assumed to be non-inductive, and for the sake of simplification the effect of the armature resistance will be neglected. This will not produce a very appreciable error. With these assumptions, the back E.M.F. will be exactly equal and opposite in phase to the applied voltage and will be wholly independent of the exciting current. The exciting current produces an M.M.F. which can be split up into two components, one producing the useful air-gap flux which generates the back E.M.F., and the other a neutralising M.M.F. equal and opposite to that set up by the armature ampere-turns. These two component M.M.F.'s are in quadrature, and their vector sum gives the total M.M.F. which must be supplied by the exciting current. The vectorial addition of the fluxes produced by these M.M.F.'s is, however, not strictly accurate, as it neglects the effect of changes of permeability consequent upon the changes in flux density. If this point be neglected, the vector diagram might be drawn to a scale of ampere-turns instead of M.M.F.'s, since they

are proportional to one another, or it may be drawn to a scale of exciting current even, since the turns on the field system are constant. Fig. 296 (a) represents the vector diagram when the power factor of the motor is unity, the compounded vectors being drawn to a scale of ampere-turns. E_1 represents the applied voltage and E_2 the back E.M.F., these two being exactly equal and opposite. The armature current, I, is in phase with E_1, since the power factor is unity, and its magnitude is determined by the load. The back E.M.F. generated is proportional to the rate of change of M.M.F., and this quantity leads the actual M.M.F. by 90°, as was shown when dealing with reactance. The ampere-turns producing the back E.M.F., therefore, lag behind E_2 by 90° and are represented by AT_v. The ampere-turns required to balance the armature ampere-turns are in phase with the armature current and are represented by AT_a. The vector sum of these two is given by AT_t, which is proportional to the exciting current required. If the armature current be zero, AT_a disappears and AT_v becomes equal to AT_t. The applied voltage being maintained constant, AT_v will have a constant value independent of the load, and can be determined from the open circuit magnetisation curve when the machine is run as an alternator. Similarly, if AT_v be made zero, then AT_a is equal to AT_t. These conditions are imitated by running the motor as an alternator on short circuit. The magnitude of AT_a can then be determined for various values of the armature current.

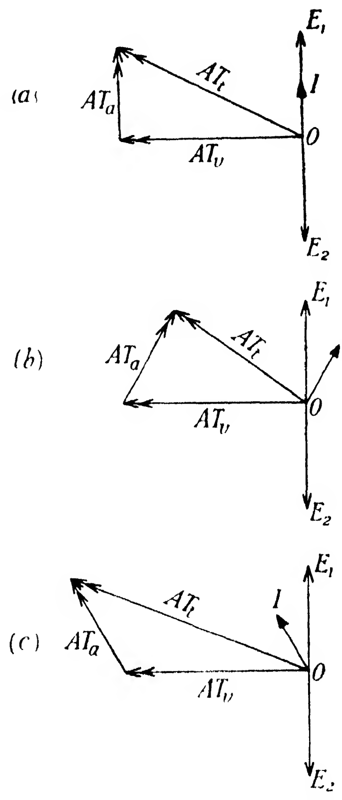


FIG. 296.—Ampere-turn Diagrams of Synchronous Motor.

If the armature current lags behind the applied voltage the conditions are represented by Fig. 296 (b), where it is seen that for the same power input the exciting current is reduced. If the motor is running with a leading current the conditions are those shown in Fig. 296 (c), which shows at a glance why the motor must be over-excited.

These ampere-turn diagrams should be compared with those in Fig. 247 relating to the alternator, remembering that the generated E.M.F. in the alternator corresponds to the back E.M.F. in the synchronous motor.

In actually drawing a diagram of this kind it is much simpler to draw it to a scale of exciting current rather than ampere-turns, as the number of field turns is not then required.

In order that Fig. 296 (*a*), (*b*), and (*c*) should represent the same load, it is necessary that the power component of I should be equal in all three cases. An increase of load would result in an increase in the power component of I.

Armature Reaction.—The armature reaction of a synchronous motor can be investigated in the same way as is adopted in the case of an alternator. Reference should be made to Figs. 243 and 244, remembering that in the synchronous motor the current is in phase with the applied voltage if the power factor be unity, and consequently leads the generated back E.M.F. by an angle $180° + \theta$. As a first approximation, therefore, if the currents in Figs. 243 and 244 be reversed, the diagrams will represent the action in the synchronous motor, and proceeding in this manner it is seen that when the motor is operating with a lagging current the magnetic flux is increased in magnitude, and when a leading current is taken the magnetic flux is decreased, this being the reverse of what happens in an alternator. In addition, the flux is, of course, distorted in both cases. A lagging current thus tends to increase the back E.M.F., whilst a leading current tends to decrease it.

Overload Capacity.—If the applied voltage and the exciting current of a synchronous motor be kept constant, the effect of an increase of load is to retard the back E.M.F. thus causing an increased resultant voltage and an increased current. The power input, however, does not go on increasing indefinitely, since the gradual retardation of the back E.M.F. vector causes the current to lag more and more behind the voltage, and there comes a time when the power factor decreases at a greater rate than the current increases. A further increase of load then causes a further retardation of the back E.M.F., resulting in a decrease in the power absorbed and in the driving torque. The reduced driving torque retards the motor again, which reduces the power input, and so this action goes on until the motor falls out of step and comes to rest. The difference between the maximum load capable of being overcome and the normal full load of the motor, expressed as a percentage of the full load, is called the *overload capacity* of the motor. In Fig. 290 the relation between the power input and the angle θ is plotted for a constant excitation and back E.M.F. If these curves were continued for larger values of θ it would be seen that the power developed reaches a maximum value, after which it commences to decrease for a further increase of θ. Similarly, it would be seen that in Fig. 291 there are two values of θ corresponding to each value of the power input. The smaller of these two angles (shown in the diagram) represents the stable running condition, whilst the larger

angle represents the unstable condition when the motor is falling out of step and pulling up.

An increase in excitation results in a roughly proportional increase in the overload capacity until magnetic saturation occurs. This can be determined by redrawing Fig. 290 for a number of different excitations and finding the maximum power input in each case. (The losses of the motor are neglected.) The relation between the percentage overload capacity and the exciting current, obtained in this way, is shown in Fig. 297.

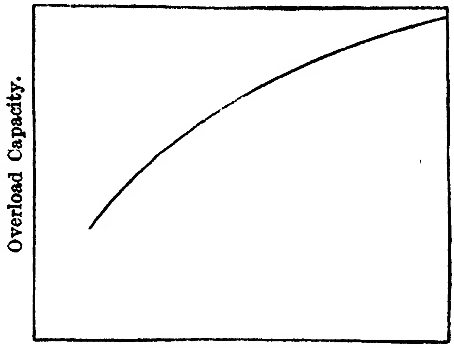


FIG. 297.—Relation between Excitation and Overload Capacity.

The presence of reactance in the motor armature is also bad, and any reduction in this direction is accompanied by an increase in the overload capacity. Thus whilst reactance is beneficial from the synchronising point of view, it is objectionable if the motor is called upon to withstand sudden heavy overloads.

Hunting.—Sudden changes of load on synchronous motors sometimes set up oscillations which are superposed upon the normal rotation, giving rise to periodic variations in speed of a very low frequency. This effect is known as *hunting* or *phase-swinging*, and can be detected by ear on account of the different notes which are set up by the fluctuating speed of the motor. Occasionally the trouble is aggravated by the motor having a natural period of oscillation approximating to the hunting period, when it is possible for the motor to be phase-swung into the unstable region, thus causing it to fall out of step.

The first effect of a sudden increase of load is a retardation of the armature causing the vector E_2 [see Fig. 289 (*a*)] to take up some such position as is shown in Fig. 289 (*b*). But during this period of retardation the armature is running at a speed slightly less than that of synchronism, and when E_2 has reached a position such that the power input is exactly that required to overcome the load, it is still running at this slightly reduced speed. Owing to the inertia of the armature, however, this reduced speed is maintained for a very short interval longer, during which the angle θ continues to increase. The power input is now greater than is required for the load on the motor, and the armature ceases to be retarded and commences to accelerate. This causes the vector E_2 to gain on E_1 and the angle θ commences to decrease. When the stable position is reached the armature is running a trifle faster than the absolutely correct synchronous speed, and, due to its inertia again, it continues

to run a little fast after this point has been reached. The angle θ has now become too small, resulting in a decreased power input which is no longer capable of maintaining the rotation against the resisting torque of the load. The armature is thus again subjected to a retardation and the whole cycle of events is repeated. The net result is that the motor perpetually increases and decreases in speed, the power input periodically varying in unison. The frequency of these changes is usually quite capable of being detected by ear and also by observation of the wattmeter pointer, which oscillates about a certain mean position on the scale.

In order to prevent hunting, modern synchronous motors are fitted with damping grids or amortisseurs (see p. 248). Whenever any motion takes place, other than the absolutely synchronous rotation, the flux in the poles is distorted, and the movement of

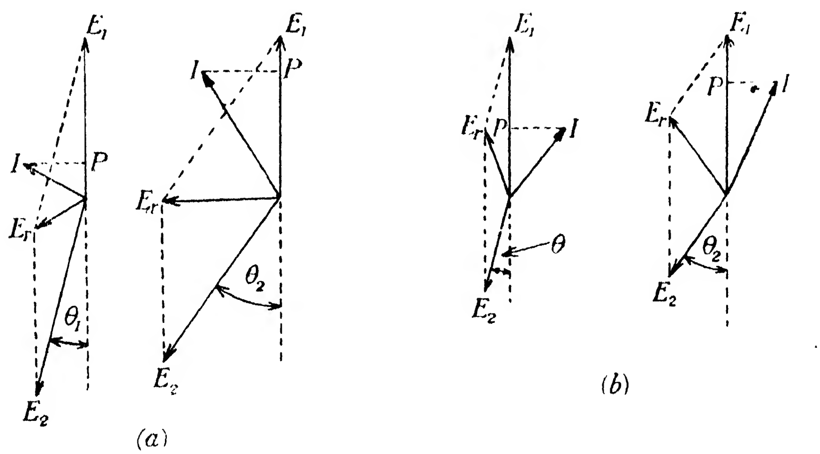


FIG. 298.—Showing Greater Change of Torque with θ for Strong Excitation.

this flux across the bars of the damping grids sets up eddy currents which tend to damp out the superposed oscillatory movement. If the rotation is absolutely uniform, there is no relative movement of the flux and the damping grid, and so no eddy currents are set up and no losses occur.

Another method of reducing hunting is to work with relatively strong fields, since if E_2 be made large a given change in the angle θ results in a greater change in the torque produced, as can be seen by referring to Fig. 298. The force tending to pull the motor into step is thus greater when strong fields are employed than when weak excitations are worked with.

Starting.—As the ordinary synchronous motor is not self-starting, it is necessary to make arrangements for starting it by some auxiliary means. In some instances there is a small C.C. exciter direct coupled to the motor shaft, and this can be employed to start up the set.

This method, however, necessitates a C.C. supply for the purpose of running up the exciter, and if a C.C. supply is available there is no need for the exciter.

A second method of starting up is to employ a small auxiliary motor of the induction type (see Chapter XXVI). As these motors always run at a speed slightly less than that of synchronism, taking into consideration the number of poles, it is seen that if the auxiliary motor has the same number of poles as the main synchronous motor the set can never be run up to the correct speed for synchronising. In order to get over this difficulty, the auxiliary induction motor is made with fewer poles than the main motor, so that the set can be synchronised as it runs through the correct speed.

Synchronous motors direct coupled to C.C. generators are frequently used for sub-station work in transforming from H.T., A.C. to L.T., C.C. The C.C. bus bars are never dead except when a complete shut down occurs, and the motor-generator set can be run up from the C.C. side, using the generator as a motor. The correct speed for synchronising is obtained by shunt regulation, and after the A.C. motor has been thrown on to the bus bars the field of the C.C. machine is strengthened, which tends to lower the speed. But since the A.C. machine must run at synchronous speed, it follows that the back E.M.F. of the C.C. machine when running as a motor becomes greater than the bus bar pressure, and so it commences to generate. In the event of a complete shut down, these sets would be unable to commence running again, and so one set at least must be provided with independent means for starting up.

Polyphase synchronous motors can also be started up by the addition of a special winding on the field system known as a squirrel cage. This consists of a number of bars let into the poles after the manner of damping grids, all the bars being joined at each end by a stout copper ring going right round the field system. The action of such a winding is discussed in full in Chapter XXVI, where squirrel cage induction motors are dealt with. It is sufficient to say here that the currents in the armature set up a rotating magnetic flux which cuts the squirrel cage winding on the field system and induces eddy currents in it. A torque is developed and the motor runs up to a speed a *little less* than that of synchronism. The motor will now be subjected to a true synchronous motor torque trying to pull it into step, just as is the case when hunting is set up. When synchronous speed is attained the exciting current may be switched on and the machine continues to run as a synchronous motor. This method of starting does away with the necessity for synchronising gear, and in addition the squirrel cage winding acts like a damping winding when running and serves to prevent hunting. On account of the large starting current required by this method, it is usual to start up the motors through auto-transformers.

Application to Sub-station Work.—As the synchronous motor is essentially a constant speed motor, it is useless where speed regulation is required, but it finds a considerable application in sub-station work in conjunction with C.C. generators. Since the motors can be wound for high voltages, there is no necessity for transformers, as the motors can be run direct off the H.T. mains. Again, the fact that a constant C.C. voltage is required calls for a constant speed machine, and the difficulties of starting are easily overcome either by using a self-starting motor as described in the last paragraph, or more frequently by using the C.C. generator as a starting motor. In a modern sub-station a number of such sets are run in parallel on both sides, the power factor being regulated by adjustment of the motor field.

CHAPTER XXII

POWER FACTOR IMPROVEMENT

Effect of Low Power Factor.—In view of the fact that the great majority of loads contain more inductance than capacity, the current in most transmission lines lags behind the voltage, giving rise to power factors of less than unity. This means that the line current for a given power transmitted is greater than it need be and causes two results. First, the losses in transmission are increased, or, conversely, the size of wire is increased, leading to the employment of larger quantities of copper, and, second, the voltage regulation is made poorer, due to the increased voltage drop in the line with the larger currents. To illustrate this fact, the case will be considered where a given amount of power is transmitted at a power factor of 0·7, and comparison will be made with what would be the case if the power factor were unity. The current is inversely proportional to the power factor and is $\frac{1}{0 \cdot 7}$ of its minimum value, or 43 per cent. greater than it need be. If the size of wire be kept constant throughout, the losses, being proportional to the square of the current, are $\left(\frac{1}{0 \cdot 7}\right)^2$, or approximately double their value in the ideal case. If the losses are kept the same, approximately double the amount of copper must be employed. The latter arrangement leaves the regulation unaltered, but the former results in an increased voltage drop. These figures, which are quite practical ones, will suffice to show the magnitude of the effect and the desirability for improving the power factor wherever possible.

General Method of Improvement.—The power factor of a system can be improved in two main ways. The first is to use only that apparatus which works at approximately unity power factor, and some supply authorities make it a rule not to allow motors of more than a given output and working below a certain power factor to be connected to their mains. In general, however, this method is impracticable. The second method is to add to the existing load, apparatus which will take a leading current of such a magnitude as

to neutralise the lagging current brought about by the general character of the load. This method is now widely adopted.

Magnitude of the Required Idle Current.—For the purposes of investigation the load current, which is imagined to lag behind the line voltage, can be split up into a power component and an idle component, the latter lagging by 90° behind the line voltage. In Fig. 299, I represents the load current lagging behind the line voltage E by an angle ϕ such that $\cos\phi$ is equal to the power factor. The power component of the current is equal to $I_P = I\cos\phi$ and the idle component is equal to $I'_L = I\sin\phi$. In order to neutralise this idle current, it is necessary to add a leading current, I_C, of the same magnitude. The vector sum of I and I_C is then given by I_P, which is less than I. The current is thus brought into phase with the voltage, raising the power factor to unity, and its magnitude is decreased at the same time without any reduction in the amount of power transmitted. The magnitude of the required leading idle current is, therefore,

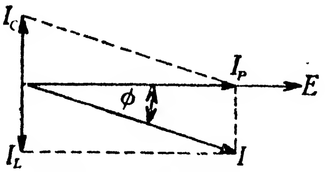


FIG. 299.—Showing Effect of Added Leading Current on Power Factor.

$$
\begin{aligned}
I_C &= I\sin\phi \\
&= I \times \sqrt{1-\cos^2\phi} \\
&= I \times \sqrt{1-(\text{power factor})^2}.
\end{aligned}
$$

The relation between the power factor and the value of I_C expressed as a percentage of the line current I is represented in

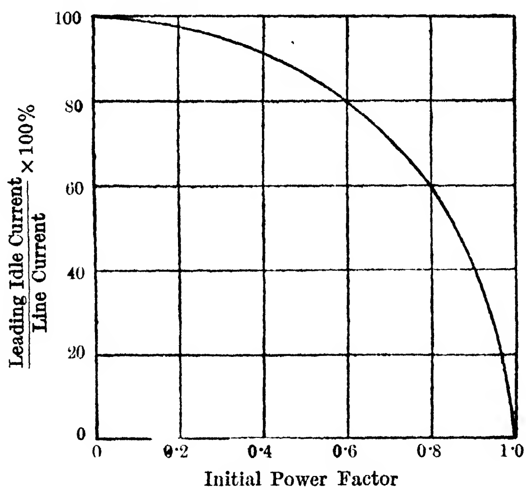


FIG. 300.—Relation between Added Leading Current and Initial Power Factor.

Fig. 300. The varying steepness of this curve shows that different amounts of leading current are required at different power factors

in order to effect the same improvement. When in the neighbourhood of unity power factor a greater amount of additional leading current is required for a given improvement than is the case at low power factors. This really means that it is easier to raise the power factor when it is very bad than when it is high.

The above reasoning applies equally well to a polyphase system, provided that in Fig. 300 the added leading idle current and the line current refer to the same circuit.

Improvement by Condensers.—The first obvious way of providing a leading current in addition to the existing load current is to connect a number of condensers in parallel across the mains at the receiving end. The capacity C necessary to bring the power factor up to unity is easily calculated from a knowledge of the power transmitted, voltage, frequency and initial power factor. Considering first a single phase case where these quantities are represented by P, E, f and $(p.f.)$ respectively, the magnitude of the required condenser current is

$$I_C = I \times \sqrt{1 - (p.f.)^2}.$$

Then

$$\begin{aligned} 2\pi f C E &= I \times \sqrt{1 - (p.f.)^2} \\ &= \frac{P}{E\,(p.f.)} \times \sqrt{1 - (p.f.)^2} \\ &= \frac{P}{E} \times \sqrt{\frac{1}{(p.f.)^2} - 1} \end{aligned}$$

and

$$C = \frac{P}{2\pi f E^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1}.$$

It is thus seen that a high frequency is desirable when condensers are used for improving the power factor. At first sight, it also appears as if a high voltage is very favourable to this method, since the capacity is inversely proportional to the square of the voltage, but the use of higher voltages necessitates a greater thickness of insulation and considerably increases the cost per mfd., so that the total cost is not affected to any enormous extent.

In a three phase transmission, the power is given by

$$P = \sqrt{3} E I \times (p.f.),$$

where E and I are the line voltage and line current respectively. The latter is equal to $\dfrac{P}{\sqrt{3}E \times (p.f.)}$, and the idle component of the line current is equal to

$$\begin{aligned} &\frac{P}{\sqrt{3}E \times (p.f.)} \times \sqrt{1 - (p.f.)^2} \\ &= \frac{P}{\sqrt{3}E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}. \end{aligned}$$

Assuming the condensers to be connected in mesh across the lines, the current per condenser circuit is $2\pi fCE$. The condenser current per line is the vector sum of the currents of two condenser circuits, and is, therefore, $\sqrt{3} \times 2\pi fCE$. If the final power factor is to be unity, this must be equal to $\frac{P}{\sqrt{3}E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}$, and, therefore,

$$\sqrt{3} \times 2\pi fCE = \frac{P}{\sqrt{3}E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}$$

and
$$C = \frac{P}{6\pi fE^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1}.$$

Since there are three condenser circuits, the total capacity required is

$$\frac{P}{2\pi fE^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1},$$

which is the same as in the single phase case for the same line voltage.

If the condensers are connected in star instead of mesh, the voltage across each condenser is reduced to $\frac{1}{\sqrt{3}} = 0{\cdot}577$ times its former value, and the cost per mfd. is correspondingly reduced. But the current per condenser circuit is now equal to $2\pi fC\frac{E}{\sqrt{3}}$, and this must neutralise a lagging idle current of $\frac{P}{\sqrt{3}E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}$. The capacity per condenser circuit is, therefore,

$$C = \frac{P}{2\pi fE^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1},$$

and the total capacity required is

$$\frac{3P}{2\pi fE^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1},$$

or three times as much as when the condensers were connected in mesh. Provided the condensers will stand the voltage, therefore, the mesh connection is the more preferable of the two.

In any case, this method is too expensive in practice, and other methods are usually adopted.

Synchronous Motor as Rotary Condenser or Phase Advancer.—Since a synchronous motor can be made to take a leading current by over-exciting it, such a motor might be employed to produce the required leading current for the purpose of improving the power

factor. The magnitude of the current taken by a synchronous motor depends to an enormous extent upon the excitation, and full load current can easily be attained even when the motor is running light. The function of a synchronous motor used in this manner is to act as a variable rotary condenser, and the general term *phase advancer* is applied to machines which have for their object the improvement of the power factor. The motor is not for the purpose of overcoming any load, although in practical cases it may be utilised for this purpose as well, but only in a subsidiary way. The primary object is to produce a leading current to neutralise the existing lag.

Considering first an ideal motor with no losses, the current taken will lead the voltage by 90° if it is over-excited, and the magnitude of this current will depend upon the value of the excitation. The current can thus be adjusted to suit the load and the power factor by varying the excitation of the machine, this being much more convenient than varying the number of condensers, which would be necessary if they were employed.

If the losses of the synchronous motor are taken into consideration, it means that a certain power component must be added, vectorially, to the leading current already mentioned, the result being that the motor current leads the voltage by an angle which is less than 90°. The extra power taken from the mains is, of course, wasted.

This method is equally applicable to the case of polyphase systems, being, in fact, cheaper in first cost, since polyphase synchronous motors cost less per k.V.A. than single phase ones. Since the construction of these motors is the same as for alternators, it follows that they can be designed for any line voltages which are obtained directly from alternators.

One of the disadvantages of this method of power factor improvement lies in the starting of the motor. Unless a self-synchronising motor be adopted, some auxiliary means of running up the motor must be provided.

Situation of Phase Advancer.—The correct situation for the phase advancer is at the receiving end of the line as close to the load as possible. If placed at the generator end of the line, it would improve the power factor of the generators and reduce their current, but it would not relieve the line at all, nor would it affect the voltage regulation of the line. When placed at the receiving end of the line, however, it not only improves the power factor of the generators and reduces their armature currents, but it reduces the main line current and improves the regulation. The ideal spot for the motor is alongside the actual load taking the lagging current, but this is impracticable in the great majority of cases. In the case of H.T. transmissions where the voltage is stepped down and distributed from a number of sub-stations, the phase advancer is placed in the

sub-station. Its effects are thus felt throughout the whole of the H.T. system, but the L.T. network is unaffected.

There is an obvious disadvantage in placing the phase advancer alongside the load, even when the latter is sufficiently large to warrant the employment of a phase advancer all to itself, since the load is usually on the consumer's premises, whilst the phase advancer is naturally the property of the supply company.

Idle Current Generator.—When a synchronous motor is employed as a phase advancer, it may be regarded as an idle current generator. Considered as a motor, it receives a small in-phase current which supplies the losses of the machine together with an idle current leading the voltage by 90°. This leading motoring current may be regarded as a generator current by reversing its direction, so that the synchronous motor may be said to receive an in-phase motoring current to supply its losses and to generate an idle current lagging by 90° behind the voltage. There is, of course, no power associated with this idle current which supplies the purely inductive portion of

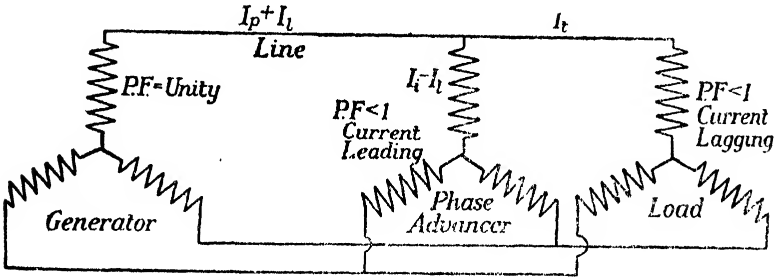


FIG. 301.—Phase Advancer in Transmission Line.

the load. The main generators supplying the power may thus be regarded as working on a non-inductive load. There is a kind of local circuit formed by the inductive portion of the load and the synchronous motor in which resonance is set up, this resonating current being superposed on to the main in-phase current flowing between the generators and the load. The general idea of the system applied to a three phase case is shown in Fig. 301, where the generator is working at a power factor of unity and is delivering a current $(I_p + I_l)$, the latter term representing the power current supplied to the phase advancer to overcome its losses. The synchronous motor is working at a very low power factor and receiving a power current I_l, whilst generating a purely lagging current I_i. The load, the power factor of which is less than unity, is receiving a current I_t which is the vector sum of I_p and I_i, the angle of lag being determined by the power factor.

K.V.A. Capacity of Phase Advancer.—Since a phase advancer is only required for the purpose of producing a purely idle current, the only power which it receives is that necessary to overcome its

losses. The machine will also work on a very low power factor, so that it is obviously unfair to rate it on a kilowatt basis. The proper unit for specifying the capacity[1] of the machine is the kilovolt-ampere. In a single phase case, neglecting the losses, the idle current is given by $I \times \sqrt{1-(p.f.)^2}$ and the total k.V.A. capacity of the phase advancer is

$$EI \times \sqrt{1-(p.f.)^2} \times 10^{-3}.$$

Expressed as a percentage of the k.V.A. capacity of the alternators, this is

$$\sqrt{1-(p.f.)^2} \times 100,$$

and this expression applies equally to the case of a polyphase system.

The size of the phase advancer is thus seen to be unaffected by the value of the current or the voltage, provided the power transmitted and the power factor are constant. A high voltage is associated with a low current and *vice versa*, but it is their product which determines the size of the machine, just as in the case of alternators.

Effect of Motor Losses.—If the synchronous motor is running unloaded the only power which it takes is that required to overcome its losses. The angle of lead of the current is thus not quite 90°, but rather less. The heating of the machine is dependent upon the losses, and since it is to be heavily over-excited, special care must be taken in the design of the field system to prevent over-heating. The fact that the power factor is greater than zero affects the k.V.A. capacity to a slight extent and also modifies the vector diagram, the general form of which is shown in Fig. 302, which is drawn for a single phase case. The line current, I_l, is drawn lagging behind the line voltage, E, by an angle ϕ_l, where $\cos \phi_l$ is the power factor of the load. The synchronous motor which is used as a phase advancer takes a current, $I_{p.a.}$, leading the line voltage by an angle, $\phi_{p.a.}$, where $\cos \phi_{p.a.}$ is its power factor. The magnitude of $I_{p.a.}$ is so adjusted that the vector sum of I_l and $I_{p.a.}$ gives a current in the mains, I_m, in phase with the voltage. In order to bring this about, the leading idle

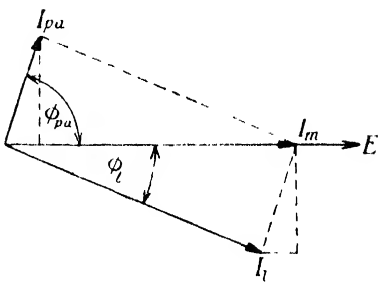


FIG. 302.—Power Factor Improvement by Synchronous Motor.

[1] The term "capacity" here has no reference whatever to the condenser effect, the unit of which is the farad.

current of the phase advancer must be equal to the lagging idle current of the load, so that

$$I_{p.a.} \sin \phi_{p.a.} = I_l \sin \phi_l$$

and

$$I_{p.a.} = I_l \frac{\sin \phi_l}{\sin \phi_{p.a.}}.$$

The k.V.A. capacity of the synchronous motor is thus

$$EI_{p.a.} \times 10^{-3}$$
$$= EI_l \frac{\sin \phi_l}{\sin \phi_{p.a.}} \times 10^{-3}$$
$$= EI_l \sqrt{\frac{1 - (p.f._l)^2}{1 - (p.f._{p.a.})^2}} \times 10^{-3}.$$

The power factor of the synchronous motor will usually lie somewhere between 0·1 and 0·2, so that, taking the larger value, the expression $\sqrt{1 - (p.f._{p.a.})^2}$ becomes equal to $\sqrt{1 - 0{\cdot}2^2} = 0{\cdot}98$. The effect of taking the losses into account is thus seen to increase the k.V.A. capacity by only 2 per cent., and it can, therefore, be neglected in the majority of cases.

As an actual example, the case will be taken where a load of 1,000 k.W. is transmitted at a power factor of 0·7, the phase advancer running at a leading power factor of 0·15 so as to bring the resultant power factor up to unity. The total k.V.A., excluding the phase advancer, is $\frac{1000}{0{\cdot}7} = 1430$, and the idle k.V.A. is $\sqrt{1430^2 - 1000^2} = 1022$. If the power factor is 0·15, then $\sin \phi_{p.a.} = \sqrt{1 - 0{\cdot}15^2} = 0{\cdot}989$, and the capacity of the phase advancer is $\frac{1022}{0{\cdot}989} = 1033$ k.V.A., the power taken by it being $1033 \times 0{\cdot}15 = 155$ k.W. The total power transmitted is now 1155 k.W. instead of 1000 k.W., an increase of 15·5 per cent., but the total current in the mains is reduced in the ratio of 1430 to 1000, and as the copper losses are proportional to the square of the current they are reduced to $\left(\frac{1000}{1430}\right)^2$, or 0·49 of their initial value. There is, therefore, a saving of 51 per cent. of the copper losses in the mains against an additional loss of 15·5 per cent. of the total useful load. The question as to whether there is a resultant saving or not depends upon the resistance of the mains and must be settled for each individual case

Partial Improvement of Power Factor.—Instead of raising the power factor from its initial value to unity, it might be raised to some such value as, say, 0·9 by the employment of a smaller phase advancer. A considerable improvement is still effected if this be done, and it is found to be much more expensive to raise the power factor by a

given amount when it is near unity than when it has a much lower value. It is thus possible to instal a phase advancer having a k.V.A. capacity insufficient to raise the power factor to unity, but sufficient to raise it to some predetermined lower value. This machine will be considerably smaller than one designed to raise the power factor to unity and may be more economical to instal on account of the reduced capital outlay required.

The k.V.A. capacity of the phase advancer for a partial improvement can be calculated in the following way. Let $p.f._l$ be the power factor of the load in the first instance, $p.f._r$ be the resultant power factor after the phase advancer has been installed, and $p.f._{p.a.}$ be the leading power factor of the phase advancer itself. The idle component of the load current is $I_l \times \sqrt{1-(p.f._l)^2}$, where I_l is the load current. The power component of the current in the mains after the improvement has been effected is

$$I_l \times p.f._l + I_{p.a.} \times p.f._{p.a.}$$

where $I_{p.a.}$ is the current taken by the phase advancer, and the total final current is

$$\frac{I_l \times p.f._l + I_{p.a.} \times p.f._{p.a.}}{p.f._r}.$$

The idle component of the resultant current is

$$\frac{I_l \times p.f._l + I_{p.a.} \times p.f._{p.a.}}{p.f._r} \times \sqrt{1-(p.f._r)^2}$$

$$= (I_l \times p.f._l + I_{p.a.} \times p.f._{p.a.}) \times \sqrt{\frac{1}{(p.f._r)^2} - 1}.$$

The difference between the idle components before and after the phase advancer is installed represents the idle current taken by the latter, and this is equal to $I_{p.a.} \times \sqrt{1-(p.f._{p.a.})^2}$. Therefore,

$$I_{p.a.} \times \sqrt{1-(p.f._{p.a.})^2}$$

$$= I_l \times \sqrt{1-(p.f._l)^2} - (I_l \times p.f._l + I_{p.a.} \times p.f._{p.a.}) \times \sqrt{\frac{1}{(p.f._r)^2} - 1},$$

and

$$I_{p.a.} \times \sqrt{1-(p.f._{p.a.})^2} + I_{p.a.} \times p.f._{p.a.} \times \sqrt{\frac{1}{(p.f._r)^2} - 1}$$

$$= I_l \times \sqrt{1-(p.f._l)^2} - I_l \times p.f._l \times \sqrt{\frac{1}{(p.f._r)^2} - 1}.$$

$$I_{p.a.}\left[\sqrt{1-(p.f._{p.a.})^2} + p.f._{p.a.} \times \sqrt{\frac{1}{(p.f._r)^2} - 1}\right]$$

$$= I_l\left[\sqrt{1-(p.f._l)^2} - p.f._l \times \sqrt{\frac{1}{(p.f._r)^2} - 1}\right].$$

$$\frac{I_{p.a.}}{I_l}=\frac{\sqrt{1-(p.f._l)^2}-p.f._l\times\sqrt{\dfrac{1}{(p.f._r)^2}-1}}{\sqrt{1-(p.f._{p.a.})^2}+p.f._{p.a.}\times\sqrt{\dfrac{1}{(p.f._r)^2}-1}}$$

$$=\frac{p.f._l}{p.f._{p.a.}}\left[\frac{\sqrt{\dfrac{1}{(p.f._l)^2}-1}-\sqrt{\dfrac{1}{(p.f._r)^2}-1}}{\sqrt{\dfrac{1}{(p.f._{p.a.})^2}-1}+\sqrt{\dfrac{1}{(p.f._r)^2}-1}}\right].$$

But the ratio $\frac{I_{p.a.}}{I_l}$ is the ratio of the k.V.A. of the phase advancer to the k.V.A. of the load, so that, expressing the k.V.A. capacity of the phase advancer as a percentage of the k.V.A. of the load, it becomes

$$\frac{p.f._l}{p.f._{p.a.}}\times\left[\frac{\sqrt{\dfrac{1}{(p.f._l)^2}-1}-\sqrt{\dfrac{1}{(p.f._r)^2}-1}}{\sqrt{\dfrac{1}{(p.f._{p.a.})^2}-1}+\sqrt{\dfrac{1}{(p.f._r)^2}-1}}\right]\times 100 \text{ per cent.}$$

Expressing the k.V.A. capacity of the phase advancer as a percentage of the k.W. transmitted to the load, it becomes

$$\frac{1}{p.f._{p.a.}}\times\left[\frac{\sqrt{\dfrac{1}{(p.f._l)^2}-1}-\sqrt{\dfrac{1}{(p.f._r)^2}-1}}{\sqrt{\dfrac{1}{(p.f._{p.a.})^2}-1}+\sqrt{\dfrac{1}{(p.f._r)^2}-1}}\right]\times 100 \text{ per cent.}$$

The above expression can be utilised to work out in a particular example the sizes of the various phase advancers for the purpose of raising the power factor from 0·7 to different final power factors, the power factors of the synchronous motors being taken as 0·15. The required k.V.A. per 100 k.W. of load are shown in the following table, together with the increase in the k.V.A. for each additional increase of 0·05 in the power factor, showing the relative expense of increasing the power factor from 0·95 to unity.

Final Power Factor ...	0·75	0·80	0·85	0·90	0·95	1·00
k.V.A. per 100 k.W. load	12·2	24·5	37·0	50·3	66·5	103·0
Additional k.V.A. ...	12·3 (0·75–0·80)	12·5 (0·80–0·85)	13·3 (0·85–0·90)	16·2 (0·90–0·95)	36·5 (0·95–1·00)	

Most Economical Final Power Factor.—The question as to what is the most economical final power factor is a very important and practical one and is ultimately decided by considerations of cost. When the power factor is raised it involves an extra expenditure on account of the phase advancer, but there is a reduction in the cost of the mains, or, alternatively, they are capable of transmitting a greater load. If the load is expanding it may be more economical to instal a phase advancer in preference to laying down additional mains.

Starting from the low power factor due to the load and gradually raising it, the saving in the mains at first far outweighs the extra cost of the phase advancer in the majority of cases, but as the power factor is raised still further the cost of the phase advancer begins to approximate to the saving in the mains, and finally any additional saving in the mains is only obtained by a greater expenditure in increasing the size of the phase advancer. There is a point, therefore, beyond which it is not economical still further to improve the power factor, and this usually occurs at a power factor somewhere about 0·95.

Phase Advancer with Mechanical Load.—A synchronous motor used as a phase advancer can be utilised to develop mechanical power in the same way as an ordinary synchronous motor. All that is necessary is that the motor whould be over-excited to a certain extent. The idle component of the motor current tends to eliminate the lagging idle current of the remainder of the load, whilst the power component of the motor current serves to develop the torque required for the load. Whilst the motor adds to the useful power load on the supply, it materially increases the total power factor.

An example will be worked out wherein a 600 k.V.A. synchronous motor developing 360 B.H.P. is installed to improve the power factor of a three phase transmission line having a load of 1000 k.V.A. at a power factor of 0·8, the line voltage being 6600.

The line current is, excluding the synchronous motor,

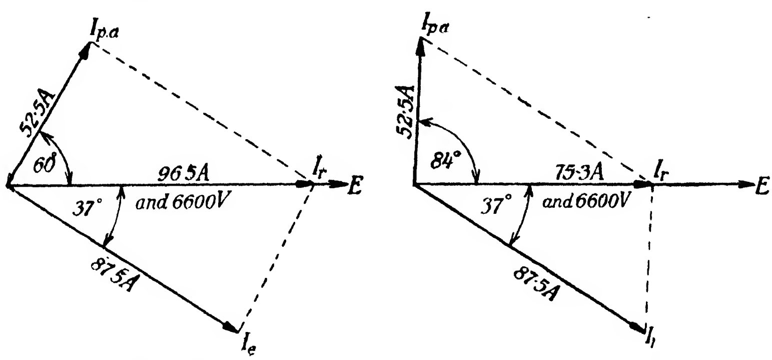
$$\frac{1000 \times 10^3}{\sqrt{3} \times 6600} = 87{\cdot}5 \text{ amperes,}$$

and lags behind the phase volts by an angle of 37°, since cos 37° = 0·8 [see Fig. 303 (*a*)]. Assuming an efficiency of 0·9 for the synchronous motor, its input is $\dfrac{360 \times 0{\cdot}746}{0{\cdot}9} = 298{\cdot}4$ k.W., and its power factor is $\dfrac{298{\cdot}4}{600} = 0{\cdot}50$. The motor current is $\dfrac{600 \times 10^3}{\sqrt{3} \times 6600} = 52{\cdot}5$ amperes, and leads the phase volts by an angle of 60°, since cos 60° = 0·50. The lagging idle current of the load is 87·5 × sin 37° = 52·5 amperes, and the leading idle current of

the synchronous motor is $52{\cdot}5 \times \sin 60^{\circ} = 45{\cdot}5$ amperes. The resultant idle current (lagging) is $52{\cdot}5 - 45{\cdot}5 = 7{\cdot}0$ amperes. The resultant wattful current is $87{\cdot}5 \times 0{\cdot}8 + 52{\cdot}5 \times 0{\cdot}5 = 96{\cdot}25$ amperes. The resultant line current is $\sqrt{96{\cdot}25^2 + 7^2} = 96{\cdot}5$ amperes, and the final power factor is $\frac{96{\cdot}2}{96{\cdot}5} = 0{\cdot}997$, say, unity. It is thus seen that for an increase of $\frac{96{\cdot}5 - 87{\cdot}5}{87{\cdot}5} \times 100 = 10{\cdot}3$ per cent. in the current the power transmitted has been increased by $\frac{800 + 298{\cdot}4}{800} \times 100 = 37{\cdot}3$ per cent., the efficiency of transmission being considerably improved.

The case will now be considered where the synchronous motor is run unloaded, assuming it to run at a power factor of 0·1, thus taking 60 k.W. to drive it light. The motor current is $\frac{600 \times 10^3}{\sqrt{3} \times 6600} = 52{\cdot}5$ amperes, the same as before, but the idle component is now $52{\cdot}5 \times \sqrt{1 - 0{\cdot}1^2} = 52{\cdot}2$ amperes, and the power component $52{\cdot}5 \times 0{\cdot}1 = 5{\cdot}25$ amperes. The resultant idle current (lagging) is $52{\cdot}5 - 52{\cdot}2 = 0{\cdot}3$ amperes, and the resultant wattful component is $87{\cdot}5 \times 0{\cdot}8 + 5{\cdot}25 = 75{\cdot}25$ amperes. The final line current is $\sqrt{75{\cdot}25^2 + 0{\cdot}3^2} = 75{\cdot}3$ amperes, the power factor being unity. The line current has thus been reduced by $\frac{87{\cdot}5 - 75{\cdot}3}{87{\cdot}5} \times 100 = 14$ per cent. at the expense of an additional power loss of $\frac{60}{800} \times 100 = 7{\cdot}5$ per cent. of the total power load.

The vector diagram for one phase, both loaded and unloaded, is shown in Fig. 303 (*a*) and (*b*).



(*a*) Loaded (*b*) Unloaded

FIG. 303.—Synchronous Motor as Phase Advancer

Kapp Vibrator.—This piece of apparatus is a particular kind of phase advancer and is used to inject a leading E.M.F. into the

rotor circuit of an induction motor. As will be shown in Chapter XXVI, these motors generate a rotor current which ordinarily lags behind the flux by 90°, and if this angle can be reduced the stator current becomes automatically advanced in phase with a consequent improvement in the power factor and overload capacity.

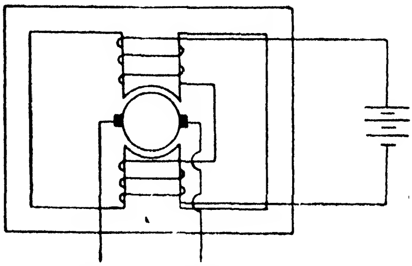


FIG. 304.—Kapp Vibrator.

The Kapp vibrator is connected in series with the rotor circuit of the motor and the starter and actually reduces the lagging current taken by the motor. Its action is thus totally different from that of the over-excited synchronous motor which compensates for the lagging current, whereas this machine prevents its production in the first place.

The vibrator consists of an armature having a commutator on which two brushes press at opposite ends of a diameter, as shown in Fig. 304. This armature is placed in an ordinary bipolar field system excited from an auxiliary battery. The armature, being connected in series with the rotor circuit of the motor, receives an A.C. of very low frequency (see Chapter XXVI for the theory of the induction motor) and experiences a torque due to the combined action of the field and the armature current. The armature will thus commence to rotate. When the current reverses the torque reverses and the armature slows down until it comes to rest at the moment when the current reaches its maximum value. During the next quarter of a period when the current is dying down the armature commences to rotate in the opposite direction. When the current becomes zero, the velocity reaches a maximum, and for the next quarter of a period whilst the current is growing it exerts a retarding effect on the armature. As a net result, the armature never becomes displaced to any great extent, but merely vibrates to and fro. An alternating back E.M.F. proportional to the speed is generated in the armature. This reaches a maximum when the velocity is a maximum, but the current at this instant is zero. The back E.M.F. is thus in quadrature with the current, and as no mechanical power is supplied to the vibrator, the energy associated with its movement must therefore be derived from an electrical source and obviously comes from the rotor of the induction motor. The vibrator armature executes approximately a simple harmonic motion and has its maximum velocity in the middle of its swing. At this moment its kinetic energy is also a maximum, whilst at the extremity of its swing its velocity and kinetic energy are zero. When the current is decreasing the velocity and kinetic energy are increasing. Energy is being transferred to the vibrator and with-

drawn from the electrical circuit. The current, therefore, dies down at an increased rate. Again, when the current is increasing the velocity and kinetic energy are decreasing. The energy is being transferred back to the electrical circuit, so that the growth of the current is accelerated. The action of the vibrator is thus to advance the phase of the current all the time, and this reacts on the main stator current, advancing its phase and improving the power factor of the motor.

Since the rotor circuits are always wound for low voltages, even when the stators are run from H.T. mains, there is no trouble from commutation. In fact, metal brushes are used. When the main induction motor is started up, however, high voltages are induced temporarily in the rotor circuit, so that arrangements are made to cut out the vibrator during this period.

When applied to a three phase motor, three vibrator armatures are employed, these being arranged inside a common field system, as shown in Fig. 305.

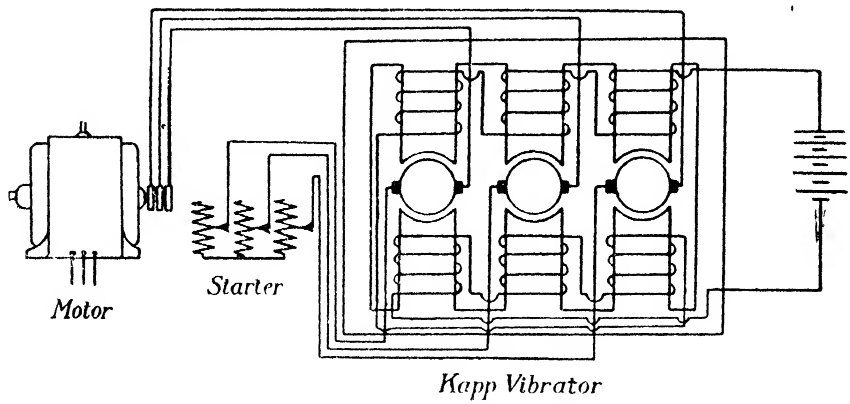


FIG. 305.—Three Phase Kapp Vibrator.

Leblanc's Exciters.—In the previous paragraph it was shown that if a leading E.M.F. were injected into the rotor circuit of an induction motor the general power factor would be improved. This can be done by coupling a number of exciters to the shaft of the motor, one for each phase. These exciters are made like single phase series motors provided with compensating windings (see Chapter XXX). The action can be most easily followed in the case of a two phase rotor (see Fig. 306). W_1 and W_2 represent the two phase windings of the rotor and E_1 and E_2 the armatures of the two exciters. The current from the phase W_1 passes from the slip ring through the compensating winding C_1, through the armature E_1, and then through the field F_2 of the exciter in the second phase, and thence to the star point. The current from the phase W_2 passes through the compensating winding C_2, through the armature

E_2, and then through the field F_1 of the exciter in the first phase and to the star point. The resistances R_1 and R_2 are those of the starter for the main motor, the exciters being short-circuited during starting up. When the motor is running, the armature E_1 has generated in it an E.M.F. which is in phase with the current in

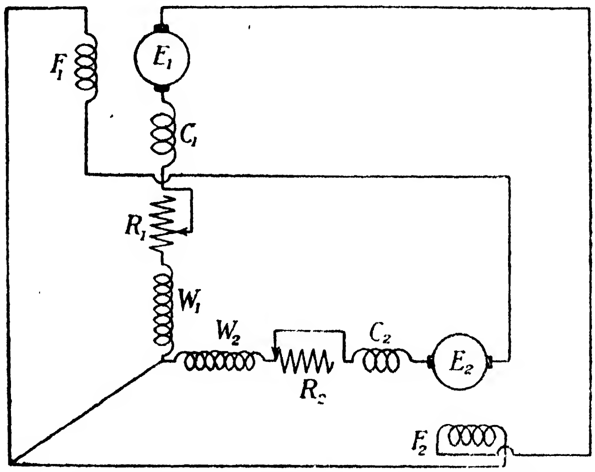


FIG. 306.—Leblanc's Exciters for Two Phase Rotor.

the second phase. If the polarity of the poles is suitably arranged this E.M.F. will lead the current in the first phase by 90°. In a similar manner, the armature E_2 generates an E.M.F. in the second phase, again leading the current by 90°. The introduction of this leading E.M.F. advances the phase of the whole rotor current, which in turn advances the stator current, so that, if desired, the induction motor can be made to take a leading current from the supply.

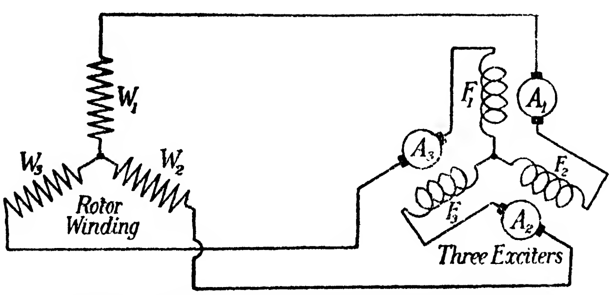
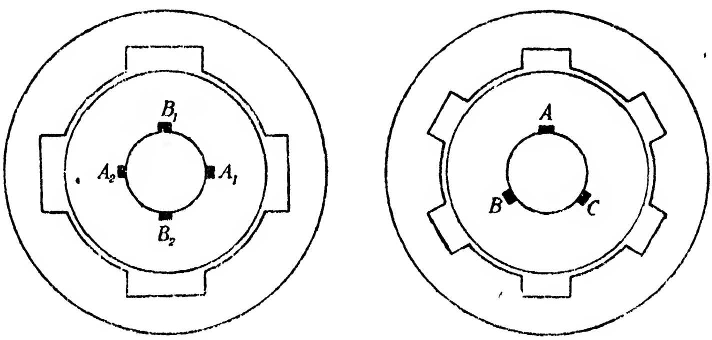


FIG. 307.—Leblanc's Exciters for Three Phase Rotor.

In the case of a three phase rotor three exciters are required, and this constitutes the main drawback to the method. In this case each exciter receives its magnetising current from the next phase, as shown in Fig. 307, which shows the connections of the main field and armature windings.

Scherbius Phase Advancer.—The three exciters described above can be combined in one machine, the armature of which is made like an ordinary drum wound C.C. armature. This is surrounded by a ring of laminations having inwardly projecting poles, but without any field windings. The commutator is provided with four brushes spaced 90° apart if used on a two phase circuit, and with three brushes spaced 120° apart if used on a three phase circuit. This is shown diagrammatically in Fig. 308 (*a*) and (*b*). These brushes are connected, through the starting resistances, to the slip rings of the induction motor. The action of the two phase machine will be studied as being the simpler, remembering that the three phase machine behaves in a similar manner. Considering the first phase where the current is led into and out of the armature at A_1 and A_2, an alternating magnetic flux will be set up having for its axis the line of the brushes A_1 and A_2. Similarly, the second



(*a*) Two Phase

(*b*) Three Phase

FIG. 308.—Scherbius Arrangement.

phase will set up a magnetic flux 90° out of phase with the first along the axis B_1B_2. These two component fluxes do not really exist separately, but it is convenient to treat them as if they did. Actually, there is only one resultant flux. Each phase winding in the armature will have an E.M.F. induced in it, due to rotation in the flux produced by the other phase. This is equivalent to exciting each phase by means of a magnetising current obtained from the other phase. The similarity to the arrangement in Fig. 306 is now apparent. A great advantage is obtained over the separate Leblanc exciters, inasmuch as the armature currents themselves excite the field. It will also be seen that the stator iron stampings are not really necessary, except in so far as they may reduce the reluctance of the magnetic circuit when an open slot winding is used on the armature. If a winding with closed slots is used, the magnetic circuit may lie wholly within the armature, the external stator being altogether dispensed with. The use of the external frame,

however, possesses several advantages from the designer's point of view.

If the phase advancer be stationary it acts like a three phase choking coil, but if it is driven from the shaft of the motor the power factor of which it is going to raise, then a leading E.M.F. will be induced in each phase, the effect of which is to advance the phase of the current.

The main advantage possessed by the phase advancers described above over the over-excited synchronous motor is that they are machines of relatively small output. The reason for this is that the phase advancer stands in the same relation to an induction motor as an exciter does to a synchronous motor. An exciter of comparatively small capacity can over-excite a synchronous motor so as to make it supply a wattless load fifty times as great, measured in k.V.A., as the rating of the exciter. A phase advancer of only 30 k.V.A. capacity is capable of effecting a total change of 1000 idle k.V.A. in the main circuit, since it only deals with the rotor circuit of the induction motor, and the k.V.A. in the rotor circuit is small compared with that drawn by the stator circuit from the supply.

CHAPTER XXIII

ROTARY CONVERTERS

Methods of Transformation.—A number of pieces of apparatus are at present on the market for converting alternating into continuous current. The main application of such machinery is in the equipment of a sub-station where A.C. is received from the H.T. mains and an L.T., C.C. is delivered to the consumers. There are three chief types of converting plant, viz.,

1. Motor Generators.
2. Rotary Converters.
3. Motor Converters.

The first type consists of a C.C. generator mechanically coupled to an A.C. motor which drives it. As A.C. motors can be built for high voltages, there is no need for transformers on this system. Two rotary machines are employed.

The second type consists of a single machine, the construction and performance of which will be discussed in the following paragraphs, but since the voltage ratio between the A.C. and C.C. ends is fixed, this method also necessitates the use of transformers for the purpose of stepping down the voltage. Thus one rotary machine is employed together with one or more statical transformers.

The third type of converting plant consists of two rotary machines, direct coupled, as will be discussed in Chapter XXIX, and may be described as a cross between a motor-generator and a rotary converter, from which the name of *motor converter* is obtained.

Other forms of converting apparatus are the *mercury vapour converter* and the *electrolytic rectifier* or *electric valve.*

Principle of the Rotary Converter.—The ordinary C.C. dynamo really generates an alternating E.M.F. which is made continuous by the action of the commutator. If a pair of slip rings be mounted on the armature of such a generator at the non-commutator end, these slip rings being connected to conductors situated diametrically opposite each other in a bipolar case, the machine will also act as an A.C. generator. Moreover, it is possible to make it generate both C.C. and A.C. at the same time by mechanically driving it. An A.C. generator is, however, capable of acting as a synchronous motor, so that the rotation may be produced by supplying the A.C. end with power in an electrical form instead of driving the machine mechanically. Driven in this way, the machine is known as a *rotary converter.* The rotation of the armature induces an

alternating back E.M.F. in it, and this is converted into a continuous E.M.F. by the action of the commutator. If no current is delivered by the C.C. end, the motoring current taken by the A.C. end is only that required to overcome the losses of the machine and maintain its rotation. When the C.C. end is connected to some form of load resistance, a drag on the armature conductors is produced and is the equivalent of putting a load on the motoring A.C. end, which consequently takes a larger current. The power input to the rotary converter is obviously equal to the power output together with the power wasted in its losses.

This machine must not be confused with a motor-generator employing a single armature core and field system. The latter machine has a double armature winding and the conductors carrying the motoring current are quite distinct electrically from those carrying the generated current. In a rotary converter the same conductors carry both currents superposed on one another, and since, generally speaking, the motoring current is flowing in the opposite direction to the generated current, the resultant current at any instant is the difference of the two.

Inverted Rotary Converter.—Instead of supplying the A.C. end with a motoring current and generating a continuous E.M.F. at the C.C. end, the reverse may be done. The commutator is then supplied with C.C., in which case the machine runs as a C.C. motor and acts like an A.C. generator. When run in this reverse manner, the machine is known as an *inverted rotary converter.*

Polyphase Rotary Converter.—If an inverted rotary converter be supplied with three slip rings connected to conductors situated one-third of a cycle apart it will generate a three phase supply. Conversely, when run in the ordinary way it may be supplied with a polyphase current if suitable slip ring connections are made. The number of slip rings necessary for the various polyphase supplies are the same as in the case of a rotating armature alternator or synchronous motor.

Since the performance of a single phase rotary converter is very unsatisfactory, these machines are seldom employed, and, as a general rule, the larger the number of phases, the better do the machines work.

Ratio of Transformation.—The brushes on the commutator are placed so as to obtain the maximum voltage generated, and, in a single phase bipolar machine, once during each half-revolution or half-cycle the conductors connected to the slip rings come under the brushes as shown diagrammatically in Fig. 309. In every other position the voltage across the slip rings is less than it is at this instant. In the position shown, therefore, the voltage across the slip rings is also a maximum, and, neglecting the losses in the machine, this is equal to the commutator voltage. The maximum value of the A.C. voltage is thus equal to the C.C. voltage in a single

phase rotary converter, and, assuming a sinusoidal wave form, the R.M.S. value of the A.C. voltage is equal to $\frac{1}{\sqrt{2}}$ times the C.C. voltage.

Still neglecting the losses, the watts at the two ends of the machine are equal, or

$$E_C I_C = E_A I_A \cos\phi,$$

and

$$I_A = I_C \times \frac{E_C}{E_A}\frac{1}{\cos\phi}$$

$$= I_C \times \frac{\sqrt{2}}{1} \times \frac{1}{\cos\phi}.$$

If the machine is operating on unity power factor the alternating current is equal to $\sqrt{2}$ or 1·414 times the continuous current.

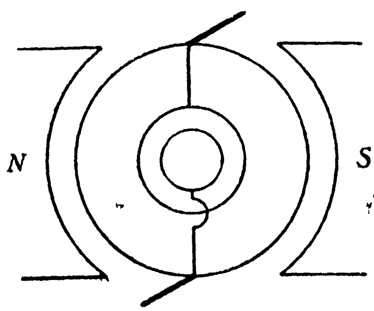


FIG. 309.—Armature Position for Maximum Voltage. Single Phase.

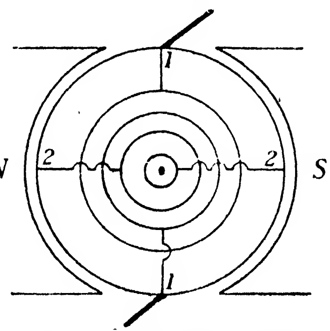


FIG. 310.—Two Phase Rotary Converter.

In the case of a two phase rotary converter (bipolar machine) there are four slip rings connected to conductors situated 90° apart (see Fig. 310). Since the armature winding is necessarily of the distributed type as used in C.C. machines, the two phases are linked together in the armature of the rotary converter itself, and consequently the phases of the supply must be linked at the centre and not at one end. The three wire system of transmission is therefore inadmissible. The two slip rings belonging to each phase being connected to conductors situated diametrically opposite, the voltage per phase is the same as in the single phase case, viz., $\frac{1}{\sqrt{2}}$ or 0·707 times the C.C. voltage.

Equating the C.C. and A.C. power, we get

$$E_C I_C = 2E_A I_A \cos\phi$$

and

$$I_A = I_C \times \frac{E_C}{E_A} \times \frac{1}{2\cos\phi}$$

$$= I_C \times \frac{1}{\sqrt{2}\cos\phi}.$$

On unity power factor, the A.C. line current is therefore 0·707 times the continuous current.

In a three phase rotary converter, the conductors connected to any two slip rings cannot be connected to the brushes at the same time, since the slip ring connections are spaced 120° apart instead of 180°, as in the single and two phase machines. The position of the slip ring connections for maximum voltage must therefore be determined. In the single phase case it was seen that this

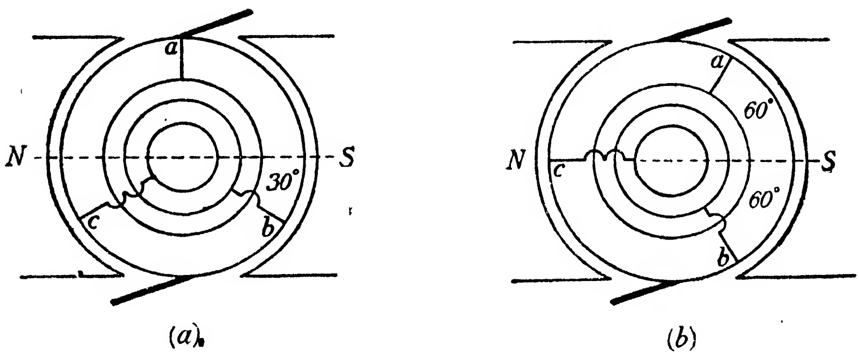


FIG. 311.—Three Phase Rotary Converter.

position was the one where the slip ring conductors enclosed the maximum number of lines of force. In other words, the vertical distance between *a* and *b* in Fig. 311 must be a maximum. Calling the radius of the circle unity, this distance is $1 + \sin 30° = 1·5$ in Fig. 311 (*a*) and $2 \sin 60° = 1·732$ in Fig. 311 (*b*). The latter position will be found to give the maximum value for all the various positions, the length of the vertical line being $\frac{\sqrt{3}}{2}$ times the full diameter of the circle.[1]

[1] This can be shown by the calculus as follows:—

$$\text{Vertical distance} = \sin\theta + \sin(120° - \theta)$$
$$= \sin\theta + \sin 120° \cos\theta - \cos 120° \sin\theta$$
$$= \tfrac{3}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta = y.$$

For $y = \text{max.}, \ \frac{dy}{d\theta} = 0,$

$$\frac{dy}{d\theta} = \tfrac{3}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta = 0.$$
$$\tfrac{3}{2}\cos\theta = \frac{\sqrt{3}}{2}\sin\theta,$$
$$\sqrt{3}\cos\theta = \sin\theta,$$
$$\tan\theta = \sqrt{3} \text{ and } \theta = 60°.$$

The maximum A.C. line voltage is therefore $\frac{\sqrt{3}}{2}$ times the C.C. voltage, and the R.M.S. value of the A.C. voltage is $\frac{\sqrt{3}}{2\sqrt{2}} = 0{\cdot}612$ times the C.C. voltage.

Equating the C.C. and A.C. power again, we get

$$E_C I_C = \sqrt{3} E_A I_A \cos\phi$$

and

$$I_A = I_C \times \frac{E_C}{E_A} \times \frac{1}{\sqrt{3}\cos\phi}$$

$$= I_C \times \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{3}\cos\phi}$$

$$= I_C \times \frac{0{\cdot}943}{\cos\phi}.$$

Similar calculations can be worked out for six and twelve phase rotary converters.

The various ratios for different numbers of phases, assuming 100 volts and 100 amperes at the C.C. end, are tabulated as follows :—

	C.C.	Single Phase.	Two Phase.	Three Phrase.	Six Phase.	Twelve Phase.
Volts between Slip Rings ...	100	70·7	70·7	61·2	35·4	18·3
Current per Slip Ring at Unity Power Factor	100	141·4	70·7	94·3	47·2	23·6

These ratios of transformation are somewhat affected by the ratio of pole arc to pole pitch, just as this ratio affects the E.M.F. generated in an alternator (see p. 245). As a general rule, the A.C. voltage is raised by reducing the ratio of pole arc to pole pitch and *vice versa*, this being done at the expense of the wave form. The ratios calculated above have been worked out on the assumption of a sinusoidal wave form which involves a ratio of pole arc to pole pitch of approximately 0·7.

Effect of Output on Ratio of Transformation.—Owing to the presence of resistance and reactance in the armature, the ratio of transformation does not remain constant for all values of the load. A certain amount of voltage is lost in this way, the magnitude depending upon the value of the load. If the A.C. supply pressure be maintained constant, the C.C. voltage gradually falls as the load comes on, after the manner of a shunt dynamo, and the relation between the C.C. load current and C.C. terminal voltage is called the characteristic of the rotary converter.

Construction.—The construction of a simple rotary converter differs very little from that of a C.C. dynamo, with the exception that slip rings are mounted at the non-commutator end of the armature. They are, however, frequently complicated by the addition of boosters for the purpose of improving the regulation, the two armatures being carried on the same shaft. Interpoles are also usually employed on modern machines for the purpose of improving the operation. Owing to the fact that the alternating and continuous currents tend to neutralise each other in the armature, the heating is considerably reduced, which means that a larger output can be obtained from a particular armature with a given temperature rise than would be the case if the currents flowed in separate conductors instead of being superposed on one another in the same bar. The size of a rotary converter for a given output and speed is thus less than that of the corresponding C.C. dynamo. Another advantage lies in the fact that a single magnetic field system serves the machine, whilst two field systems are required in the case of a motor-generator set. For these reasons, the rotary converter, together with its complement of transformers, has a higher efficiency than the motor-generator, a gain of 3–4 per cent. being obtained on a 1,000 k.W. set.

Number of Poles.—The relation between the number of poles and the frequency and speed is the same as in the case of a synchronous motor or an alternator. Rotary converters work better on low frequencies, and consequently a frequency of 25 has become more or less a standard for rotary converter work. Successful operation can also be obtained at a frequency of 50, but this is about the limit from a practical point of view. For a given frequency the speed depends to a large extent upon the armature diameter, and, as this goes up with the output, the speed must come down with a corresponding increase in the number of poles. The following table shows the number of poles which may be expected for various outputs :—

Output.	25 Cycles.	50 Cycles.
100— 200 k W.	4 Poles	6— 8 Poles
300— 500 k.W.	6 ,,	8—10 ,,
600— 900 k.W.	8 ,,	12—16 ,,
1000—1200 k.W.	12 ,,	20—24 ,,
1500 k.W.	16 ,,	—

Armature Windings.—Drum wound armatures are employed with windings exactly like those in C.C. dynamos. Either two circuit (wave) or multiple circuit (lap) windings may be used, but it is the usual practice to adopt the two circuit winding for outputs up to about 100 k.W., whilst multiple circuit windings are used for the larger outputs.

When two circuit windings are adopted there is only one connection required per slip ring, corresponding to the single brush spindle in the C.C. dynamo. When multiple circuit windings are adopted there must be as many connections to each slip ring as there are pairs of poles, corresponding to the number of brush spindles in parallel in the C.C. dynamo. The total number of tappings required is therefore equal to the number of slip rings in a two circuit winding and the number of slip rings multiplied by the pairs of poles in a multiple circuit winding.

Consider as an example the case of a 6-pole multiple circuit armature having 144 conductors. Used on single phase, there would be 6 tappings spaced 24 conductors apart; as a two phase machine there would be 12 tappings spaced 12 conductors apart; as a three phase machine there would be 9 tappings spaced 16 conductors

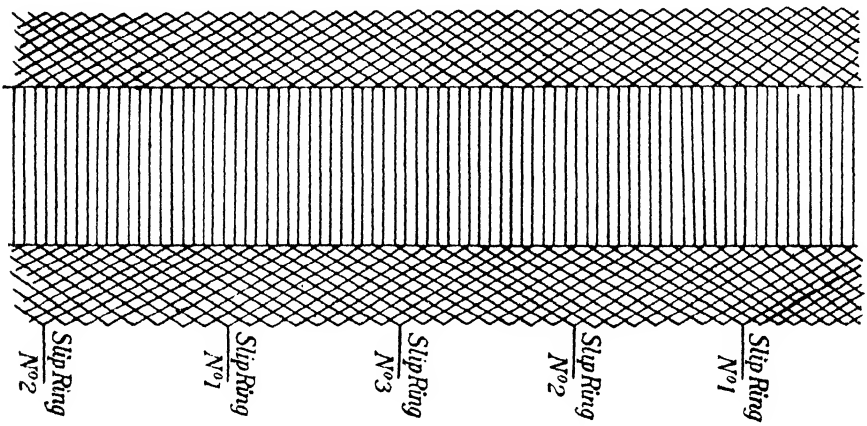


FIG. 312.—Three Phase Armature Winding for Rotary Converter.

apart; and as a six phase machine there would be 18 tappings spaced 8 conductors apart.

A development of a portion of this winding when used for three phase is shown in Fig. 312.

Transformer Connections for Six Phase Rotary Converter.—Owing to their improved performance, six phase rotary converters are largely adopted on three phase systems, since the six phases can be produced merely by connecting up the single three phase or the three single phase transformers in certain ways.

The first way is called the double mesh method of connection. Each phase of the transformer is provided with a double secondary and each set of three secondaries is connected in mesh or delta. The three primaries are usually connected in mesh as well, since the continuity of supply is still maintained by these connections, even if one transformer should break down. Fig. 313 shows the diagram of connections and also the vector diagram. The potential

of the neutral point of each mesh coincides, since the systems are symmetrically linked together in the armature itself. If the C.C. voltage is E_C, the transformer secondary voltage is $0{\cdot}612E_C$. Calling the ratio of A.C. line voltage to C.C. voltage R, the transformer ratio must be made $\dfrac{R}{0{\cdot}612} = 1{\cdot}63R$

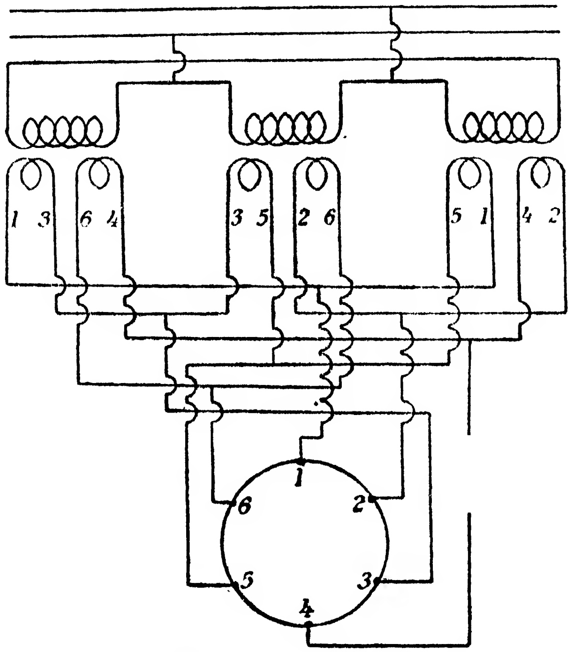
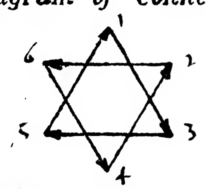


Diagram of Connections



Vector Diagram

FIG. 313.—Double Mesh Connections for Six Phase Rotary Converter.

The second method is called the double star method of connection and is occasionally employed. Double secondary windings are again required, each set being connected in simple star. The two star points are joined together and the six free ends connected as shown in Fig. 314, which also shows the vector diagram. The transformer secondary voltage per phase is now $\dfrac{0{\cdot}612}{\sqrt{3}}E_C = 0{\cdot}354E_C$, and if the primaries are still connected in mesh, the transformer ratio must be made $\dfrac{R}{0{\cdot}354} = 2{\cdot}83R$.

The third method, called the *diametral method*, is really a development of the previous one and is now largely adopted. Referring to Fig. 314, it is seen that each pair of secondaries can be replaced by a single winding, the middle points being taken to a common star point. But this latter connection is unnecessary, as in a three phase transmission scheme, the armature itself fixes the

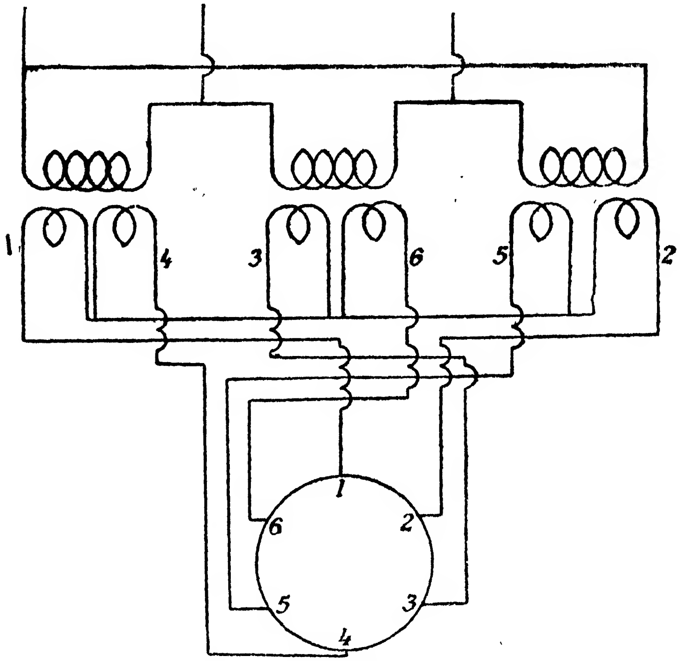
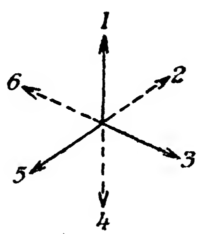


Diagram of Connections



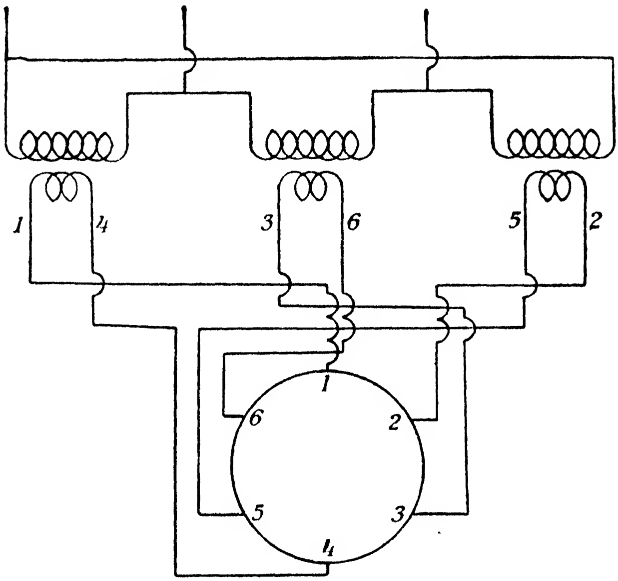
Vector Diagram

FIG. 314.—Double Star Connections for Six Phase Rotary Converter.

neutral point by its various connections. This method, therefore, only requires three single secondaries instead of six, these being connected as indicated in Fig. 315, which shows the vector diagram in addition. The latter also shows that the middle points of the three secondaries are all at the same potential. Since the single secondary takes the place of two in the second method, the trans-

former secondary voltage must be doubled, thus halving the transformer ratio, which now becomes $1{\cdot}41R$.

Twelve Phase Rotary Converter operated from Three Phase Supply.—By using a combination of the double mesh and the diametral systems of connections, it is possible to operate a twelve phase rotary converter from three phase mains. Referring to Fig. 316,



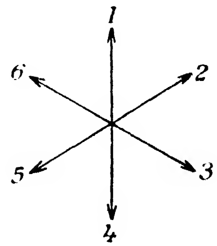


FIG. 315.—Diametral Connections for Six Phase Rotary Converter.

which represents the armature, it is seen that six of the phases can be supplied by the double mesh connections, whilst the intermediate six phases can be supplied by diametral connections. The phase of the voltage across each of the latter pair of points coincides with that across one leg of the mesh, so that there are really only voltages of three different phases required. These can be obtained from three single phase transformers having three secondaries each.

The two secondaries belonging to the double mesh will have the same number of turns, whilst the third one will need to have rather more turns. In fact this secondary will require to give $\frac{2}{\sqrt{3}} = 1{\cdot}15$ times the voltage given by either of the other two, this ratio being obtained from the relative lengths of the lines 2, 8 and 1, 9 or 3, 7

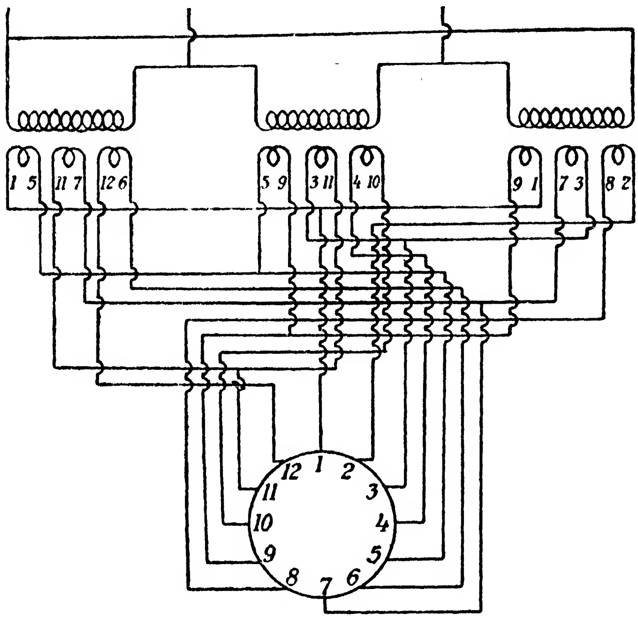
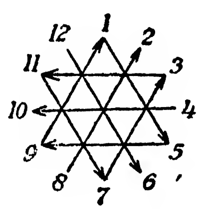


Diagram of Connections



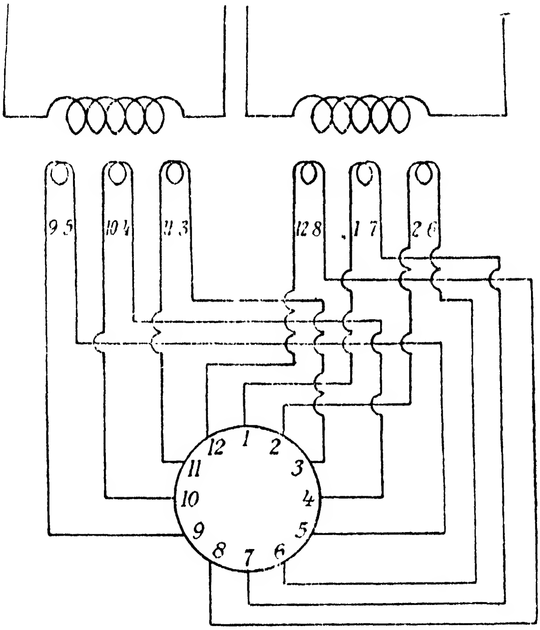
Vector Diagram

FIG. 316.—Connections of Twelve Phase Rotary Converter operated from Three Phase Supply.

in Fig. 316. It is seen from the vector diagram that the neutral points of each mesh and of the diametrally connected secondaries have the same potential, which is halfway between the potentials of the two sets of brushes on the commutator.

Twelve Phase Rotary Converter operated from Two Phase Supply.—By employing a somewhat similar arrangement to that shown in Fig. 316, a twelve phase rotary converter can be supplied from two

phase mains (see Fig. 317). Each transformer has three secondaries as before, arranged so as to give the same relative voltages as in the three phase case. Two of them give the same voltage as each other, whilst the third gives $\frac{2}{\sqrt{3}} = 1{\cdot}15$ times the voltage of the other two. The commutator voltage is $\sqrt{2} = 1{\cdot}41$ times this latter voltage.



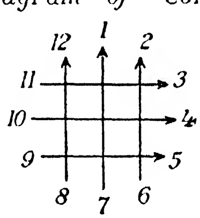


FIG. 317.—Connections of Twelve Phase Rotary Converter operated from Two Phase Supply.

Three Wire Rotary Converters.—Rotary converters can be used to supply a three wire C.C. system without employing separate balancers, provided that the neutral point of the transformer low tension windings is available. With a balanced load, this neutral point maintains a potential midway between the potentials of any two slip rings connected to opposite points on the armature. The potential of this neutral point is thus constant, and is also

midway between the potentials of the two sets of brushes on the commutator. The middle wire of the C.C. system is therefore connected to the neutral of the A.C. side as shown in Fig. 318, which represents a six phase three wire rotary converter. The out-of-balance current of the three wire system is thus led back to the star point of the transformer secondaries, where it distributes itself amongst the various phases, producing an unbalanced effect

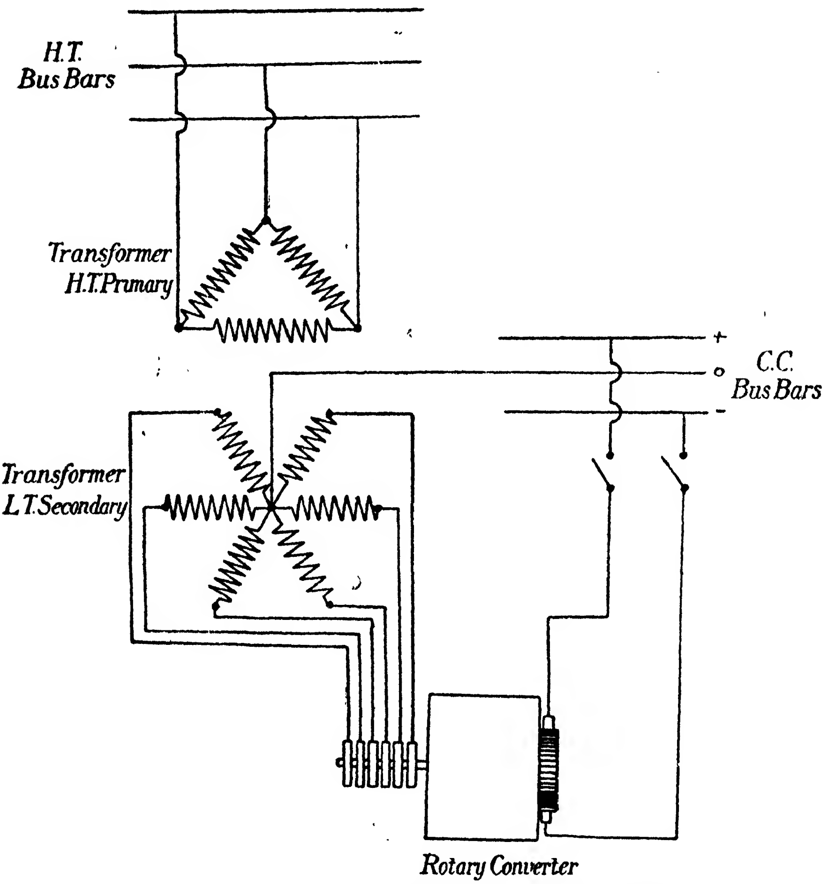
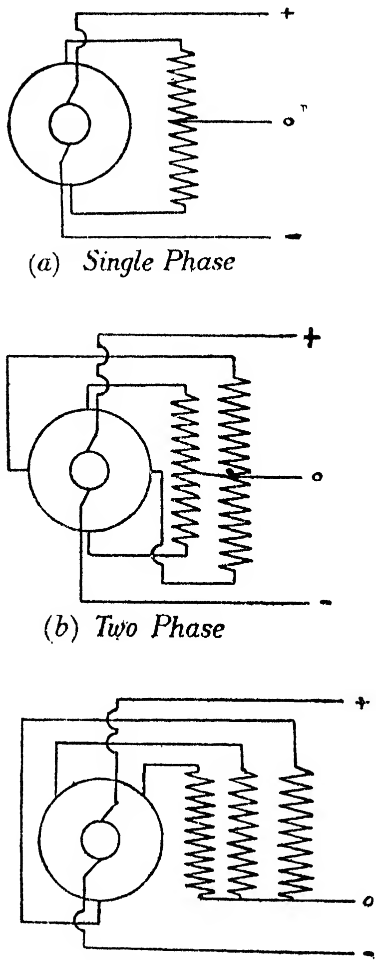


FIG. 318.—Connections for Three Wire Rotary Converter.

similar to an ordinary unbalanced load. This disturbs the regulation to a certain extent, but out-of-balance currents up to 25 per cent. of the full load current can be dealt with in this manner without detriment. If close regulation is desired, the series field windings and the interpole windings can be split into two halves, one being connected on the positive side of the machine and the other on the negative.

Static Balancers.—If the neutral point of the transformer secondaries is not available, an artificial neutral point can be made by a

system of choking coils connected across the slip rings. Such an arrangement is called a *static balancer*. In the case of a single phase machine a single choking coil is all that is necessary, the middle point being connected to the middle wire [see Fig. 319 (*a*)]. In a two phase machine two choking coils are necessary, their middle points being joined together to form the neutral point [see Fig. 319 (*b*)]. Three and six phase rotary converters require three choking coils [see Fig. 319 (*c*), representing a three phase machine]. Strictly speaking, a six phase machine requires six choking coils, but three are sufficient to provide the neutral point, the other three being neglected as far as the static balancer is concerned. The neutral point of the static balancer takes up a constant potential midway between those of the brushes on the commutator, exactly as in the three wire rotary converter, where the neutral point of the transformer secondaries is available. Any out-of-balance current is dealt with in the same way as described in the previous paragraph.



(*a*) *Single Phase*

(*b*) *Two Phase*

(*c*) *Three Phase*

FIG. 319.—Connections of Static Balancers.

Effect of Excitation.—When run from the C.C. side a rotary converter acts as a C.C. shunt motor, the speed varying with the excitation if the A.C. side is on open circuit. When driven from the A.C. side, it runs as a synchronous motor without any speed variation. When connected to the supply on both sides, either may act as a motor driving the other part as a generator, depending upon the voltages and the excitation. If running with a weak field, the C.C. side may try to run faster than the speed of synchronism, the result being to make the A.C. side act as a generator. If over-excited, the C.C. side tries to run below the speed of synchronism, and is consequently driven by the A.C. side, which is now motoring whilst the C.C. side is generating. At one excitation, therefore, the C.C. current will die down to zero and then reverse in direction as the excitation is still further increased,

whilst just before this point is reached it will be found that both sides are motoring, neither being strong enough to make the other side generate. Raising the C.C. line voltage is the equivalent of weakening the field and has the same effect. The variations in the main currents for different excitations is shown in Fig. 320.

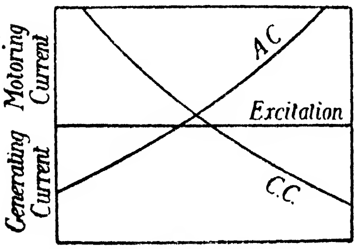


FIG. 320.—Effect of Excitation.

Armature Current and Heating.—The resultant current in the armature of a rotary converter is the difference between the continuous and the alternating currents flowing at the instant under consideration. This current will vary from instant to instant and also will be different in different conductors, depending upon their proximity to the slip ring tappings.

Consider a single phase bipolar case. The A.C. is a maximum when the conductors connected to the slip rings come under the commutator brushes, assuming this current to be in phase with the E.M.F. In the position shown in Fig. 321 (*a*), therefore, the conductors *aa'* are just carrying their maximum A.C., but at this instant these conductors pass under the brushes and the C.C. flowing through them reverses. The A.C. and C.C. components of the current in the conductors *aa'*, together with their resultant, are shown in Fig. 321 (*b*). For an efficiency of 100 per cent., the R.M.S. value of the A.C. is $\sqrt{2}$ times and the maximum value double that of the C.C. In the case of the conductors *aa'*, the C.C. reversal occurs at the peak of the A.C. wave, and consequently the resultant current takes on a momentary value far in excess of the normal C.C., but this state of affairs does not apply to all the other conductors. For example, the conductors *dd'* also have the maximum A.C. flowing in them in the position shown in Fig. 321 (*a*), but the C.C. reversal does not take place until a quarter of a period later. The resultant current in these conductors is shown in Fig. 321 (*e*), whilst the currents in intermediate conductors such as *bb'* and *cc'* are shown in Fig. 321 (*c*) and (*d*). The maximum height of the A.C. wave remains the same throughout, but the different positions at the time of the C.C. reversal cause vast changes in the resultant armature current. The most complete neutralisation occurs at those conductors situated midway between the slip ring tappings, and these conductors carry the minimum resultant current. As the slip ring tappings are approached, the R.M.S. value of the resultant current increases until it attains a maximum at the conductors nearest to the tappings themselves.

Next consider a three phase bipolar case. In Fig. 322 (*a*) maximum voltage exists between the slip rings (1) and (2) in the position shown. Again assuming unity power, the current in this

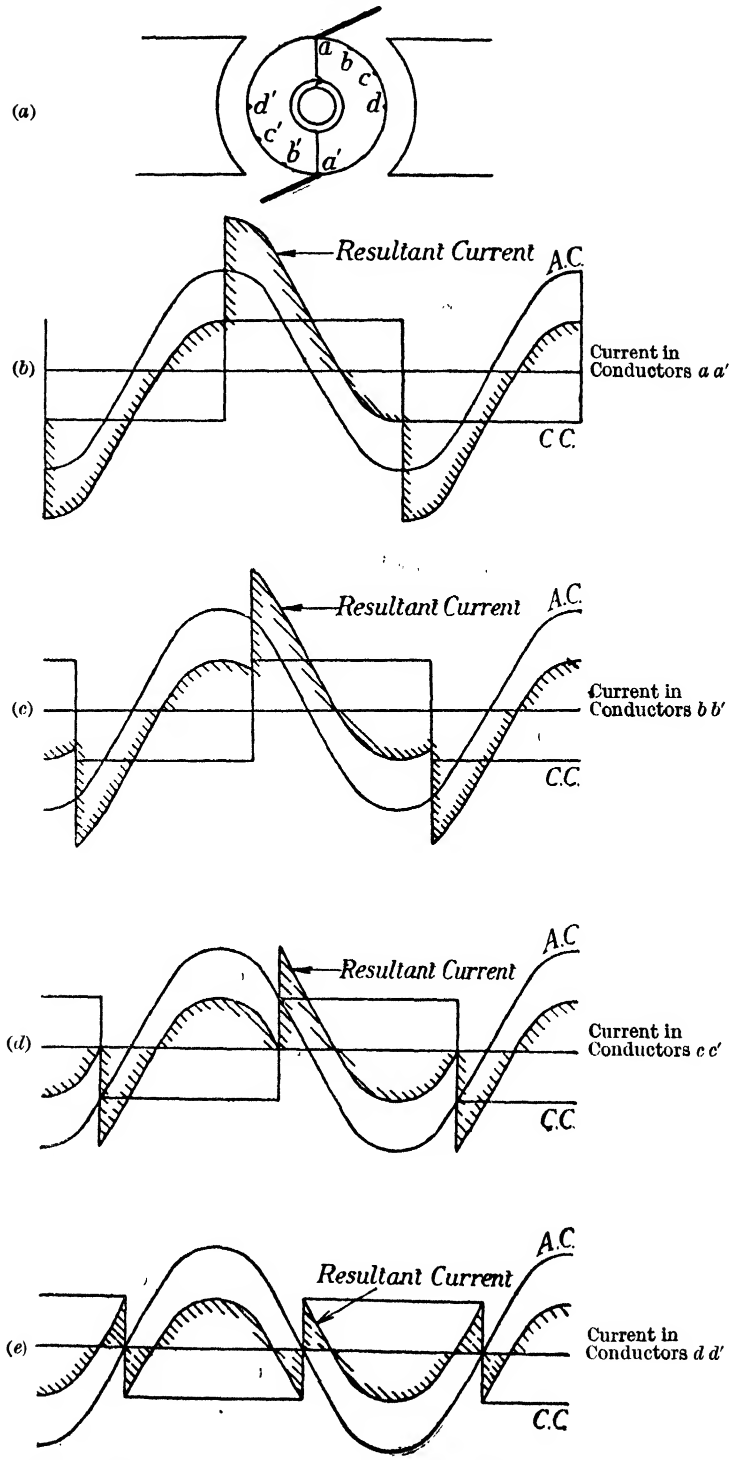


FIG. 321.—Armature Currents in Single Phase Rotary Converter.

section of the mesh-connected armature will also be a maximum in the same position. (The fact that the line current is 30° out of phase with the voltage across slip rings does not matter. It is the armature current that is desired.) The conductor *a*, immediately to the right of slip ring No. 1, will therefore come under the brushes 30° after the point of maximum A.C. The C.C. and A.C. components of the current in this section of the armature, together with

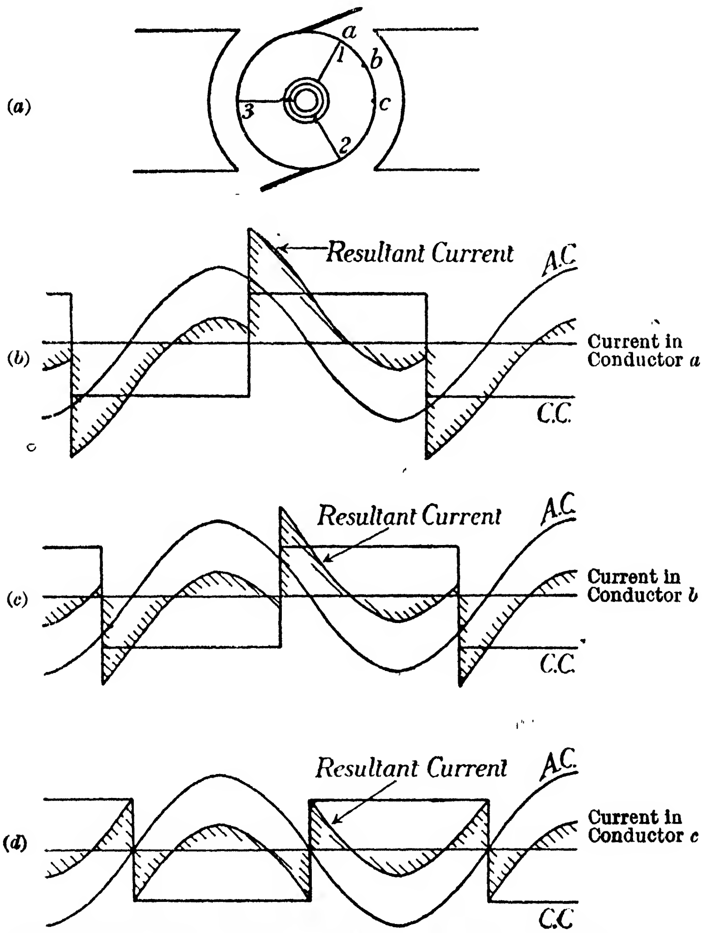


FIG. 322.—Armature Currents in Three Phase Rotary Converter.

their resultant, are shown in Fig. 322 (*b*). Considering conductor *b*, situated 30° behind *a*, the maximum A.C. occurs at the same instant as in *a*, but the C.C. reversal in this case occurs 60° after the instant of maximum A.C. The currents are shown in Fig. 322 (*c*). Conductor *c* is situated 60° behind *a* and is midway between the two slip ring tappings. The component currents and their resultant in this case are shown in Fig. 322 (*d*). Here the most complete neutralisation is obtained, and, in consequence, this conductor

carries the minimum current. In fact, whatever the number of phases, the conductor situated midway between two slip ring tappings will carry the minimum current.

The maximum height of the A.C. wave compared with the

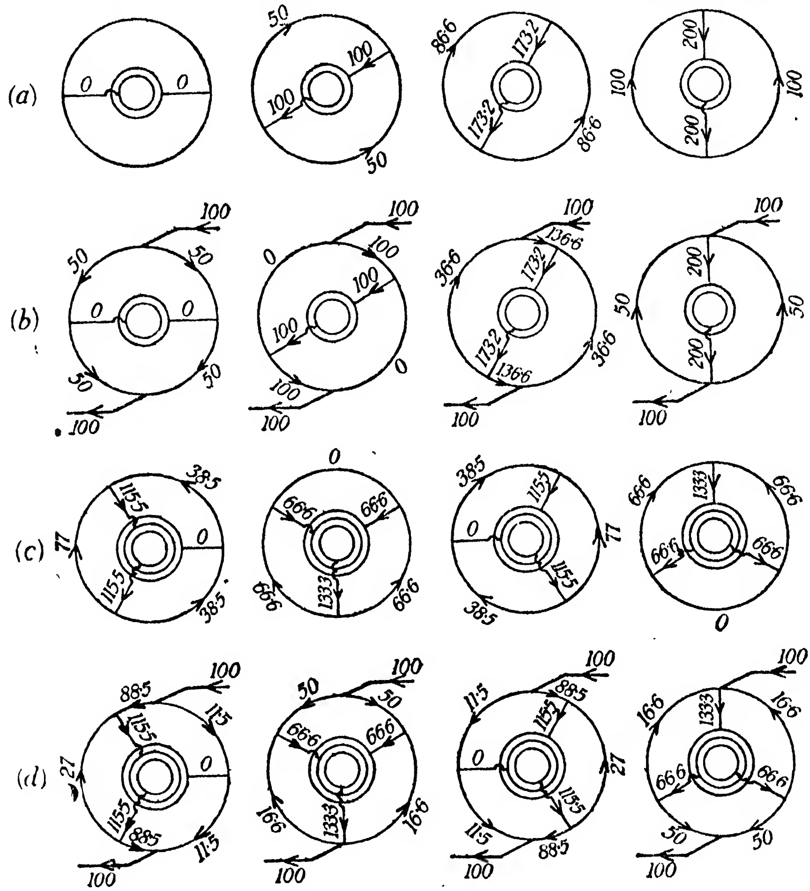


FIG. 323.—Armature Currents in Rotary Converter.
(a) Single Phase. A.C. only. (b) Single Phase. C.C. on A.C.
(c) Three Phase. A.C. only. (d) Three Phase. C.C. on A.C.

height of the C.C. curve is calculated from the voltage ratio. The R.M.S. line voltage is $\frac{\sqrt{3}}{2\sqrt{2}}$ times the C.C. voltage, and the R.M.S. line current is, therefore, $\frac{2\sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2\sqrt{2}}{3}$ times the C.C. The maximum line current is $\frac{2\sqrt{2}}{3} \times \sqrt{2} = \frac{4}{3}$ times the C.C., and the maximum armature A.C. is $\frac{4}{3\sqrt{3}} = 0{\cdot}77$ times the line value of the C.C. and 1·54 times the armature value.

On studying Figs. 321 and 322 it is seen that the further the conductor is situated from the midway position the larger is the current which it has to carry. In this respect the three-phaser has an obvious advantage over the single phase machine, particularly as the heating effect is proportional to the square of the current.

The armature currents for various positions relative to the poles can be studied further by referring to Fig. 323, which represents the armature of a bipolar rotary converter in various positions, the poles being supposed to lie along a horizontal axis. Fig. 323 (*a*) represents the A.C. only in a single phase winding. In Fig. 323 (*b*) the C.C. is superposed, the resultant current being shown. Fig. 323 (*c*) represents the A.C. only in a three phase winding, whilst Fig. 323 (*d*) shows the resultant effect of superposing the C.C. on the A.C.

One conclusion to be drawn is that the closer the slip ring tappings are made the less will be the armature heating, and this forms one of the chief reasons for the modern preference for six and twelve phase rotary converters.

Effect of Number of Slip Rings on Output.—It was shown above that the greater the number of slip rings the less was the amount of heating produced in the armature conductors. This is equivalent to saying that for the same temperature rise of the armature the output is greater the greater the number of slip rings, or, conversely, for a given output and temperature rise, a six phase machine is smaller than one with fewer phases.

Starting.—There are three methods of starting up a rotary converter, viz. :—

(1) By means of an auxiliary starting motor.
(2) From the C.C. side.
(3) From the A.C. side.

The first method is to employ a small auxiliary induction motor (see Chapter XXVI), exactly as in the case of a synchronous motor. This is described on p. 331. After being synchronised, the C.C. side is paralleled on to the bus bars.

The second method is to run the set up from the C.C. side as an ordinary shunt motor, synchronising when the correct speed is attained. When this method is employed in a sub-station a complete shut-down renders it impossible to start up again, since the C.C. bus bars would be dead. To obviate this danger, it is the usual practice to have at least one of the rotary converters started up by means of an auxiliary motor. In addition, the sub-station is usually linked up to others from which it is possible to obtain the necessary C.C. supply.

In order to avoid the large currents in the L.T. circuit it is usual to synchronise on the H.T. side of the transformers. Fig. 324 shows the chief connections for starting up a rotary converter in this manner. When the load is subjected to heavy fluctuations a some-

what different procedure is commonly adopted. The rotary converter is run up to a speed slightly above that of synchronism. The C.C. side is then opened and the A.C. side immediately closed. The machine then pulls itself into step, when the C.C. switches are again closed.

The third method is applicable to polyphase rotary converters, the procedure being the same as in the case of self-starting synchronous motors (see p. 331). Unfortunately, however, the polarity of the C.C. side is not definite, this depending upon the position of the armature at the instant of switching in. To see the reason for this, consider a machine started up and ready to be synchronised. If the armature be now suddenly retarded so that it loses half a cycle, the current being reversed at the same instant, the conditions for synchronising are still maintained, but the polarity on

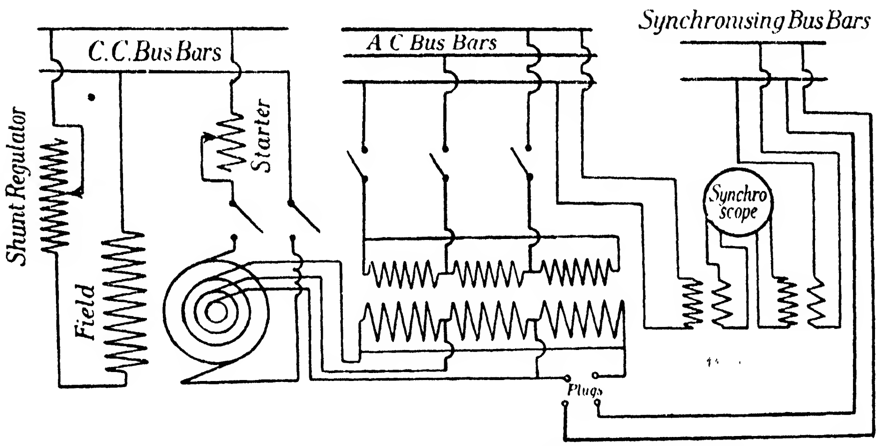


FIG. 324.—Three Phase Rotary Converter started from C.C. Side.

the commutator is also reversed. When the armature is started up by means of the eddy currents in the pole shoes, either of these two conditions may be set up, depending upon the moment at which synchronism is reached. To determine the polarity on the commutator, a moving coil voltmeter is employed, the pointer of which will indicate a voltage of a very low frequency (the difference between that of the supply and that of the machine) when near synchronism. The field switch is then closed when the voltmeter indicates that the polarity is correct and the voltage is a maximum. Another method of correcting the polarity is to provide the field with a double pole change-over switch. If the C.C. voltage is found to be reversed, this switch is thrown over and then back again. Since the rotation is maintained in the same direction, the first reversal causes the exciting current to demagnetise the field, whilst the second change-over causes it to be built up in the other direc-

tion, the armature having been retarded half a period in the meantime.

Parallel Running.—Rotary converters driven from the same A.C. system are often required to run in parallel on the C.C. side. If they are directly paralleled at the commutator and slip rings, however, a complete local circuit is formed by two converters by way of the A.C. and C.C. bus bars, and large cross currents are liable to be set up in this local circuit, due to slight differences in the operation of the two machines. To avoid this, the rotary converters are usually operated from separate transformers. When the same bank of transformers is used, the rotary converters should be operated from independent secondaries.

For good parallel running it is desirable that there should be a comparatively large voltage drop from no-load to full load, as in the case of alternators. When run in conjunction with a battery, it is usual to include a reverse current cut-out on the C.C. side to prevent the rotary converter running back should a large drop in voltage occur on the A.C. side.

Hunting.—Rotary converters, like synchronous motors, are subject to hunting troubles, and the way in which hunting is set up has already been described (see p. 329). The armature currents of a polyphase converter set up a field which, under perfect conditions, is stationary in space. When hunting occurs, however, this field oscillates to and fro and sets up an E.M.F. in the coils undergoing short-circuit by the brushes on the commutator, and thus is the cause of violent sparking. To avoid this trouble, which in early machines was frequently of considerable magnitude, damping grids, or amortisseurs, are fitted on to the poles. Any oscillation of the flux across the pole face, produced by irregularities in the speed, generates an E.M.F. in the damping grids, setting up eddy currents which tend to damp out the oscillations which produce them.

In addition to adopting the above expedients, the wave form employed should be as close an approximation to a sine wave as possible.

Overload Capacity.—Well-designed rotary converters have a high overload capacity, since the heating is less than in a corresponding C.C. generator. Also the armature reaction due to the motoring and generating action is a differential effect, and as interpoles are used the sparking is not so acute as it otherwise might be. A good machine will have an overload capacity of 100 per cent. for short periods, this being an important fact when considering the spares required in a sub-station.

Power Factor.—The power factor of a rotary converter depends upon its excitation, just as in the case of a synchronous motor, and, as a general rule, it is advisable to operate it at as near unity power factor as possible. If there is other apparatus connected to the line, taking a lagging current, the resultant power factor of the

system can be improved by over-exciting the rotary converter so as to make it take a leading current at a power factor of, say, 0·9. Unfortunately, however, this affects the armature heating to a serious extent, particularly in the neighbourhood of the slip ring tappings. In fact, in a six phase converter, the heating in such a coil is increased by about 80 per cent. if the power factor is changed from unity to 0·9 leading. As this is the equivalent of a very considerable overload, it is very necessary to operate the machine at as near unity power factor as possible when on or about full load.

It is thus seen that from a practical standpoint the rotary converter is not nearly so good as a synchronous motor-generator set for the purpose of improving the power factor.

Compounding.—Rotary converters may be compounded in just the same way as C.C. shunt generators, this winding being in addition to that of the interpoles. In such a case, the C.C. bus bars should include an equalising bar for the purpose of paralleling the various series coils of the different converters running in parallel.

Armature Reaction.—In a C.C. generator the armature reaction acts in such a direction as to require a forward lead of the brushes, whilst in a motor the armature reaction is in the opposite direction. In the rotary converter the C.C. and A.C. currents may be considered as being superposed on one another in the armature so that there will be two armature reactions tending to neutralise one another. In a polyphase machine the A.C. and C.C. currents are roughly of the same magnitude, so that the two reactions will be approximately equal. Successful commutation may therefore be obtained for all loads up to full load, and even for overloads, by fixing the brushes in the no-load neutral position. This neutralisation of the armature reactions has the effect of considerably raising the limiting load from the sparking point of view, this being much higher than would be the case if the machine were used as a C.C. generator.

Voltage Regulation.—The C.C. voltage obtained from a given rotary converter depends upon the impressed A.C. voltage and drops slightly as the load comes on. In an ordinary generator this can be corrected by adjusting the field strength, but in the present instance this is of no avail, since it merely changes the power factor on the A.C. side. In order to obtain a certain amount of voltage regulation on the C.C. side, various methods have been devised of which the following are the chief :—

(1) Reactance regulation.
(2) Booster regulation.
(3) Induction regulator control.
(4) Split pole regulation.

Reactance Regulation.—A change in the excitation of a rotary converter does not affect its C.C. voltage, but it alters the power factor and the armature current. If a choking coil be placed in

series with the rotary converter it will absorb a variable voltage depending upon the value of the current. The phase of this voltage will also depend upon the current in the converter armature, which in turn depends upon the excitation. The terminal voltage of the converter is the vector difference of the supply voltage and that absorbed by the choking coil, and by adjusting the excitation the phase and magnitude of the voltage absorbed by the choking coil can be regulated so that a practically constant voltage is obtained at the C.C. end for all loads.

In order to maintain a constant C.C. voltage, the voltage across the converter slip rings must rise slightly as the load comes on, and the way in which this is brought about is shown in Fig. 325, which represents the vector diagram of the quantities concerned. Fig. 325 (*a*) is drawn for a weak excitation, the armature current, I, lagging behind the slip ring voltage, $V_{s.r.}$. The voltage drop across the choking coil is represented by $V_{c.c.}$ and leads the current by 90°, whilst the voltage supplied by the transformer secondary is

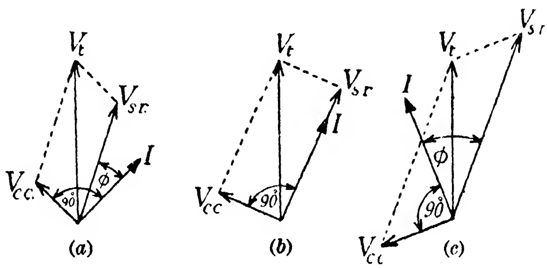


FIG. 325.—Reactance Regulation.

represented by V_t and is assumed to be constant throughout. V_t is the vector sum of $V_{s.r.}$ and $V_{c.c.}$. The weak excitation and lagging armature current correspond to the conditions existing on light loads. Fig. 325 (*b*) represents the conditions on a larger load. The excitation of the rotary converter has been increased so as to bring the power factor up to unity. $V_{c.c.}$ still leads the current by 90°, whilst its magnitude is reduced on account of the improved power factor, but is increased on account of the increased load. It is the displacement in phase, however, which is the chief cause of the increased slip ring volts $V_{s.r.}$, as is seen from the diagram. Fig. 325 (*c*) represents the conditions on full load. The rotary converter is now over-excited, so that its armature current leads the slip ring voltage. The voltage absorbed by the choking coil is now increased, both on account of the increased load and the decreased power factor. But, again, the chief feature of the diagram is the advance in phase of this voltage which still leads the current by 90°. The result is clearly shown to be a further increase in the voltage supplied to the slip rings, and this increase can, by

a suitable design, be made to counteract the increased drop in voltage due to the load coming on. It is also possible to obtain a rising C.C. voltage, as in the case of an over-compounded C.C. generator, by increasing the size of the reactance.

The desired reactance may be obtained in two ways. A special reactance coil may be inserted in each phase between the transformer and the slip rings, or, alternatively, the reactance may be supplied by the transformer itself.

The disadvantage of this method of control is that the rotary converter cannot always be worked on the best power factor, since the regulation depends to a large extent upon the angle of phase difference between the slip ring voltage and the armature current.

Booster Regulation.—The additional voltage required at the slip rings of the rotary converter is here obtained by means of a booster, which is carried on the main shaft and is situated between the slip rings and the armature of the rotary converter as shown in Fig. 326. The booster consists of a rotating armature A.C. generator having the same number of phases and poles as the converter itself. Each phase of the booster armature is connected in series with one of the slip ring leads, the far end being taken to one of the tappings on the converter armature. The function of the booster is to generate a small voltage proportional to the C.C. load, this small voltage being added to that at the slip rings in order to counteract the natural drop as the load comes on. To effect this the booster receives its excitation from the C.C. end of the converter, its fields being connected in series with the load, like the compounding coils on the converter itself. On no-load the excitation of the booster is zero, so that it supplies no voltage at all As the load increases the field current increases and the generated E.M.F. boosts up the supply so as to maintain a practically constant voltage at the commutator for all loads. If desired, a larger booster voltage may be induced, so that the C.C. terminal pressure rises with the load, as in the case of an over-compounded generator.

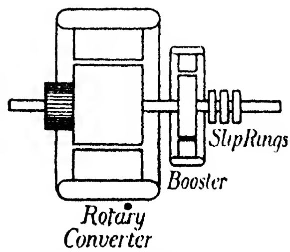


FIG. 326.—Rotary Converter with Booster.

With booster control, the power factor is independent of the load, so that unity power factor can be obtained, or, if desired, a leading current can be drawn from the mains.

Induction Regulator Control.—The simplest method of hand regulation is to employ a small boosting transformer, as shown in Fig. 176, in addition to the main transformers, so that boost can be applied gradually as the load comes on. Unfortunately, either very heavy currents or very high voltages have to be dealt with, according as the boost is obtained on the H.T. or L.T. side of the

main transformers. The method is therefore comparatively expensive, but close regulation can be obtained by it, such as is desired if the rotary converter is operating on a lighting load.

Instead of using a boosting transformer of the type shown in Fig. 176, another piece of apparatus called an *induction regulator* may be employed. This really consists of a slip ring induction motor (see Chapter XXVI) in which the rotor is held stationary, arrangements being made for rocking it to and fro through a certain angle. This induction regulator acts just like a transformer, the stator winding being used as the primary and the rotor winding as the secondary. Connections for a three phase rotary converter are made as shown in Fig. 327. The phase of the voltage generated in the induction motor rotor depends upon the position of the rotor, so that by slowly rotating it by hand the main transformer voltage can be boosted up or down at will. The two extreme positions of the induction regulator are those in which the boost voltage is

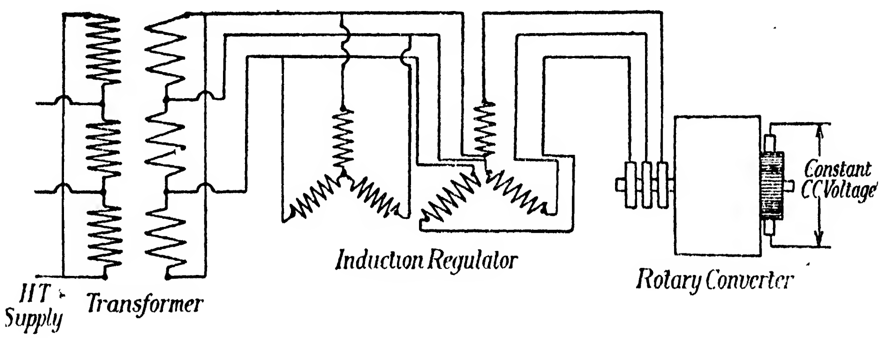


FIG. 327.—Induction Regulator Control.

directly added and subtracted, intermediate voltages being obtained in between. This method of control is not as satisfactory as some of the other methods, whilst the regulator itself is expensive.

Split Pole Regulation.—The regulation in this method is provided by distorting the voltage wave form by means of split poles. The main portion of the pole receives a constant excitation, whilst the auxiliary part receives a variable excitation. In this way, the distribution of the flux along the pole face is disturbed and the wave form is distorted. The voltage on the A.C. side is determined by the R.M.S. value, whilst the commutator voltage depends upon the maximum value of the wave, so that by making the wave more peaked the C.C. voltage is kept up as the load comes on. Split pole converters are practically limited to frequencies of 25 cycles per second, since for higher frequencies commutation troubles are set up.

Rotary Converters v. Motor-generators.—In comparing rotary converters with motor-generators for transforming purposes, the

former must be considered in conjunction with the necessary transformers and control gear, whilst the latter is a more self-contained unit in which a voltage variation is quite easily obtained by means of shunt regulation. On the other hand, the motor-generator set consists of two rotating machines as compared with one in the case of the rotary converter, and usually works out a little more expensive. On the score of efficiency, also, the two systems are much the same, the rotary converter probably having a little advantage again, but up to the last few years the motor-generator was supposed to score from the point of view of reliability. Both systems are in common use.

CHAPTER XXIV

RECTIFIERS AND VALVES

Mercury Vapour Converter.—The essential part of a mercury vapour or mercury arc converter consists of an arc which is struck between a graphite anode (+) and a mercury cathode (—). With such electrodes it is found that a current will only flow one way, and that if the graphite is made the cathode no current flows at all. If such an arc is set up by an alternating pressure, one half of the current wave will be cut off, leaving a uni-directional current having a wave form similar to that shown in Fig. 328 (*a*). In order to make use of both half-waves, a second anode is provided, this being connected to the other A.C. line wire, a common cathode being employed. With this arrangement the theoretical wave form obtained is that shown in Fig. 328 (*b*).

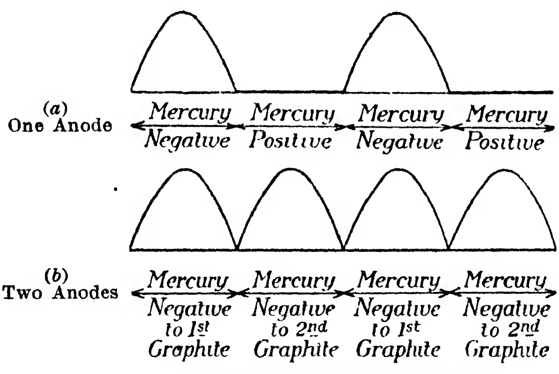


FIG. 328.—Current Wave Form.

Description of Apparatus.—The arc is formed in an exhausted glass vessel, *T*, called the rectifier tube (see Fig. 329). This contains two graphite anodes, *AA*, each connected with the exterior by means of a platinum wire fused through the glass. At the bottom is a pool of mercury, *C*, forming the cathode, also connected with the outside by a platinum wire fused through the glass. Unfortunately, however, the arrangement is not self-starting, so an auxiliary electrode, *S*, is used as a temporary anode for starting the arc. This consists of another little pool of mercury lying close to the

main pool as shown, and is connected to one of the main anodes through a resistance, *R*. In order to start the arc, the whole glass bulb is tilted to one side, thus causing momentary contact between the mercury in the two pools. An arc is thus struck producing sufficient mercury vapour to start the main arcs from the graphite anodes. The resistance, *R*, is included to limit the violence of the starting arc.

The A.C. supply is brought to the two ends of an auto-transformer, each end being further connected to one of the anodes. One of the C.C. supply leads is connected to the mercury cathode, whilst the other is connected through a choking coil to the middle point of the transformer. The choking coil is for the purpose of smoothing out the irregularities of the current wave form, and does

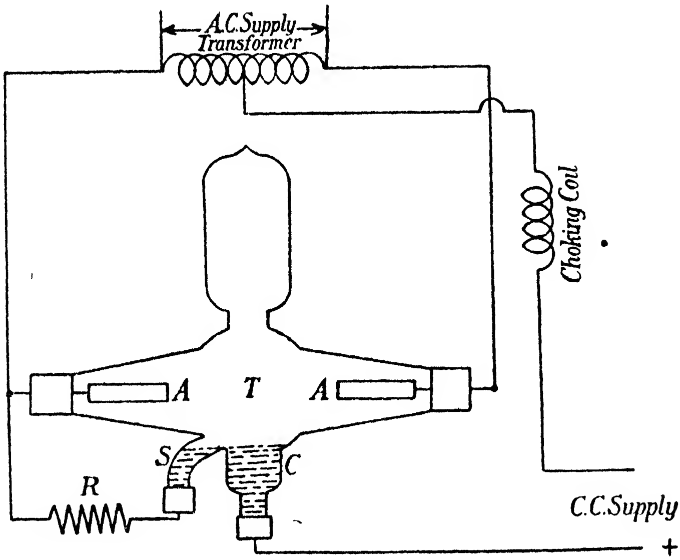


FIG. 329.—Mercury Vapour Converter.

not cause any loss, since it is connected on the C.C. side. The C.C. supply is thus obtained from the left- and the right-hand side of the transformer alternately.

Three Phase Converter.—In the case of a three phase converter the negative pole is formed by the neutral point of the star-connected transformer secondaries, as shown in Fig. 330. The bulb is now provided with three anodes, but otherwise the arrangement is the same.

Performance.—The rectified voltage gradually falls as the load comes on in common with various types of rotating machines, in addition to which there is a constant voltage drop of 15 volts in the mercury arc. There is also the loss in the transformer and an I^2R loss. An efficiency of 80–90 per cent. can be obtained and

maintained over a large range of loads, whilst power factors of 0·9–0·95 are reached.

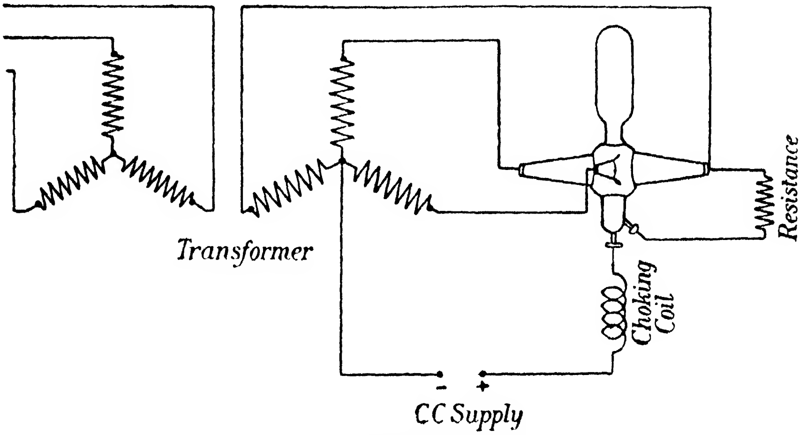


FIG. 330.—Three Phase Mercury Vapour Converter.

These mercury vapour converters can be constructed with glass bulbs for currents up to 80 amperes, whilst it has been suggested

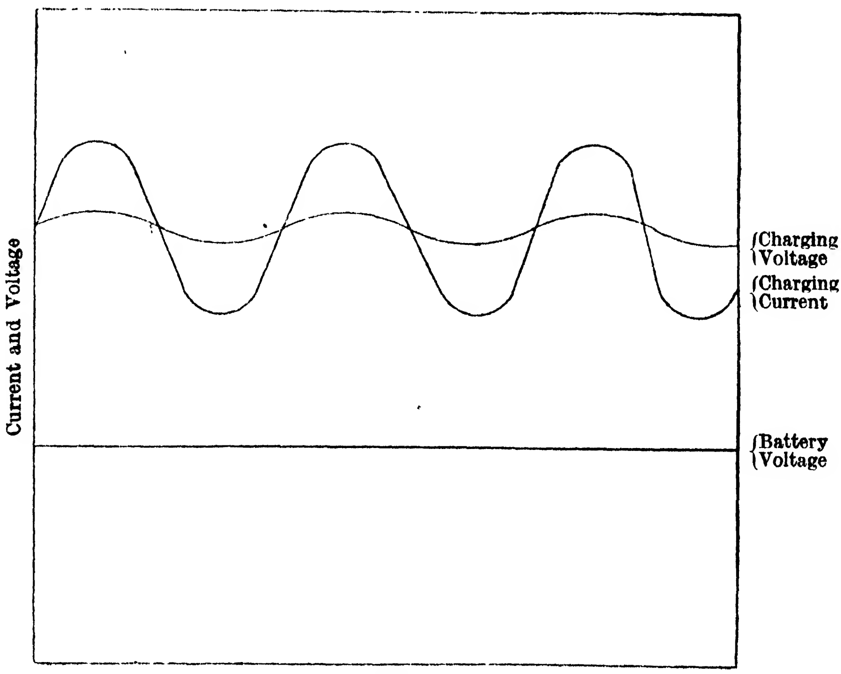


FIG. 331. Curves for Converter Charging Accumulators.

to use metal bulbs for larger currents. They find a considerable application in battery charging and projection arcs when only an

A.C. supply is available. As an illustration, the curves in Fig. 331 represent the current and voltage variations when used for charging a battery of accumulators.

Electrolytic Rectifiers or Electric Valves.—When two metallic electrodes are dipped in an electrolyte and the whole is subjected to a difference of potential a *valve effect* is produced. This means that the resistance of the apparatus when the current is flowing one way is vastly different from what it is when the current is reversed. If such an arrangement is subjected to an alternating pressure, half of the current wave is eliminated and a uni-directional current of varying magnitude results. Aluminium is usually employed as the cathode (—), whilst lead or iron is generally used as the anode (+).

Nodon Valve.—In the Nodon valve the cathode consists of a hollow cylinder of aluminium alloyed with a small proportion of foreign material. Surrounding this is a somewhat larger cylindrical lead plate, used as the anode. These are immersed in a solution of neutral ammonium phosphate, the whole being contained in an iron pot. In order to keep the apparatus cool when working, water is passed through the central hollow aluminium cylinder, but in the larger sizes air cooling is employed, this being provided by means of a small fan.

Arrangement of Cells.—If only one cell is used, an intermittent current is obtained having a wave form like that shown in Fig. 328 (*a*), but the disadvantages of this are overcome by a grouping invented by Grätz. This employs four cells and is a kind of Wheatstone bridge arrangement, as shown in Fig. 332. The A.C. supply is fed into a transformer the secondary of which is connected across the two points *AC*. A pulsating uni-directional current is then obtained from the points *BD*, which are connected to the load. When the point *A* is positive, the current flows through the cells 1, 3 along the path *ABDC*. When the point *C* is positive, the current flows through the cells along the path *CBDA*. Thus the point *B* is always positive and the point *D* always negative. With this arrangement two cells are always active, whilst the other two are inactive, the result being a uni-directional current having a wave form like that shown in Fig. 328 (*b*).

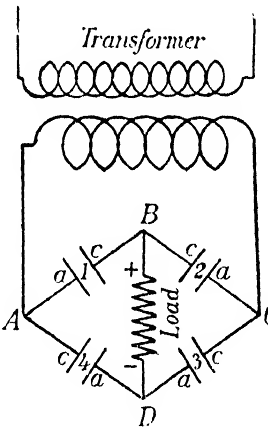


FIG. 332.—Arrangement of Nodon Valves.

Groups of such cells can also be arranged for two- and three-phase currents, in which cases the rectified current has a much more uniform value.

Performance.—Efficiencies of 75 per cent. can be reached whilst high power factors of 0·9 and over are obtained. They are not suitable for voltages much above 100, but they have been constructed for currents of 300 amperes. With currents of this magnitude, however, trouble is experienced on account of the temperature rise, and special cooling arrangements must be adopted.

CHAPTER XXV

ROTATING FIELDS

Production of a Rotating Field.—A rotating magnetic field is one in which the flux rotates round a fixed axis and can be produced by merely rotating a magnet or a coil of wire in which a C.C. is flowing.

A rotating field can also be produced by a system of stationary coils supplied with polyphase currents. Consider Fig. 333, which represents a simple 2-pole (not a 4-pole) two phase system, the two

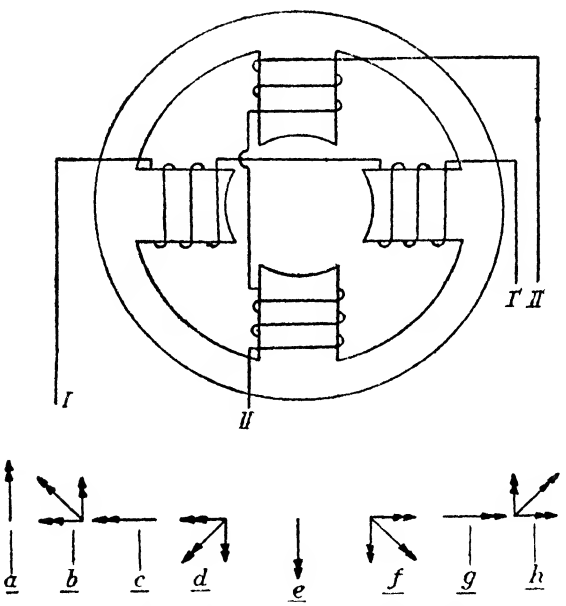


FIG. 333.—Production of Rotating Field.

horizontal projections being supplied from one phase and the two vertical projections from the other. The combined effect of these two phases is to produce a resultant field the axis of which changes from instant to instant, and it will be seen that this resultant field gradually rotates whilst maintaining a constant magnitude. At the instant when the current in the first phase is zero that in the other is a maximum. The combined M.M.F. is therefore vertical,

since there is no horizontal component, and the resultant flux is also vertical, as shown at (*a*). In order to determine the axis of the resultant field, each phase will be imagined to produce its own flux, the two component fluxes then being added together vectorially. This is not theoretically accurate, but it is a convenient method of dealing with the problem and leads to accurate results. As the current in the first phase grows that in the second phase dies down. The vertical field is now not so great, but there is a horizontal field to be added to it vectorially, the result being a magnetic flux having an axis lying along an inclined line, as shown at (*b*). After a time, the current in phase I reaches a maximum whilst that in phase II has died down to zero. The resultant flux is now horizontal, as shown at (*c*). The axis of the resultant flux at succeeding instants is shown at (*d*), (*e*), (*f*), etc., from which it is seen that the magnetic flux gradually revolves, completing one revolution in the time taken to pass through one cycle in a bipolar case. In a multipolar case the magnetic flux swings past a pair of poles for every cycle, and consequently the speed of revolution is $\frac{1}{n}$th that in the bipolar case, where n is the number of pairs of poles.

Care must be taken in determining the number of poles, which is the same as the number of poles *per phase*.

Two Phase Rotating Field.—In order to determine the magnitude of the resultant field, a sinusoidal current wave form will be assumed and also a sinusoidal flux distribution. This latter assumption will be found to be unjustified later on, but it enables the resultant flux to be calculated with greater ease.

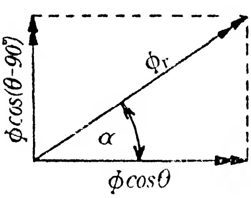


FIG. 334.—Combination of Fluxes (Two Phase).

Let Φ represent the maximum flux produced by each phase separately. Considering an instant when the current in the first phase has advanced through an angle θ from its maximum value, the flux due to this phase is represented by $\Phi \cos \theta$, and lies along a horizontal axis. At the same instant the flux due to the other phase, which is lagging behind, is represented by $\Phi \cos (\theta - 90°)$ and lies along a vertical axis. The resultant flux, Φ_r (see Fig. 334), is equal to

$$\begin{aligned}\Phi_r &= \sqrt{\{\Phi \cos \theta\}^2 + \{\Phi \cos (\theta - 90°)\}^2} \\ &= \Phi \sqrt{\cos^2 \theta + \cos^2 (\theta - 90°)} \\ &= \Phi \sqrt{\cos^2 \theta + \sin^2 \theta} \\ &= \Phi.\end{aligned}$$

It is thus seen that the resultant flux due to the two phases is equal to the maximum flux produced by each phase separately and is constant in magnitude and independent of the angle θ.

The angle through which the axis of the resultant flux has rotated, corresponding to an advance in phase of the current of $\theta°$, can also be calculated. This angle is represented by α in Fig. 334 and

$$\tan\alpha = \frac{\Phi\cos(\theta - 90°)}{\Phi\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta.$$

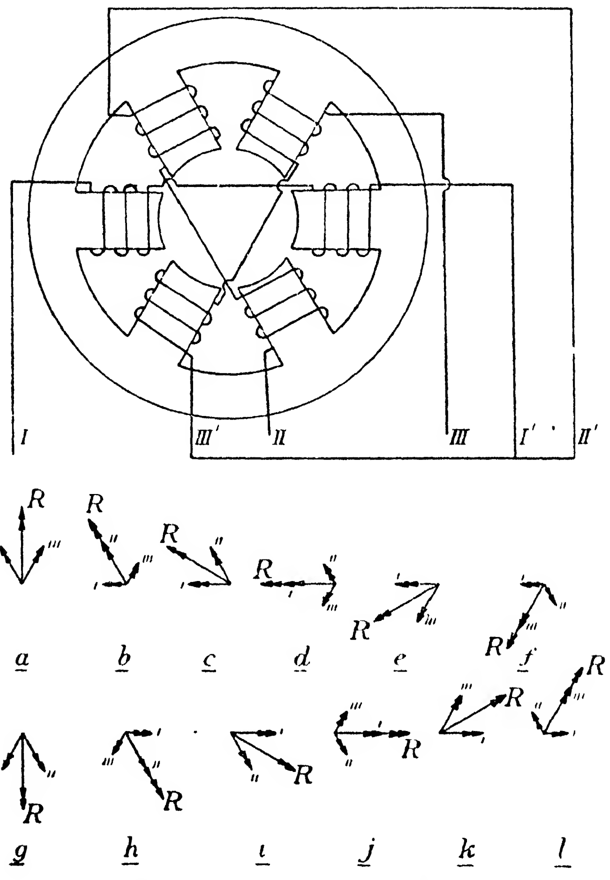


FIG. 335.—Three Phase Rotating Field.

Therefore α is equal to θ and an advance of $\theta°$ in phase corresponds to a rotation of $\alpha°$ of the magnetic flux. The flux is thus seen to rotate uniformly, having a constant magnitude all the time.

Three Phase Rotating Field.—A simple bipolar arrangement for three phases would be provided with six projections (see Fig. 335)

instead of four, as in the two phase case. The coils belonging to each phase are wound on to two opposite projections, care being taken to obtain a space displacement of 120° instead of 60°. This is done by connecting the three front ends, I, II, III, to the supply, the three rear ends, I′, II′, III′, being joined together to form a star point. A mesh connection can also be adopted if desired.

Commencing with the current in phase I at zero, the current in phase II (lagging by 120°) is $-\frac{\sqrt{3}}{2}$ times its forward maximum value and the current in phase III (lagging by 240°) is $+\frac{\sqrt{3}}{2}$ times its forward maximum value. The flux vector diagram representing these conditions is shown at (*a*) in Fig. 335. After an interval of 30°, the current in phase I has risen to half its maximum value, the component flux due to this phase doing the same. The flux due to the second phase has now reached its maximum, whilst that due to phase III has died down to half its maximum value, as shown at (*b*). The flux due to phase III continues to die away, falling to zero 30° later, when the resultant flux is due to phases I and II only, as shown at (*c*). The values of the component fluxes at further successive intervals of 30° are shown at (*d*), (*e*), (*f*), etc., from which it is seen that the resultant magnetic field makes one complete revolution per cycle as in the two phase case. The magnitude of the resultant flux is also seen to be constant for all the instants illustrated, and it will now be shown that it is constant throughout.

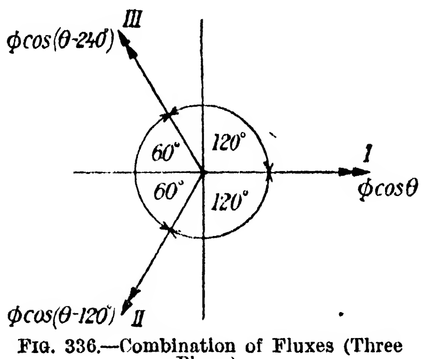


FIG. 336.—Combination of Fluxes (Three Phase)

Using the same notation as before, the values of the component fluxes at any instant are $\Phi\cos\theta$, $\Phi\cos(\theta-120°)$, and $\Phi\cos(\theta-240°)$. These fluxes act along axes which are mutually inclined to each other at 120°, so that in order to obtain an expression for the resultant flux all the instantaneous horizontal components and all the instantaneous vertical components will be determined and the two resultants added together vectorially.

The resultant horizontal component (see Fig. 336) is

$$\Phi\cos\theta - \Phi\cos(\theta-120°)\cos 60° - \Phi\cos(\theta-240°)\cos 60°$$

$$= \Phi\left[\cos\theta + \tfrac{1}{4}\cos\theta - \frac{\sqrt{3}}{4}\sin\theta + \tfrac{1}{4}\cos\theta + \frac{\sqrt{3}}{4}\sin\theta\right]$$

$$= \Phi \times \tfrac{3}{2}\cos\theta.$$

The resultant vertical component is

$$0 - \Phi\cos(\theta - 120°)\sin 60° + \Phi\cos(\theta - 240°)\sin 60°$$

$$= \Phi\left[\frac{\sqrt{3}}{4}\cos\theta - \tfrac{3}{4}\sin\theta - \frac{\sqrt{3}}{4}\cos\theta - \tfrac{3}{4}\sin\theta\right]$$

$$= \Phi \times (-\tfrac{3}{2}\sin\theta).$$

The resultant field is, therefore,

$$\Phi_r = \sqrt{\Phi^2 \times (\tfrac{3}{2}\cos\theta)^2 + \Phi^2 \times (-\tfrac{3}{2}\sin\theta)^2}$$

$$= \tfrac{3}{2}\Phi \times \sqrt{\cos^2\theta + \sin^2\theta}$$

$$= \tfrac{3}{2}\Phi.$$

The resultant field is again constant and independent of θ as in the two phase case and is $\frac{3}{2}$ times the maximum field produced by each phase separately. It will be shown later that this constant has a value of 2 when the non-sinusoidal flux distribution is taken into account. The flux also rotates with uniform speed, making one complete revolution per cycle in a bipolar case.

Flux Distribution with Concentrated Winding.—It is usual to arrange the magnetising winding in slots in preference to putting it on salient pole pieces, a uniform air-gap being thus obtained when a cylindrical rotor is placed in the field system. In a bipolar arrangement each coil will lie in two slots and will span an arc of 180°. That portion of the air-gap lying between the two slots on one side forms one pole, whilst the other portion of the air-gap lying between the two slots on the other side forms the other pole. There is thus one coil per pair of poles. The M.M.F. is proportional to the ampere-turns, and has the same value

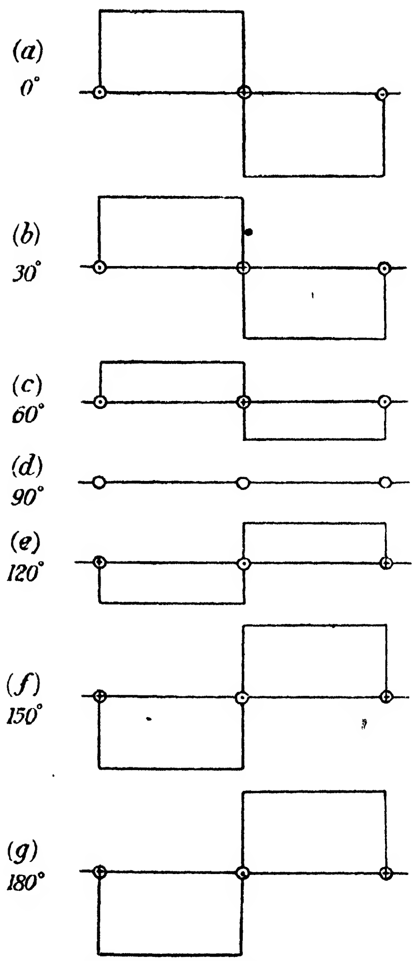


FIG. 337.—Flux Distribution with Concentrated Winding.

over the whole arc of 180°. Over the remaining 180° it is equal in magnitude, but opposite in sign. As the reluctance is approximately uniform all round the air-gap, the flux density is constant all over the pole, and is represented by Fig. 337. When the magnetising current dies down, the flux density diminishes proportionally, successive intervals of 30° being shown. After 90° the flux reverses and grows sinusoidally with time until it reaches a maximum, as shown in Fig. 337 (*g*). The action is then repeated.

For two phases four slots would be necessary, and for three phases six slots, these numbers being multiplied by the number of pairs of poles if a multipolar field is required.

Flux Distribution with Distributed Winding.—It is not usual, however, to concentrate the whole of the winding for one pole into a single slot, the general practice being to distribute it over a number of slots. This modifies the shape of the flux wave to a certain extent, since the coils in the different slots embrace slightly different arcs. Consider a case where there are five slots per pole for one phase (see Fig. 338). The flux set up by each of the five coils will

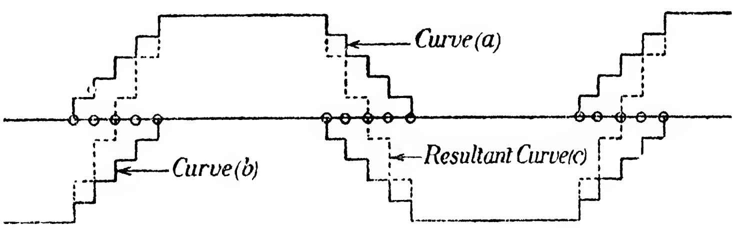


FIG. 338.—Flux Distribution with Distributed Winding.

be confined to the space lying between the two conductors of that coil. Since the current in each coil is the same, there will be five times the ampere-turns per coil acting on the space between the inner conductors. Between these and the next adjacent conductors there will only be four times the ampere-turns per coil, since the inner conductors are inoperative on their outside. Finally, in the space between the outer conductors and the next ones inside there is only that due to the single outside coil. The curve of flux distribution for this pole is shown in curve (*a*), Fig. 338. Next considering the flux over the adjacent pole pitch, the same considerations lead to a curve of flux distribution as shown in curve (*b*), and combining these two flux curves together the resultant flux distribution is obtained as shown in curve (*c*). Owing to the fringing of the lines, it is sufficiently near to consider the sloping zig-zag lines as straight lines, in which case the curve of flux distribution takes the form shown in Fig. 339. The horizontal distance between *a* and *b* is equal to the pole pitch, whilst that between *c* and *d* is equal to half the pole pitch in a two phase case and one-third the pole pitch in a three phase case. The space between *d* and *e* is filled up by the

remaining phases, each of which produces a component flux wave of its own. These component flux waves change sinusoidally with time and are displaced with respect to one another in space as well.

Change of Flux Distribution with Time.—The component flux set up by each phase varies sinusoidally with time, as was shown in

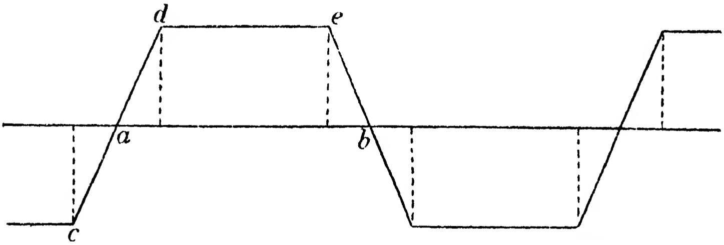


FIG. 339.—Resultant Flux Distribution for One Phase.

Fig. 337. Each phase produces a similar component flux different in phase and displaced in position, and the resultant flux is obtained by adding all these components together at any instant. The resultant flux travels along the air-gap at a speed equal to a double pole pitch per cycle.

Resultant Flux Distribution due to Two Phases.—If the whole of the surface is utilised by the magnetising winding, the coils for one phase will cover an arc corresponding to 90° electrical for each pole, since both phases cover an arc of 360° for a pair of poles. For example, in a two phase four pole field system the coils for one phase belonging to each pole would occupy an arc of 45° geometrical or 90° electrical, as shown in Fig. 340.

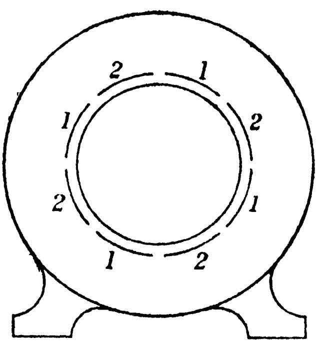


FIG. 340.—Two Phase Four Pole Field System.

The flux distribution for such a case is shown in Fig. 341. At (*a*) the current in the first phase is a maximum, whilst that in the second phase is zero. The current in the first phase now begins to decrease, whilst that in the second phase begins to grow, and 30° later the conditions are represented at (*b*). Calling the maximum height of the flux wave due to one phase unity, the two heights are now cos 60° = 0·866 and sin 30° = 0·5. The maximum flux density is now 0·866 + 0·5 = 1·366. At 45° from the start (*c*) is obtained. The maximum flux density is now cos 45° + sin 45° = 0·707 + 0·707 = 1·414. Up to this point the wave has been getting gradually more peaked, but from 45° to 90° it gets flatter again, until at 90° from the start it has regained its original shape and has

advanced 90° electrical along the air-gap. The same series of events is then repeated every quarter-cycle.

It is seen that the flux distribution is not constant, but is concentrated and spread out alternately. This can be seen from Fig. 342,

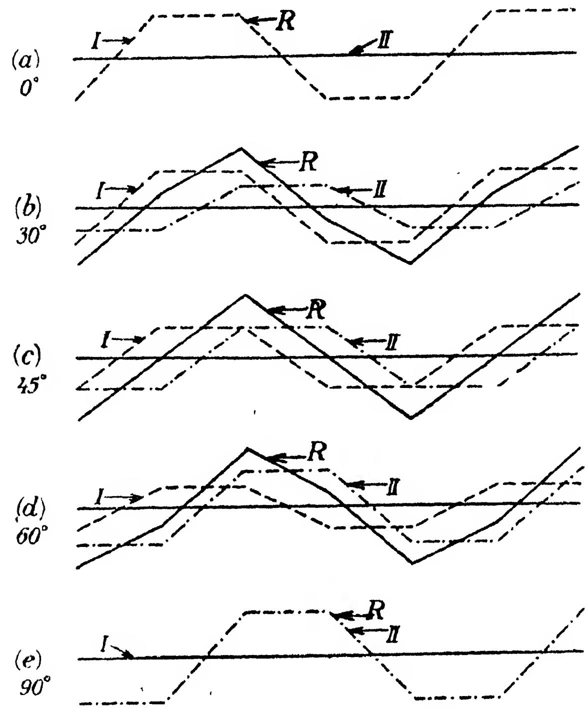


FIG. 341.—Flux Distribution with Two Phase Winding.

which shows the resultant flux distribution every 15° throughout a quarter of a cycle, the change in shape of the curve and its gradual advance being shown. In addition to the change in its distribution, the flux undergoes a slight change in magnitude which is propor-

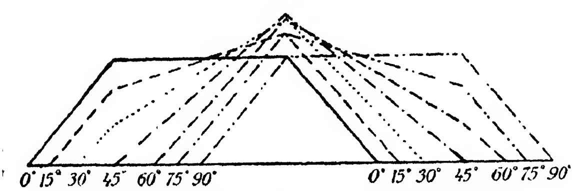


FIG. 342.—Gradual Advance of Flux (Two Phase).

tional to the area enclosed between the curve and the axis. The total flux has a maximum value when the current in one phase is a maximum. Its minimum value, which is about 6 per cent. less, occurs when the two currents are equal 45° later. The absolute

constancy of the flux as worked out on p. 382 is thus only correct for concentrated windings which are not used in practice. The distribution of the winding also causes a concentration of the flux at certain instants and results in the instantaneous maximum flux density being 1·41 times that due to one phase alone. This further reacts on the wave form of the current and distorts it to a certain extent by introducing harmonics.

Calling the height of the flat topped wave unity, the average flux density, which is given by $\frac{\text{area}}{\text{base}}$, is 0·75 when the wave is flat topped and 0·71 when the wave is peaked (triangular), having a maximum value of 1·41. The mean value is $\frac{0{\cdot}75 + 0{\cdot}71}{2} = 0{\cdot}73$, and the ratio of maximum to average flux density is, therefore, $\frac{1{\cdot}41}{0{\cdot}73} = 1{\cdot}94$.

Resultant Flux Distribution due to Three Phases.—Each of the phases will now utilise one-third of the available winding space, so that the coils cover an arc of 60° instead of 90° in the two phase case. The flux distribution is shown in Fig. 343. At (*a*) the current in phase I is a maximum, and hence the currents in phases II and III are equal to half their maximum values, but are in the reverse direction. Advancing by 120° to the right of the flux wave due to phase I, the flux wave due to phase II is plotted. This is in the reverse direction and has only half the amplitude, since a sine wave has a value of — 0·5 when it has passed its maximum value in the positive direction by 120°. Advancing by another 120° to the right, the flux wave due to phase III is obtained. This is also in the reverse direction and again has only half its maximum amplitude, since a sine wave also has a value of — 0·5 when it has passed its maximum value by 240°. Adding these three component flux waves together, the resultant flux wave is obtained. This wave approximates to a sine wave, but is rather more peaked.

As the current advances in phase the wave form gradually changes, Fig. 343 (*b*) representing the conditions 15° after it has passed its maximum The resultant flux wave has now advanced to the left and has become incidentally a closer approximation to a sine wave. Fig. 343 (*c*) represents the conditions 30° from the start. The current in phase III has now dropped to zero, whilst phase I has $\frac{\sqrt{3}}{2}$ times its maximum positive value and phase II has $\frac{-\sqrt{3}}{2}$ times its maximum positive value. The resultant wave form has now become slightly more flat-topped than a sine wave, after which it tends to get a little more peaked again. The flux wave thus oscillates between the two extremes shown in Figs. 343 (*a*)

and (c) every 30°, there being six peaked and six flat-topped values every cycle. The changes in the distribution of the flux are thus seen to be less than in the two phase case, slight variations in magnitude occurring in both instances.

Maximum Resultant Flux.—The maximum resultant flux density is twice the maximum due to each phase separately, the value $\frac{3}{2}$

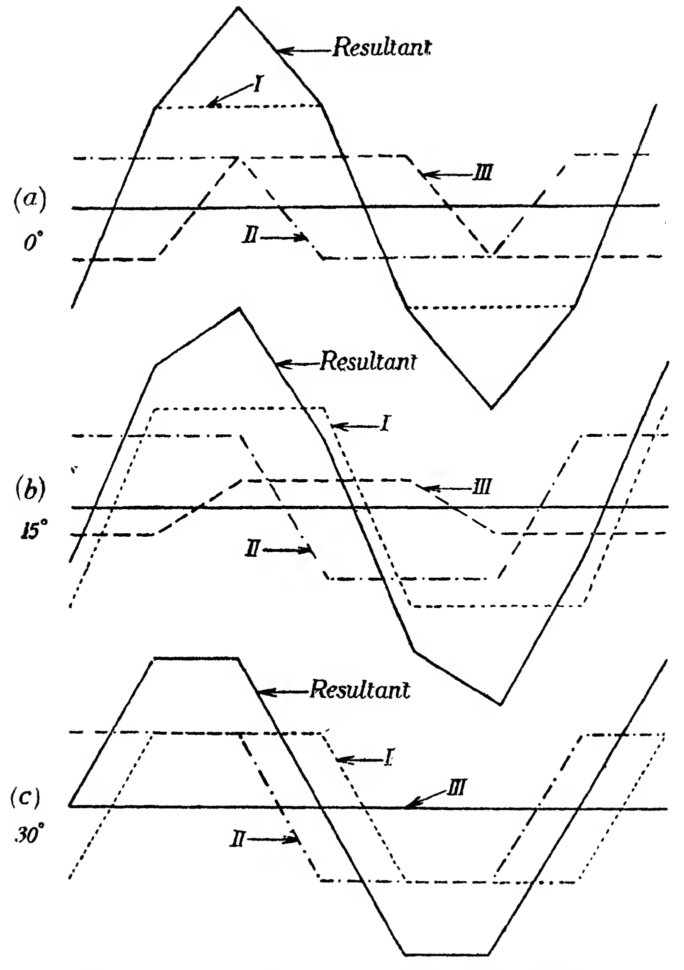


FIG. 343.—Flux Distribution with Three Phase Winding.

obtained on p. 384 only being correct for a concentrated winding. The distribution of the winding thus has the effect of raising the maximum flux density.

The total flux is proportional to the area of the curve and only varies by about 1 per cent. between its extreme values. Calling the maximum flux density due to each phase unity, the average height of the wave is 1·17. The ratio of maximum to average

flux density is therefore $\frac{2}{1\cdot17} = 1\cdot71$ instead of 1·57 for a pure sine wave.

Effect on Magnetising Current.—In order to produce a given total flux in a given air-gap, the average flux density is fixed. The effect of distributing the winding is therefore to raise the maximum flux density, and as the magnetising current is estimated from this quantity it follows that this also is increased. The actual increase in the R.M.S. value of the magnetising current is about 8 per cent., since $\frac{1\cdot71}{1\cdot57} = 1\cdot08$. The current wave form is also distorted to a certain extent.

Alternating Field produced by Single Phase Current.—A rotating field cannot be set up by a single phase current, the field in this case merely alternating in synchronism with the magnetising current. To produce the rotating effect, it is necessary to have at least one other phase, but it is not necessary that the currents should be equal. If they are unequal it means that the rotating field will not be constant in magnitude as it rotates.

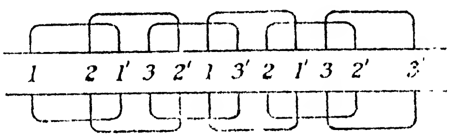


FIG. 344.—Three Phase Travelling Field.

Travelling Field.—Imagine the stator shown in Fig. 340 to be cut open and straightened out. The magnetic field will then appear to travel from one end to the other instead of rotating. In order to strengthen the effect, another winding may be added on the other side of a short air-gap, as shown in Fig. 344. The poles formed by the second winding must occur opposite to those of the first and at the same instant, but, of course, a south pole must lie opposite to a north. The corresponding phases of these two windings are connected in series.

Reversal of Rotation.—In order to reverse the direction of rotation, the connections of any two phases are interchanged. If this is done in a three phase case, it is seen that instead of advancing in the order 1, 2, 3, the field advances in the order 3, 2, 1. The same argument holds good for the travelling field, which then moves from right to left instead of from left to right.

CHAPTER XXVI

INDUCTION MOTORS.—PRINCIPLES AND CONSTRUCTION

General Construction.—An induction motor consists of two main parts, a stator and a rotor. The stator consists of a number of stampings slotted so as to receive the windings, these stampings being held between two end plates, which are carried from the outer carcase of the motor. This outer carcase serves merely as a mechanical protection for the stampings and does not carry any flux, as is the case in the outer yoke of a C.C. motor. The stator winding is called the primary, and is connected to the supply, producing the rotating field on which the induction motor depends for its action.

The rotor consists of another set of stampings mounted on a spider, which is carried from the shaft. These stampings are also slotted and receive the rotor winding, which is called the secondary, since it has currents induced in it, as in the case of the transformer. Since there are no salient pole pieces, a uniform air-gap is obtained all the way round, this being, in general, smaller than in the case of a C.C. motor of similar size.

In some induction motors the rotor windings are short-circuited on themselves, having no external connections, whilst in others the rotor conductors are brought out to slip rings (usually three) and then joined through a resistance, which is used for starting up.

Production of Rotation.—The primary (stator) winding is supplied with a polyphase current in order to produce the rotating field, and this field, issuing from the stator stampings, crosses the air-gap and enters the rotor. It there cuts the rotor conductors, which form the secondary winding, and induces an E.M.F. in them. The rotor circuit being closed, this E.M.F. sets up a current in the rotor, which consequently absorbs a certain amount of power, this reappearing as heat. The winding then tends to place itself in such a position that it generates a minimum amount of electrical power, and in order to do this it follows the rotating field and commences to rotate. The induced E.M.F. is thereupon reduced, since this is proportional to the rate at which the rotor conductors are cut by the rotating field, and this in turn causes a drop in the rotor

current. The power wasted in the rotor circuit is thus reduced, when rotation is set up.

Another way of looking at the problem is to consider the interaction of the fluxes set up by both the stator and the rotor windings. Fig. 345 (*a*) represents a portion of the air-gap showing the stator flux by itself rotating in a counter-clockwise direction. These lines of force are cutting the rotor conductors from right to left, and this is equivalent to a movement of the rotor conductors from left to right. E.M.F.'s are induced in these rotor conductors which are short-circuited, producing currents flowing away from the observer. A hypothetical secondary flux is thus set up, consisting of clockwise lines of force surrounding the rotor conductors, as shown in Fig. 345 (*b*). Combining the stator and rotor fluxes,

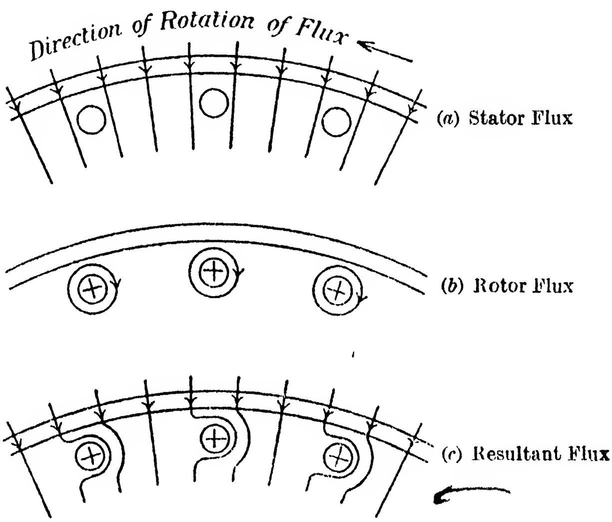


FIG. 345.—Distortion of Flux by Rotor Current.

we get the resultant flux actually existing. This is shown in Fig. 345 (*c*), and as the field tends to straighten itself out the rotor conductors are urged in a counter-clockwise direction and follow the field in its rotation.

The same result can be obtained straight away by applying Fleming's left-hand rule, which shows that the rotor conductors are subjected to a motoring force in a counter-clockwise direction.

Slip.—The rotating field revolves with the speed of synchronism, and if the rotor conductors were to follow at exactly the same speed there would be no relative movement of the field and the rotor. In this case, there would be no E.M.F. induced in the rotor and the rotor current would drop to zero. The distortion of the field causing the motoring action now disappears, and the rotor immediately commences to slow down. As soon as it does this, however,

the rotor conductors begin to be cut by the rotating field at a rate depending upon the difference in the speeds of rotation of the flux and the rotor. The slower the speed of the rotor the greater is the rotor E.M.F. and current, and, consequently, the greater is the motoring force. Actually, the speed of the rotor adjusts itself, so that the magnitude of the rotor current is just sufficient to exert the necessary torque to overcome the mechanical resistance to motion of the rotor. If it is running too fast, the rotor current is too small to maintain the rotation, and the speed falls. If it is running too slow, the rotor current is greater than necessary, and produces an acceleration which speeds the rotor up.

The difference between the speed of the rotating field and the actual speed of the rotor is called the *slip* of the motor, and may be expressed in revolutions per minute, but a much more common way is to express it as a percentage of the synchronous speed. With modern induction motors, the slip generally lies between zero and about 5 per cent.

On account of the fact that it must run at a speed rather less than that of synchronism, the induction motor is sometimes termed an *asynchronous motor.*

If f represents the frequency of supply and n the revolutions per second of the rotor, then the frequency of slip is $(f - n)$ in a two pole motor and $(f - pn)$ in a multipolar motor, where p is the number of pairs of poles. Expressing this as a fraction of the supply frequency, it becomes $\frac{f - pn}{f}$ or $\frac{f - pn}{f} \times 100$ per cent.

Rotor Current.—When the rotating field is set up by switching on the stator current, the motor acts like a polyphase transformer with a short-circuited secondary. A current is produced in the rotor winding, which is usually wound for three phases, and this develops the torque necessary to set the rotor in motion. The magnitude of this current depends upon the induced E.M.F. per phase and also upon the impedance per phase. The induced E.M.F. is proportional to the rate of cutting lines of force, and this in turn is proportional to the difference in speed of the rotating field and the rotor, *i.e.* the slip. (The resistance of the rotor winding is constant, but the reactance also depends upon the slip, since the frequency of the rotor currents is the same as the frequency of slip.) If the slip is doubled the induced E.M.F. is also doubled, but the impedance is not doubled, although it is very materially increased. As a result, the rotor current is increased, but it does not increase proportionally to the slip. Another effect of the reactance in the rotor circuit is to cause the rotor current to lag behind the induced rotor E.M.F. by a certain angle, this having the twofold effect of reducing the power factor and the torque developed per ampere.

Stator Construction.—The construction of the stator of a modern induction motor follows the general lines of that of the stator of a

rotating field alternator of a similar size (see Figs. 201, 202 and 203). It consists in the main of a ring of stampings held between end cheeks and supported from the outer casting, which serves only as a mechanical support and fulfils no magnetic function. This outer casting is perforated with a number of large holes for ventilation and supports the two end shields, which also carry the bearings. These are of the rigid type, since the spherical seating adopted in C.C. motors is not suitable on account of the much smaller air-gaps employed in induction motors. In the larger sizes pedestal bearings are adopted. The stator is wound in the same way as

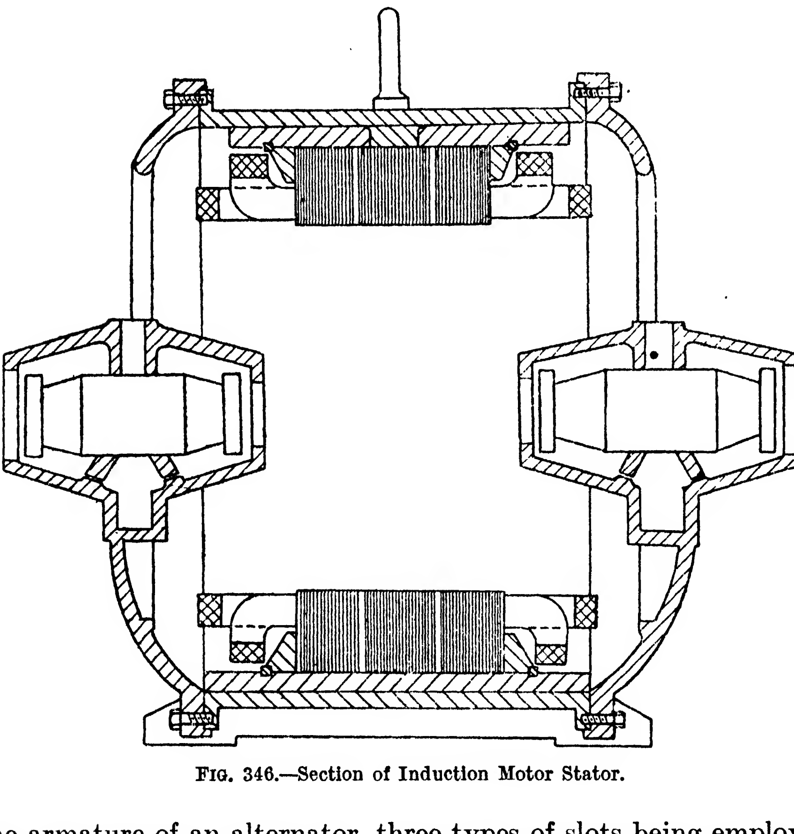


FIG. 346.—Section of Induction Motor Stator.

the armature of an alternator, three types of slots being employed, as shown in Fig. 210. Of these, the open and semi-closed are most used on account of the greater ease of winding and the reduced leakage flux which is set up. Radial ventilating ducts are spaced every few inches along the stator core, these being arranged opposite to those in the rotor, so that a through passage is provided for the air right from the centre of the machine to the outside.

A section of a typical induction motor stator is shown in Fig. 346.

Stator Winding.—The stator can be either bar- or coil-wound,

exactly as in the case of alternators. The winding also is distributed over a number of slots per pole so as to utilise the surface of the stator to a greater extent. The appearance often differs considerably from that of alternators, inasmuch as induction motors generally have a smaller number of poles. It is usual to have one coil per pair of poles for each phase, so that a four pole three phase motor will have six coils in all. The arrangement of the coils in this case is indicated in Fig. 347, which shows three shapes of coils. These have an actual space displacement of 30° (or 60° electrical), but the connections to the middle coil are reversed in order to get the correct phase angle. This arrangement enables the winding to be split up into two portions, so that in case of a burn-out in one coil it is not necessary to strip the whole stator winding.

The practice of winding one coil per pair of poles is not adopted in the case of two pole motors, as it would mean that a very large bunch of end connections would have to lie on top of one another. By using two coils per pair of poles, no extra ampere-turns are

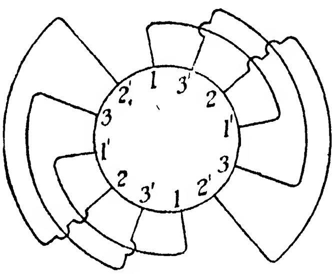


FIG. 347.—Arrangement of Coils in Four Pole Three Phase Stator.

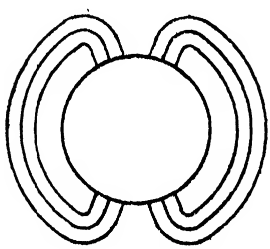


FIG. 348.—End Connections for Two Pole Three Phase Stator.

required, and half the end connections are carried round one side of the stator, whilst the other half are carried round the other side. The end connections for one phase of a three phase two pole stator having six slots per pole per phase are shown in Fig. 348.

Construction of Wound Rotors.—The usual type of wound rotor consists of a ring of sheet iron stampings mounted directly on the shaft in the smaller sizes and built on to a cast iron spider in the larger sizes. The rotor winding is carried in slots arranged along the outer periphery. Ventilating holes are punched in the plates to provide an entrance for the air, which is thrown outwards radially by means of the ventilating ducts. The three phase winding is connected in star, and the three ends are brought out to three slip rings. In small motors, these usually lie on the outside of the bearing, the shaft being made hollow to allow the three conductors to pass through the bearing. The current is collected from these slip rings by means of carbon or copper brushes, from which it is led to a resistance which is used for starting purposes. When the

motor is running, these slip rings are short circuited by means of a collar, which is pushed along the shaft and connects all three slip rings together on the inside. The brushes are provided in a large number of cases with a device for lifting them from the slip rings when the motor has been started up, thus reducing the wear and the frictional loss.

Rotor Winding.—The simplest form of rotor winding is a coil winding of the same type as that employed on the stator. There is an exact number of slots per pole per phase, with one coil for each phase per pair of poles.

For heavier currents a bar winding is adopted in which the end connections take on the familiar appearance shown in a C.C. motor. Owing to the number of slots being a multiple of the phases times the poles, a certain amount of asymmetry is set up in the winding. For example, each end connection spans a pole pitch, but when the winding has travelled once round the armature it would close on

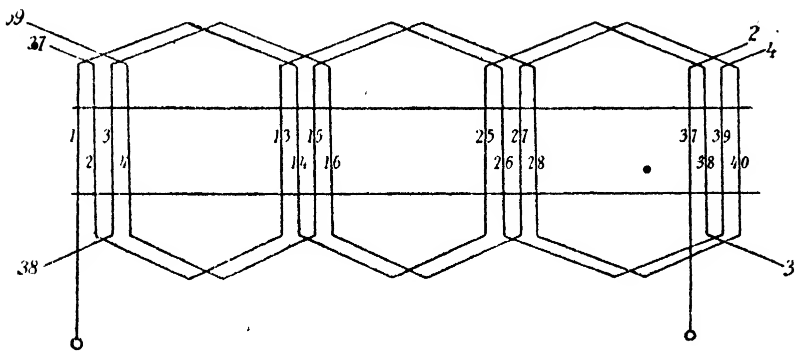


FIG. 349.—Rotor Bar Winding.

itself, and to avoid this an unsymmetrical end connection is included here, so as to enable all the conductors in one phase to be connected in series.

As an example, consider a three phase four pole winding having 24 slots in all with two conductors per slot arranged one on top of the other. There are thus 8 slots per phase and 2 slots per pole per phase. Imagine the conductors to be numbered consecutively 1, 2, 3, 4, etc., round the rotor, the odd numbers being at the top and the even numbers at the bottom of a slot. The mean pitch is 12, but an odd number must be adopted so as to enable both odd and even numbers to be included in the winding. A winding pitch of 13 and 11 will therefore be used alternately. The conductors are then connected up in the order 1, 14, 25, 38, but instead of using a short end connection and going to 49 (which is 1), a long one is used, taking the winding to 3. The winding then proceeds 3, 16, 27, 40. The winding must now retrace its path, because if it goes on it trespasses on the next phase, which commences at

conductor No. 5. An unsymmetrical end connection going to 28 (employing the mean pitch) is now used, and the remaining half of the phase is wound backwards, just like the first half. One end of the winding is connected to one of the slip rings and the other to the star point. One phase of this winding is shown in Fig. 349.

Squirrel Cage Rotors.—A simple and effective form of rotor is that known as the *squirrel cage*, which consists of a laminated core as in the wound rotor with a single conductor lying in each slot. These conductors consist of heavy copper bars lightly insulated from the core and short circuited at each end by means of a pair of stout end rings of phosphor bronze or brass. Each conductor is riveted or screwed on to both end rings, the joints being soldered in addition. The whole rotor winding thus consists of a permanently short circuited system having as many phases as conductors. Fig. 350 shows a section of a typical small squirrel cage rotor.

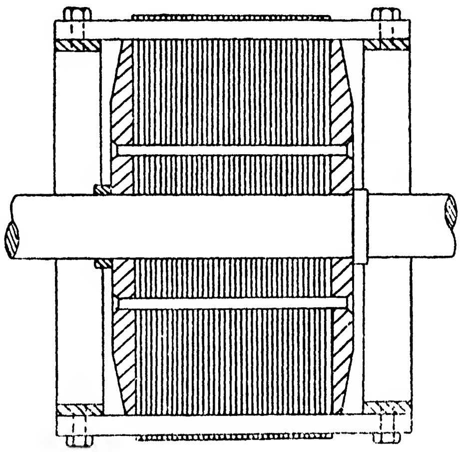


FIG. 350.—Squirrel Cage Rotor.

The great advantage of such a rotor lies in the soundness and simplicity of the mechanical design. There are no moving contacts at all, and the construction of the whole squirrel cage rotor is exceedingly simple. Its great disadvantage lies in the small starting torque and large starting current which is characteristic of the type.

Torque.—The torque developed by the rotor is proportional to the instantaneous product of the rotor current and the strength of the magnetic field cutting the rotor. If the whole motor, stator and rotor together, be imagined to rotate in the *opposite* direction to that of the rotating field and at the same speed, the field will appear to stand stationary in space and will have an approximately sinusoidal distribution over the air-gap. The rotor will appear to have a slow rotation in the opposite direction to that in which it is really rotating, the speed of this slow rotation being that of the slip. The

induced rotor E.M.F. will have a sinusoidal wave form, the maximum value occurring when it passes the centre line of the pole where the maximum flux density occurs. Owing to the rotor reactance, however, the maximum current will not occur until a little later, so that the field and the current are not quite in phase. The average product of these two quantities, which is proportional to the torque, is therefore reduced, just as in the same way that the power in a circuit is reduced when the current is made to lag behind the voltage. The amount of this reduction is given by $\cos\theta$, where θ is the angle of lag of the current behind the voltage in the rotor. Reactance in the rotor circuit is thus seen to be harmful, inasmuch as it reduces the torque which is developed.

Let e and i represent the induced rotor voltage and current per phase, f_s the frequency of slip, R and L the rotor resistance and inductance per phase respectively. Then

$$e = kf_s,$$

where k is a constant.

$$i = \frac{kf_s}{\sqrt{R^2 + (2\pi f_s L)^2}}.$$

Since $$\cos\theta = \frac{R}{\sqrt{R^2 + (2\pi f_s L)^2}},$$

the torque is proportional to

$$i\cos\theta = \frac{kf_s}{\sqrt{R^2 + (2\pi f_s L)^2}} \times \frac{R}{\sqrt{R^2 + (2\pi f_s L)^2}}$$
$$= \frac{kRf_s}{R^2 + (2\pi f_s L)^2}.$$

The maximum value of this expression is obtained when $R = 2\pi f_s L$,[1] when the maximum torque becomes

$$T_m = \frac{kRf_s}{R^2 + R^2} = \frac{kf_s}{2R} \text{ or } \frac{kf_s}{4\pi f_s L} = \frac{k}{4\pi L}.$$

Relation between Slip and Torque.—If the resistance and inductance of a given rotor be kept constant, the magnitude of the torque depends solely upon the slip, provided the magnitude of

[1] The maximum torque is obtained by differentiating with respect to f_s and equating to zero; thus:

$$\frac{\{R^2 + (2\pi f_s L)^2\} \times kR - kRf_s \times 2 \times 2\pi f_s L \times 2\pi L}{\{R^2 + (2\pi f_s L)^2\}^2} = 0.$$

$$\therefore \quad \{R^2 + (2\pi f_s L)^2\} \times kR = kRf_s \times 2 \times 2\pi f_s L \times 2\pi L.$$
$$R^2 + (2\pi f_s L)^2 = 8\pi^2 f_s^2 L^2.$$
$$R^2 = 8\pi^2 f_s^2 L^2 - 4\pi^2 f_s^2 L^2$$
$$= 4\pi^2 f_s^2 L^2.$$
$$R = 2\pi f_s L.$$

the rotating field is also constant. For low values of the slip, the reactance is negligible compared with the resistance, and the expression $\frac{kRf_s}{R^2 + (2\pi f_s L)^2}$ becomes approximately $\frac{kRf_s}{R^2}$, so that the torque is practically proportional to the slip. For large values of the slip, the reactance is large compared with the resistance, so that the expression for the torque becomes approximately $\frac{kRf_s}{(2\pi f_s L)^2}$ or $\frac{kR}{4\pi^2 L^2 f_s}$. The torque is now approximately inversely proportional to the slip. As the slip increases from zero, therefore, the torque at first increases, then reaches a maximum, when $R = 2\pi f_s L$ and then decreases.

As an example, consider the case of a motor operating on a frequency of 50, where $T_m = 1$, $k = 1$, and $R = \frac{5}{4}$.

$$\text{Since} \qquad T_m = \frac{k}{4\pi L}, \quad L = \frac{k}{4\pi T_m} = \frac{1}{4\pi}.$$

The expression $\frac{kRf_s}{R^2 + (2\pi f_s L)^2}$ can now be evaluated for various values of f_s, and this has been done in the second line of the following table. Next let the resistance be increased to four times its original value, viz., $R = 5$. The new values of the torque are shown in the third line in the table, from which it is seen that the torque now attains a maximum value for four times the slip in the previous case.

f_s	0	1	2	2·5	5	10	15	20	30	40	50
Torque for $R=\frac{5}{4}$	0	0·69	0·98	1·00	0·80	0·47	0·32	0·25	0·17	0·12	0·10
Torque for $R=5$	0	0·20	0·38	0·47	0·80	1·00	0·92	0·80	0·60	0·47	0·38

The above figures are plotted in Fig. 351, which shows the general shape of the torque-slip curves.

The value of the slip for maximum torque depends upon the relative values of R and L. If R is low compared with L, the maximum torque occurs with a small slip, and *vice versa*. For a given rotor, the inductance is fixed, but if it were possible to reduce it the maximum torque would be increased and would occur at an increased slip, assuming a constant resistance. A reduction of both resistance and inductance in the rotor would therefore increase the maximum torque and keep down the slip.

Relation between Rotor Resistance and Torque.—It was shown on p. 399 that the torque developed in the rotor was proportional to

an expression which included, amongst other terms, the rotor resistance. Of course the resistance of the rotor itself is a fixed quantity, but by inserting an external resistance in series with each phase the total amount can be varied. The maximum value of the torque is independent of the resistance, but the slip at which this maximum torque is obtained is proportional to the total resistance per phase. A means of speed regulation is thus obtained by varying the rotor resistance, since the maximum torque can be obtained at any speed from standstill up to that of synchronism simply by varying the rotor resistance. When no resistance is inserted at all, the rotor runs at full speed, and to increase this speed, *i.e.* decrease the normal slip, it would be necessary to decrease the rotor resistance, which is only possible by re-designing the winding.

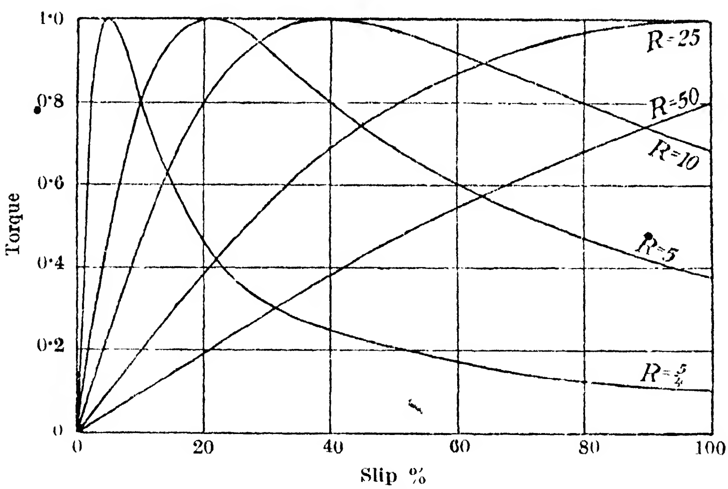


FIG. 351.—Torque-slip Curves.

If the load on the shaft necessitates a certain torque being developed, an increase in the rotor resistance will cause a corresponding increase in the slip and thus bring the speed down. To illustrate the relationship which exists between the rotor resistance and the slip for a given torque, three other curves have been drawn in Fig. 351, corresponding to $R = 10$, $R = 25$, and $R = 50$. All these curves obey the same law, their only difference being that they attain their maximum values at different points. In fact the last curve for $R = 50$ never does reach its maximum by the time standstill is reached, but it would attain the same height as the others at the hypothetical slip of 200 per cent., which, of course, is impossible in ordinary circumstances.

Another series of curves can be derived from those shown in Fig. 351, giving the relation between the rotor resistance and the

slip for a given torque. These are obtained by drawing a horizontal line through, say, 0·5 × maximum torque and noting the corresponding values of the rotor resistance and the slip. These curves are drawn in Fig. 352 for values of the torque equal to 0·25, 0·50, 0·75, and 1·00 times the maximum.

Speed Regulation.—In ordinary circumstances, an induction motor is practically a constant speed machine, since the slip is very small even at full load. In this respect it is comparable with the C.C. shunt motor, the speed of which also drops very slightly as the load comes on. The introduction of the rotor resistance, however, causes a reduction in speed, enabling all values from standstill up to very nearly synchronous speed to be obtained. It is not possible to obtain an increase in speed in this way, since the motor must run at a speed less than that of synchronism. The only way in which this can be done is by increasing the frequency, which is not

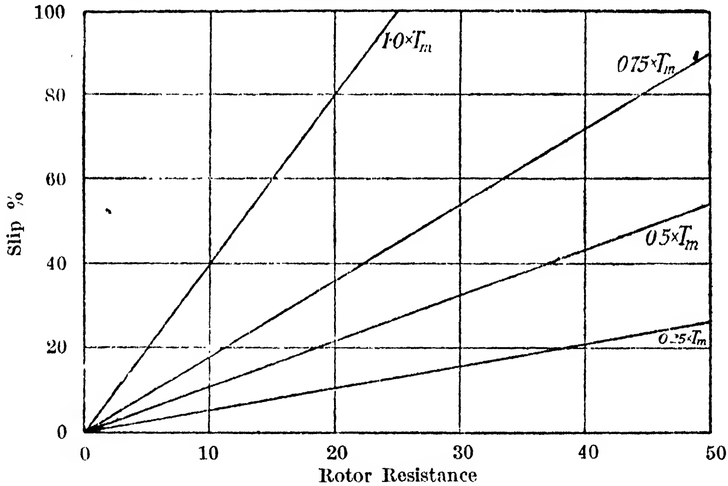


FIG. 352.—Relation between Slip and Rotor Resistance.

practicable, or by changing the number of poles. This is done in some instances, but it usually leads to a poorer performance of the motor.

The insertion of the additional resistance is brought about by bringing the three ends of the three phase rotor to the slip rings, from which external connections are made to the three phase variable resistance.

In the case of squirrel cage rotors, it is not possible to insert external resistance in circuit, and so no speed regulation is possible other than the small natural drop caused by the increase of load.

Starting Torque.—At the moment of switching on, the frequency of slip is the same as the frequency of supply, and thus the starting torque is obtained by substituting f for f_s in the expression for the torque, which now becomes

$$T_s = \frac{kRf}{R^2 + (2\pi fL)^2}.$$

The only practicable way to vary this is again to vary the resistance, and there is one particular value of R which will give the maximum starting torque. This is obtained when $R = 2\pi fL$, f being the frequency of supply. Not only should the rotor resistance and reactance be equal, but they should both be small if a large starting torque is desired, since the maximum torque obtainable is inversely proportional to the inductance. Both a higher and a lower rotor resistance result in a decreased starting torque, since if a higher resistance is employed the increased impedance brings the rotor current down, whilst if a lower resistance is employed the reduction in the power factor more than counterbalances the increase in the current. The case is, in fact, similar to that discussed on p. 71, where it was shown that the maximum power in a circuit occurs when the angle of lag is 45° and the resistance equals the reactance.

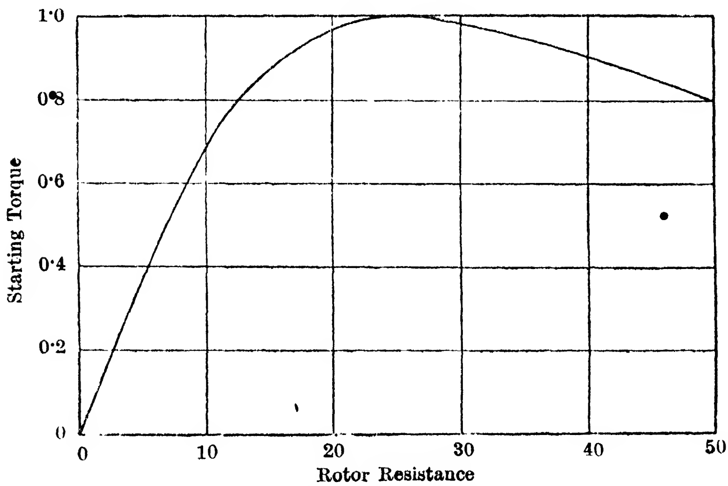


FIG. 353.—Relation between Starting Torque and Rotor Resistance.

In the present problem it is desired to get a rotor current which has a maximum component in phase with the field.

Considering the same example taken on p. 400, the values of the starting torque have been worked out for various resistances, and these values are tabulated in the following table and plotted in Fig. 353.

R	1	5	10	20	25	30	40	50
T_s	0·08	0·38	0·69	0·97	1·00	0·98	0·90	0·80

Starting Resistances.—The three phase resistances used for starting up induction motors consist of three separate variable

resistances joined together by means of a three-armed handle which forms a star point. Three movable contacts are thus formed, rigidly attached to one another so as to make them move in unison, and these move over three rows of contacts. The arrangement of the various circuits is shown in diagrammatic form in Fig. 354, whilst the internal connections of a starting resistance are shown in Fig. 355. The three terminals make connection with the ends

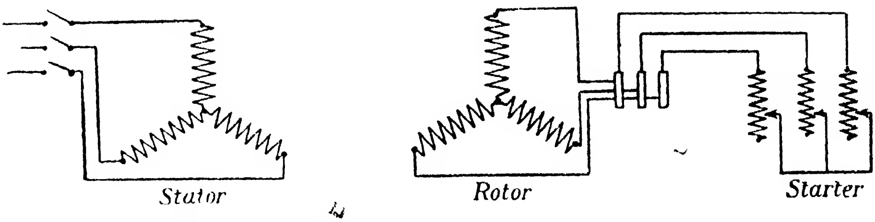


FIG. 354.—Diagram of Connections of Induction Motor.

of the three resistances and are joined to the three slip rings. The rotation of the starting handle is limited to approximately one-third of a revolution by means of stops, the position shown being that when the supply is first switched on, all the resistance being in. To start the motor up, the main switch is closed and the starting handle slowly moved round in a clockwise direction through approximately 120°. The resistance is then all cut out, only that of the leads and switch contacts remaining. As, however, these are usually comparable with the resistance of the rotor itself, it is necessary to short circuit the rotor at the slip rings themselves as well. This is done by means of a metal collar, which is forced along the shaft under the slip rings, touching them all, thereby eliminating the resistance of the brush contacts on the slip rings, which is quite appreciable. The brushes are then raised to reduce the wear and the frictional loss.

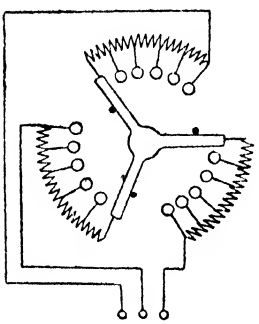


FIG. 355.—Internal Connections of Starting Resistance.

Another type of starter is the liquid resistance. This consists of a tank containing the liquid into which three blades, B (see Fig. 356), are dipped gradually by means of the handle, H, until they engage in three short circuited contacts, C, at the bottom. The short circuiting device at the slip rings is then employed as before.

Auto-transformer Starters.—The use of the starting resistance is not possible in the case of squirrel cage rotors, and so other means must be adopted to limit the starting current. This is usually done by means of an auto-transformer starter, which consists, as its name implies, of an auto-transformer of which the primary is

connected to the line, whilst the secondary, giving a reduced voltage, is connected to the stator of the induction motor. The effect of the reduced voltage on the motor is to reduce, proportionally, the strength of the rotating field. This in turn reduces the E.M.F. generated in the rotor circuit, and hence the rotor current also. Since the torque developed is proportional to the product of the rotor current and the strength of the rotating field, it is seen that it is proportional to the square of the voltage applied to the stator. A reduction in the applied voltage at starting, therefore, causes a considerable drop in the starting torque, but is necessary in view of the reduction in the starting current which it also brings about. By the action of the auto-transformer, however, the current drawn from the line is less than that supplied to the motor, so that an excess current can be taken by the motor without an overload being put upon the mains. For example, consider a motor which would take six times full load current if suddenly thrown straight on to the mains, developing twice full load torque in so doing. If the

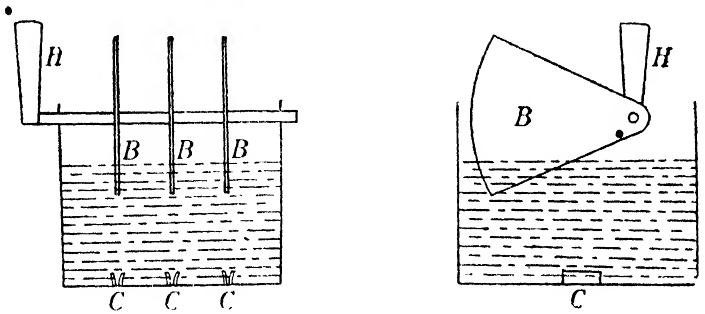


FIG. 356.—Liquid Starter.

auto-transformer had a ratio of 2 : 1 the motor current would be reduced to three times full load current and the line current to 1·5 times full load current. The torque would go down as the square of the voltage ratio, and would thus be only 0·5 times full load torque. This is a very inferior starting torque, but it is practicable, whereas without the auto-transformer, the motor would blow its fuses. To obtain full load starting torque, the motor voltage must be reduced to $\frac{1}{\sqrt{2}} = 0{\cdot}71$ times the line voltage. The motor current is now $6 \times 0{\cdot}71 = 4{\cdot}26$ times full load current and the line current $0{\cdot}71 \times 4{\cdot}26 = 3$ times full load current. The values of the starting torque available, together with the corresponding values of the voltage applied to the stator and the motor and the line currents, expressed as percentages of the normal full load values, for various tappings on the auto-transformer, are shown in Fig. 357. It is thus seen that the squirrel cage induction motor is not very suitable for starting up under full load torque

conditions in view of the heavy current taken, slip ring motors being much preferable in such cases.

The auto-transformer may have a number of tappings on it, but it is usual, particularly in the smaller motors, to use one tapping only, this being determined experimentally as the one giving the best all-round starting properties. The auto-transformer is then

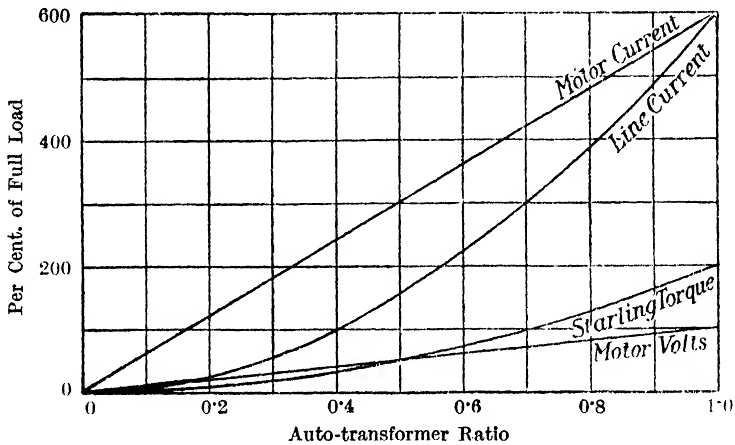


FIG. 357.—Effect of Auto-transformer.

provided with a change-over switch, one position being for starting and the other for running, as shown in Fig. 358. To start up the motor the switch is closed on the "starting" side, and when a certain speed is attained it is thrown over quickly to the "running" side.

Star-delta Connections.—Another method of reducing the voltage

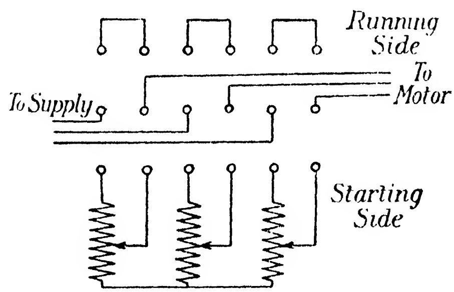


FIG. 358.—Auto-transformer Switch Connections.

on an induction motor at starting is to connect the phases in star for starting and in delta (or mesh) for running. In this way the volts per circuit are reduced to $\frac{1}{\sqrt{3}}$ times the line volts at starting, whilst when running each circuit receives the full line pressure. The motor operates as if an auto-transformer were used having a

voltage ratio of $\sqrt{3}:1$, the motor receiving 57·7 per cent. of the full voltage at starting. Since the torque is proportional to the square of the applied voltage, this arrangement results in a reduction in the starting torque to one-third of its value when switched on direct, whilst the current per phase winding is reduced in the ratio of $\sqrt{3}:1$. Since the line current is $\sqrt{3}$ times the current per phase winding when delta connected, the line current at starting is also one-third of what it would be if switched on direct. The necessary operations may be performed by means of a small drum type controller, the connections of which are shown in Fig. 359.

The disadvantages of this method of starting are, (1) only one starting position is obtained, so that the method is not suitable for

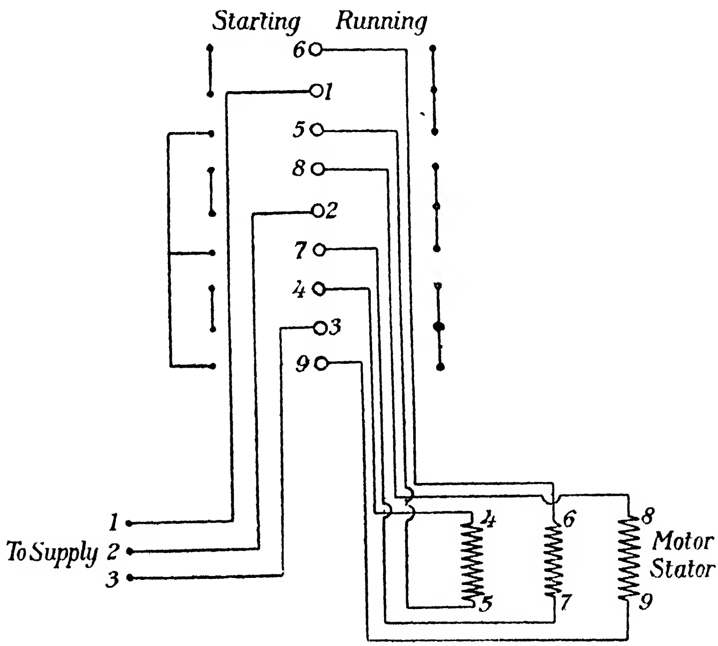


FIG. 359.—Star-delta Controller.

large motors; (2) an induction motor runs better with star than with delta connections, since any lack of electrical balance in the latter case causes circulating currents to flow round the stator winding; and (3) the motor starts up with a low torque.

Reversal of Rotation.—In order to reverse a two phase motor, all that is necessary is to reverse the connections of one of the stator phases, this causing the rotating field to rotate in the opposite direction. In the case of a three phase motor, any two of the stator leads must be interchanged, this having the same effect.

Output of Rotor.—Of the power supplied to the stator, a portion is wasted in stator copper loss and another portion in stator iron loss. The remainder is supplied to the rotor. The effect of the

copper loss is to absorb a portion of the applied voltage, the remainder, obtained by vectorial subtraction, being utilised for the production of the rotating field. This corresponds to that component of the primary voltage of a transformer which remains after the *IR* drop has been deducted. The power associated with this voltage is transferred to the secondary through the medium of the magnetic flux. The same thing applies to the induction motor. The stator current, also, may be divided into two components, one of which supplies the no-load magnetising current and the constant iron loss current. The other component provides the ampere-turns necessary to balance the ampere-turns set up by the rotor current. The power transferred to the rotor consists, therefore, of the product of this component of the current and the voltage producing the flux, multiplied by the cosine of the angle of phase difference between the two. Let I_s, E_s and $\cos\phi$ represent these quantities respectively, line values being considered. The power supplied to the rotor in a three phase case is therefore $\sqrt{3}E_sI_s\cos\phi$. If N is the ratio of the turns per phase on the stator to the turns per phase on the rotor, the voltage induced in the rotor at standstill is $\frac{E_s}{N}$. When the rotor is rotating with a frequency of slip, f_s, the induced voltage becomes $\frac{E_s}{N}\times\frac{f_s}{f}$.

The rotor current is NI_s and the angle of lag is the same as in the stator, as was explained when discussing the transformer. The power wasted in heating the rotor is therefore

$$\sqrt{3}\frac{E_s}{N}\times\frac{f_s}{f}\times NI_s\times\cos\phi$$

$$=\sqrt{3}E_sI_s\cos\phi\times\frac{f_s}{f}.$$

The power output of the rotor is obtained by subtracting the losses from the input and is

$$\sqrt{3}E_sI_s\cos\phi-\sqrt{3}E_sI_s\cos\phi\times\frac{f_s}{f}$$

$$=\sqrt{3}E_sI_s\cos\phi\times\frac{f-f_s}{f}.$$

The efficiency of the rotor is thus

$$\frac{\sqrt{3}E_sI_s\cos\phi\times\frac{f-f_s}{f}}{\sqrt{3}E_sI_s\cos\phi}$$

$$=\frac{f-f_s}{f}$$

$$=\frac{\text{actual speed}}{\text{synchronous speed}}$$

Similarly, the ratio of loss to input may be expressed by

$$\frac{f_s}{f} = \frac{\text{speed of slip}}{\text{synchronous speed}}.$$

Summarising, we get

$$\frac{\text{Output}}{\text{Input}} = \frac{\text{actual speed}}{\text{synchronous speed}}$$

$$\frac{\text{Loss}}{\text{Input}} = \frac{\text{speed of slip}}{\text{synchronous speed}}.$$

$$\frac{\text{Loss}}{\text{Output}} = \frac{\text{speed of slip}}{\text{actual speed}}.$$

It is thus seen that for a high efficiency the slip must be low. In fact, the efficiency is reduced by a percentage equal to the percentage slip.

In the above argument, the power necessary to supply the friction loss is included in the output of the rotor.

Magnetic Leakage.—Magnetic leakage occurs in both the stator and rotor. The stator leakage flux consists of a number of lines of force set up by the stator current, but not linking with the rotor winding, whilst the rotor leakage flux consists of another number of lines of force set up by the rotor current, but not linking with the stator winding.

The stator leakage causes the stator winding to possess inductance, and this absorbs a certain portion of the applied voltage, although there is no direct loss of power associated with it. This is equivalent to reducing the voltage producing the useful rotating field cutting the rotor. This in turn reduces the induced E.M.F. in the rotor, and hence the rotor current. Since the torque developed depends upon both the strength of the rotating field and the rotor current, the presence of stator leakage reduces the torque.

The effect of rotor leakage is to give inductance to the rotor winding. Owing to the increased impedance of the circuit, the rotor current is decreased, thereby decreasing the torque, whilst it also causes the rotor current to lag behind the rotor induced E.M.F. This was shown on p. 399 to result in a reduction of the torque as well, so that both the rotor and the stator leakage fluxes cause the output of the motor to be reduced and the power factor made worse.

When considering the performance of the induction motor, it is usual to imagine the whole of the leakage flux as being supplied from the stator winding, the rotor being non-inductive. This is permissible, since that portion of the induced rotor E.M.F. used up in overcoming the rotor reactance is supplied from the stator winding by transformer action. In fact, the whole of the rotor

effects come from the stator, since the latter receives the external electrical supply.

Number of Stator Conductors.—The number of stator conductors per phase required to produce the rotating field can be calculated in the same way as is done with transformers. The formula is (see p. 172)

$$E = 4{\cdot}44 f\Phi T \times 10^{-8}.$$

E is now the E.M.F. per phase, Φ the flux per pole, and T the turns per phase. Substituting conductors for turns, it becomes

$$E = 2{\cdot}22 f\Phi N \times 10^{-8}$$

and

$$N = \frac{E \times 10^8}{2{\cdot}22 f\Phi}.$$

The constant 2·22 only holds good when all the turns link with all the flux, and this is only obtained when the winding is concentrated in one slot. In a practical case, the winding is distributed over a number of slots, and this introduces a constant called the *Breadth Factor.*

Breadth Factor.—In the case of alternators, the effect of distributing the winding is to cause a slight reduction in the induced E.M.F.; in the case of induction motors, the effect of employing several slots per pole per phase is to reduce slightly the number of linkages set up. In order to generate the same flux, therefore, slightly more turns are necessary, which means that the constant 2·22 is slightly reduced. This reduction factor is called the *Breadth Factor,* and has a value of approximately 0·95 for commercial three phase motors and 0·9 for two phase motors.

The number of stator conductors per phase becomes, therefore,

$$N = \frac{E \times 10^8}{0{\cdot}95 \times 2{\cdot}22 f\Phi}$$

$$= \frac{E \times 10^8}{2{\cdot}1 f\Phi} \text{ for three phase motors}$$

and

$$N = \frac{E \times 10^8}{0{\cdot}9 \times 2{\cdot}22 f\Phi}$$

$$= \frac{E \times 10^8}{2 f\Phi} \text{ for two phase motors.}$$

Number of Rotor Conductors.—The number of rotor conductors bears no fixed ratio to those on the stator, since if a small number of turns are employed the cross section of the conductors can be made correspondingly large. The impedance of the short circuited winding is thus proportional to the square of the number of turns, assuming a constant copper space factor, whilst the induced E.M.F. is directly proportional to the number of turns. The current is

therefore inversely proportional to the number of turns, so that the total ampere-turns on the rotor are approximately constant, depending only on the available slot space.

Three Phase Rotor for Two Phase Motor.—It is not at all necessary for the rotor to be wound for the same number of phases as the stator, since the function of the stator winding is to set up a rotating field which can be obtained from any polyphase source. For example, a squirrel cage rotor is one which has as many phases as conductors, since the current in each bar has a slightly different phase from that in its neighbours.

Since three phase rotors only require three slip rings and three terminals, instead of four for two phase rotors, and have a slightly higher breadth factor, they are always used for two phase motors. Another very important advantage possessed by the three phase rotor, from the manufacturer's point of view, lies in the fact that one rotor can be utilised for both two- and three-phase machines, and since many more three phase motors are made than two phase, it is convenient to make all the rotors three phase.

Single Phase Induction Motor.—A single phase winding is only capable of producing an alternating magnetic field, and is not capable of producing a rotating field unaided. A single phase motor constructed on the rotating field principle must have some auxiliary help, therefore, to enable a rotating field to be set up. It is found that if a polyphase induction motor has all its phases broken except one, the motor will continue to run, although it will not start up again after having been shut down. The reason for this is that when running the induced currents in the rotor set up a true rotating field of their own which enables the rotation to be maintained, but the effect disappears when the motor comes to rest. This can be shown by considering Fig. 360, which indicates a two pole stator wound with a single phase winding and a rotor on which two short circuited windings are placed at right angles.

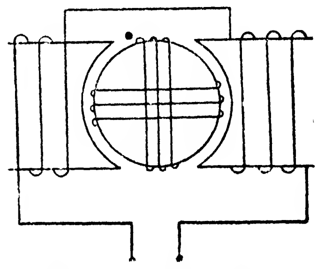


FIG. 360.—Single Phase Induction Motor.

First, consider the action of the vertical rotor winding, which acts like the secondary of a transformer and has a voltage induced in it proportional to the rate of change of the stator flux. This voltage is, therefore, in quadrature (*i.e.* 90° out of phase) with the flux. The horizontal coil has no induced voltage due to transformer action, since the turns do not link with the stator flux, but owing to the rotation of the rotor these turns cut the stator flux and have a voltage induced in them. This voltage is in phase with the stator flux. The two sets of turns will produce two fluxes having a space displacement of 90° and differing in phase by 90°. The

combination of these two sets up the desired rotating field. It is not, of course, necessary to confine the rotor winding to two simple coils at right angles, since any polyphase winding will produce the same result. In fact, a squirrel cage rotor is quite satisfactory. Since the whole of the magnetisation must be supplied from the one phase, the magnetising current is larger than in a similar polyphase motor, and the behaviour of the machine under load is very inferior as regards efficiency, power factor and overload capacity.

Starting of Single Phase Induction Motors.—In order to enable the motor to start up, it is necessary to produce an initial rotating field, and this is done by means of a "*split phase.*" A second winding is placed on the stator after the manner of the second phase of a two phase motor. This winding need not be as strong as the main winding, and is made of smaller wire, since it is only in use during the starting period. The main- and the starting-windings are connected in parallel, and both supplied from the same single phase source, but to obtain the difference in phase an additional reactance is placed in the starting winding. The currents in the two branches now set up a rotating field, not uniform in

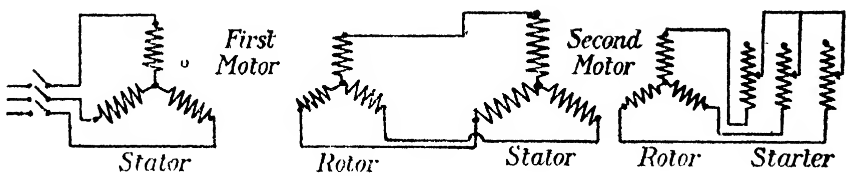


FIG. 361.—Cascade Connections.

magnitude, but sufficient to cause the motor to rotate. When a fair speed is attained, the starting winding is cut out. Instead of adding inductance to the starting winding, a condenser may be placed in series with it, whilst in some motors resistance is actually added, the idea being that since the two windings already possess a fair amount of inductance in themselves, the necessary phase displacement can be obtained by bringing the current in one branch more into phase with the voltage.

Cascade Arrangement.—A system of connections sometimes adopted with polyphase motors for obtaining two speeds is that known as the *cascade arrangement.* This employs two motors mechanically coupled together, the stator of the first being connected to the supply. The rotor of the first is connected to the stator of the second, and the rotor of the second is connected to a starting resistance in the usual way. The diagram of connections is shown in Fig. 361.

The frequency of the current supplied to the second stator is the frequency of slip of the first motor, and the frequency of slip of the second motor is the usual small value of a few per cent.

Neglecting the second slip and assuming the same number of poles for both motors, it is seen that the synchronous speed of the second motor is the same as the speed of slip of the first. If there are p pairs of poles, the speed of the first motor is

$$\frac{f-f_s}{f} \times \frac{f}{p} = \frac{f-f_s}{p}.$$

Neglecting the second slip, the speed of the second motor is $\frac{f_s}{p}$.

Equating these two, we get

$$\frac{f-f_s}{p} = \frac{f_s}{p},$$

$$f - f_s = f_s,$$

$$f_s = \tfrac{1}{2}f.$$

The synchronous speed of the set connected in cascade is thus half the synchronous speed of either when connected in the ordinary way.

If the number of pairs of poles on the two motors be different, let them be represented by p_1 and p_2 respectively. The speed of the first motor is then $\frac{f-f_s}{p_1}$, and the speed of the second $\frac{f_s}{p_2}$.

But
$$\frac{f-f_s}{p_1} = \frac{f_s}{p_2}.$$

and
$$fp_2 - f_s p_2 = f_s p_1.$$

$$f_s(p_1 + p_2) = fp_2.$$

$$f_s = f \times \frac{p_2}{p_1 + p_2}.$$

The speed is therefore

$$\frac{f_s}{p_2} = \frac{f}{p_1 + p_2}.$$

In other words, the synchronous speed of the set is the same as that of a single motor having as many poles as both component machines added together.

If the two motors are not mechanically coupled together, the arrangement is unworkable, since the moment any load is put upon one motor it stops and acts just like a transformer, whilst the other motor runs up to its normal speed.

Induction Generator.—If the rotor of an induction motor is driven mechanically from some external source at a speed higher than that of synchronism, the machine will act as a generator and the stator will supply a current to the mains. Unfortunately, such a machine is not capable of acting independently, but must run in parallel with another A.C. generator, as otherwise there will be no

rotating field set up and no generator action will take place. In other words,. the other generator must supply the A.C. magnetising current for the induction generator. The effect of raising the speed is to increase the output of the machine, the frequency remaining constant. In fact, the frequency is determined by the other generator and is independent of the speed. For this reason, the machine is sometimes called an *asynchronous generator*, since it does not run at synchronous speed.

Machines of this kind have not a wide application in practice, but the principle is sometimes employed with induction motors where a regenerative control is used, as in lift motors on the downward journey and railway motors when descending a steep gradient.

CHAPTER XXVII

INDUCTION MOTORS.—PERFORMANCE AND TESTING

No-load Conditions.—If the rotor winding be open-circuited by raising the brushes on the slip rings, no current will flow in the rotor and no torque will be developed. In these circumstances, the rotor will remain stationary, and the stator winding will take only that current necessary to maintain the rotating field and to provide for the stator iron loss. The former component will be a purely magnetising current and will lag behind the E.M.F. by 90°, whilst the latter will be a power component, relatively small in magnitude. On short circuiting the rotor, the motor runs up to full speed, the only torque which is developed being that necessary to overcome the friction and the rotor iron loss. A very small rotor current is sufficient to produce this torque, so that the slip is extremely small and the motor runs at very nearly synchronous speed. Owing to the extremely low frequency in the rotor circuit, the rotor iron loss is quite negligible. This small rotor current will cause a corresponding current to flow in the stator winding, and this component, combined vectorially with the current obtained in the stator with the rotor on open circuit, forms the no-load current of the motor.

The power factor will be considerably less than 0·5 in an ordinary case, so that when measuring the power of a three phase motor running light by means of the two wattmeter method, one wattmeter will read negative (see p. 126), and the total power will be obtained by subtracting the two wattmeter readings.

If the voltage be applied to the stator very gradually, the motor may be made to run at a very much lower speed than normal, but as this is an unstable state it has no very great practical value.

Effect of Voltage.—If the applied voltage be gradually lowered, the magnitude of the rotating field will be reduced proportionally. The rotor current required to produce the no-load torque will be, therefore, slightly increased, since the torque is proportional to the product of the field strength and the rotor current. This does not produce any appreciable effect on the no-load stator current, however, until very low voltages are reached, and in the meantime the magnetising and the stator iron loss currents are falling with

the voltage, so that the resultant stator current first of all falls as the voltage is reduced, but after a time rises again.

The power absorbed falls away as the voltage is reduced, but not so fast as the idle magnetising current, so that the power factor rises. In fact, when a motor is started up by means of an auto-transformer it happens quite frequently that the second wattmeter reads positive for a time, going behind the zero as the voltage is raised.

The slip remains practically constant over a wide range of voltage, but commences to increase at a considerable rate when very low voltages are reached.

Effect of Frequency.—The relation between the applied voltage, the flux per pole and the frequency is given by the formula

$$E = 2\cdot 1 f \Phi T \times 10^{-8}.$$

With a constant applied voltage, therefore, the flux is inversely proportional to the frequency, so that reducing the frequency below its normal value causes an increased flux, which increases the magnetic saturation in the iron and thereby reduces the permeability. The magnetising current is thus increased both on account of the increased flux and the decreased permeability. This makes the power factor poorer, and results in an increased copper and iron loss.

If the motor be designed for a low frequency in the first place, the flux density is prevented from reaching such high limits by employing a larger number of turns on the stator. With a given number and size of slots this results in a smaller size of wire being used, and this only permits a smaller current to flow. Both the output and the speed of the motor are thus reduced more or less proportionally.

For example, consider an 8-pole 50-cycle motor having a synchronous speed of 750 r.p.m. If the same carcase be employed for a 25-cycle motor with the same number of poles, the synchronous speed would be 375 r.p.m., and there would be twice as many conductors per slot half the size of those in the first case. Carrying half the current, therefore, the motor would do half the output. If the number of poles be reduced to four, the synchronous speed would be 750 r.p.m., as in the first case. The pole pitch is thus doubled and the original cross section could be used, since there will be twice as many slots per pole. The same current per conductor can now be allowed as in the 50-cycle motor, so that the output is the same for the same synchronous speed.

The above 4-pole 25-cycle motor is superior to the 8-pole 50-cycle motor, since the pole pitch is longer, and in all probability less leakage will take place. The magnetic leakage plays a most important part in determining the performance of the motor and affects not only the power factor, but also the overload capacity of the motor.

In order to get a reasonable length of pole pitch, few poles are required, and this leads to high speeds unless low frequencies are adopted. In general, motors designed for low frequencies will operate with better power factors than those intended for higher frequencies

Load Test.—In order to carry out a load test on a three phase induction motor, the motor is connected up as shown in Fig. 362, the instruments required being a voltmeter, an ammeter, and a three phase wattmeter. Instead of the latter, two single phase wattmeters may be used (see Fig. 98), or one single phase wattmeter in conjunction with the special change-over switch shown in Fig. 99.

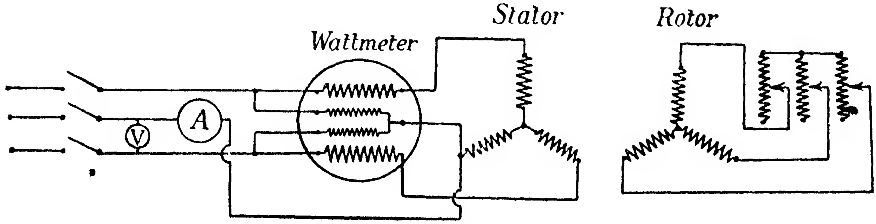


FIG. 362.—Electrical Connections for Load Test.

A power factor indicator is unnecessary, as this can be determined by the ratio of the two wattmeter readings. For low loads, the power factor will be probably below 0·5, so that one wattmeter will read negative. The volt coil of this instrument must be reversed in order to get its reading. The rotor is connected up to a three phase starting resistance if a slip ring rotor, whilst an auto-transformer should be used in the stator circuit if the rotor is of the squirrel cage type. Sometimes it is desired to measure the current in a slip ring rotor, and this is done by inserting an ammeter in one of the slip ring connections, but this is not very satisfactory, since the currents usually become unbalanced to an appreciable extent, owing to the fact that the resistance of the ammeter is quite comparable with the resistance of the rotor, which is, of course, very low. No extra apparatus, therefore, should be included in the rotor circuit other than that absolutely necessary.

The load can be taken up by another machine run as a generator (C.C. or A.C.) coupled up to the induction motor either directly or by belt. The generator is then loaded up on resistances or otherwise to obtain the required loads. The output of the generator is measured, and its efficiency at various loads must be known, so that the output of the motor can be determined. When a belt drive is employed, the loss in the belt must also be allowed for.

Another method of determining the output of the motor is to use some form of brake, such as the Prony brake or the eddy current brake.

A simple method of determining the output, when it is not kept on for more than a few minutes at a time, consists in loading up the motor by means of a simple band brake with a spring balance on each side to measure the pull on the tight and the slack side of the belt. The arrangement is shown in Fig. 363, where P_1 and P_2 represent the two spring balances, the pulls being measured in lb. The net pull at the rim of the pulley is $(P_1 - P_2)$ lb. and the torque developed is $T = (P_1 - P_2) \times r$ lb.-ft., where r is the radius of the pulley in feet. The work done per revolution is

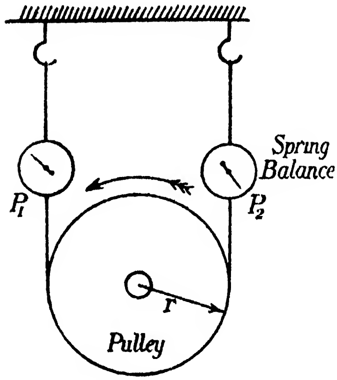


FIG. 363.—Arrangement for Band Brake.

$$2\pi r(P_1 - P_2)$$

or $2\pi T$ foot-lb., and if n is the speed in r.p.m. the work done per minute is $2\pi nT$. The B.H.P. developed is $\frac{2\pi nT}{33000}$, and this is equivalent to $\frac{2\pi nT}{33000} \times 746 = 0{\cdot}142\ nT$ watts. To determine the output, therefore, n and T must be obtained, and this involves measurements of the speed and the pulls on the two spring balances and a knowledge of the radius of the pulley.

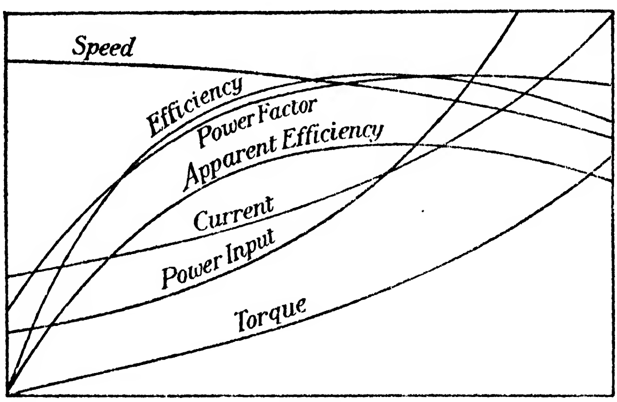


FIG. 364. Performance Curves of Induction Motor.

In carrying out the test, the tension on the brake is adjusted to a certain value, and the various measurements are taken. This is then repeated for a number of other tensions on the brake, so that a complete range of loads is obtained which should be plotted against B.H.P. as a base. The performance curves of a typical induction motor are shown in Fig. 364.

The disadvantage of this method of testing lies in the fact that if the load is considerable the band brake very soon gets extremely hot, so that the observations have to be taken quickly.

Power Factor.—The power factor when running light is very low, since the major portion of the stator current is idle magnetising current. As the load is increased, the no-load magnetising current remains approximately constant, whilst the power component increases on account of the load. The current has now, however, another idle component, due to the flux which is forced into the leakage paths, but this does not prevent the power factor from rising to a very high value. As the load is increased, the effect of this component becomes more marked, and the power factor begins to decrease again after having passed through a maximum value. The shape of the power factor curve is shown in Fig. 364.

Efficiency.—The mechanical output of the motor was shown on p. 418 to be $0{\cdot}142\ nT$ watts, whilst the input is given by wattmeter readings and is equal to $W_3 = \sqrt{3}\ EI\cos\phi$ and $W_2 = 2EI\cos\phi$ in a three- and two-phase case respectively, E and I being the line values.

The efficiency is thus

$$\frac{0{\cdot}142nT}{W_3} \text{ or } \frac{0{\cdot}142nT}{\sqrt{3}EI\cos\phi} = \frac{0{\cdot}082nT}{EI\cos\phi} \text{ (3 phase)},$$

and
$$\frac{0{\cdot}142nT}{W_2} \text{ or } \frac{0{\cdot}142nT}{2EI\cos\phi} = \frac{0{\cdot}071nT}{EI\cos\phi} \text{ (2 phase)}.$$

The efficiency curve follows the same general law as in C.C. motors, commencing at zero for no-load and rising to a maximum, after which it commences to drop slightly, as shown in Fig. 364. The efficiency and power factor curves are very similar, their chief difference lying in the fact that the former starts from zero whilst the latter does not.

Apparent Efficiency.—If the effect of power factor be not taken into account, it appears as if the input were equal to $\sqrt{3}\ EI$, and this quantity is called the *apparent power*. The ratio of the output to the apparent power input is called the *apparent efficiency*, and is, of course, equal to the product of the true efficiency and the power factor. Values of the apparent efficiency are plotted in Fig. 364 along with the other performance curves.

Heat Run.—In order to carry out a heat run on an induction motor, it is necessary to load it up in such a manner that the load can be maintained for a number of hours. The band brake method previously described is not, therefore, applicable, and some other method must be adopted. The usual practice is to maintain full load for six hours and then to determine the temperature rises in various parts of the motor, usually by thermometer. In order to avoid wasting the energy involved in the test, which is considerable in

the case of large motors, a C.C. generator is sometimes coupled to the motor and loaded back on to the C.C. supply. In this case, the power consumed is only that used up in overcoming the losses in the two machines.

Measurement of Slip.—There are three main methods of measuring slip, viz.,

1. By measurement of synchronous and actual speeds.
2. By measurement of rotor frequency.
3. By stroboscopic method.

These will be dealt with in turn.

1. The actual speed of the motor is measured and also the speed of the driving alternator or any synchronous apparatus connected to the supply, the slip being obtained from the difference of the two readings. Expressed as a percentage, this is

$$\frac{\text{Synchronous speed} - \text{Actual speed}}{\text{Synchronous speed}} \times 100.$$

If the number of poles on the alternator is not the same as the number on the induction motor, the speed of the alternator must be multiplied by the ratio of the numbers of poles. This is not a very accurate method of determining the slip, albeit a convenient one, since it depends on the difference of two quantities very nearly equal, and a small error in determining either of the speeds leads to a very large error in the result.

2. Since the rotor frequency is very low, not often exceeding three cycles per second, it can be read on a moving coil ammeter by counting the oscillations of the pointer. The method adopted, therefore, is to connect a moving coil ammeter in series with one of the leads going from the slip rings to the starter and count the beats in a given time. If a central zero instrument be used, the number of complete to and fro swings must be counted, but in the ordinary type the reverse half of the wave will produce no effect, since the pointer will be pressing against the zero stop. The percentage slip is then given by

$$\frac{\text{Rotor frequency}}{\text{Stator frequency}} \times 100.$$

If the motor is of the squirrel cage type, this method must be modified, since there are no slip rings. There will be usually a small portion of the magnetic field going right through the centre of the rotor and cutting the shaft, which will have a small E.M.F. induced in it. This can be detected by pressing a lead against each end of the shaft, the other ends being connected to a sensitive moving coil millivoltmeter. The measurement is then made in the same way as with the wound rotor.

3. In the stroboscopic method, a cardboard disc, with alternate

black and white sectors painted on it, is attached to the end of the motor shaft. This disc is illuminated by means of an arc lamp running from the same supply. If the disc is normally in a very poor light, an incandescent lamp is sufficient, this being preferable, since it is not so likely to unbalance the voltages of the system. Assuming the number of black sectors to be equal to the number of poles on the motor, one sector would move forward a pole pitch in half a cycle if there were no slip at all and the next black sector would occupy the original position of the first. But the disc is illuminated once every half-cycle, so that, viewed in the light of the arc, the disc would appear to be stationary. But since a certain amount of slip occurs, the second black sector will not quite reach the initial position of the first in half a cycle, with the result that the disc appears to rotate slowly in a direction opposite to that of the motor. A complete apparent revolution of the disc will thus correspond to a slip of as many cycles as there are pairs of poles on the motor. If the apparent revolutions of the disc per minute are counted, the frequency of slip is obtained from

$$\frac{\text{apparent r.p.m.}}{60} \times \text{pairs of poles.}$$

The percentage slip is then calculated as before. For testing motors with different numbers of poles, a series of discs is required having different numbers of sectors painted on them.

Instead of using an arc or an incandescent lamp, an electrically driven tuning fork may be used (see Fig. 365). This must have a natural period of vibration equal to the frequency of supply, and is kept in action by means of a small electromagnet operated from accumulators through an ordinary make and break. When the circuit is made, the two prongs of the fork are attracted, thus breaking the circuit and allowing the prongs to fly back. At the top end of each prong of the fork is attached a metal flag with a slot cut in it. These slots are arranged to overlap in the normal position, so that when the fork is vibrating the observer can see through the flags twice per period. The tuning fork acts in the same way as the arc lamp, therefore, in allowing the observer to see the disc on the motor shaft twice per period. This arrangement is much more convenient than that of the arc lamp, but it suffers from the disadvantage of only operating on the one frequency.

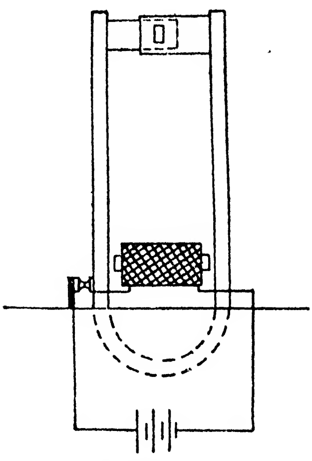


FIG. 365.—Tuning Fork for Slip Measurement.

In the slip stroboscope of Dr. Drysdale, a small synchronous motor drives a conical roller, and this in turn drives a small disc containing a number of slots. The speed of the disc is adjusted by sliding it up or down the roller until it is running at the same speed as the motor. The painted disc on the motor shaft now appears stationary and the slip is indicated by the position of the disc resting on the roller.

Measurement of Resistance.—The resistance of the stator windings can be measured conveniently by passing a C.C. through them, obtained from a few accumulators, and determining the current and voltage. If the windings are connected in star, the resistance per phase is half that observed, since there are two circuits in series, whilst if the circuits are connected in delta there are two windings in series connected in parallel with the third. If r be the resistance per circuit, then the joint resistance, as determined by the voltmeter and ammeter, will be

$$R = \frac{1}{\frac{1}{r} + \frac{1}{2r}} = \frac{2r}{3}$$

and $$r = \tfrac{3}{2}R.$$

The same can be done for the rotor circuits if it is a slip ring motor, but care must be taken to measure the voltage drops across the rings directly and not across the brushes or terminals, since the effect of the brush resistance is very appreciable, and this is cut out when the motor is running normally.

Simple Circle Diagram.—In order to simplify the development of the circle diagram, it will be assumed, to commence with, that the motor has no losses in the stator. Furthermore, only one phase will be dealt with. This is quite legitimate, since each of the phases acts in the same way. Under these conditions, the stator will set up a flux lagging by 90° behind the applied E.M.F. just as in the case of a transformer. (In the case of a three phase star-connected motor the applied voltage per phase is $\frac{1}{\sqrt{3}}$ times the line voltage and the stator flux dealt with is the flux per phase.) Part of this flux cuts the rotor conductors, whilst the remainder goes through leakage paths. In the actual motor, the rotor sets up a leakage flux of its own, but the ampere-turns required to produce this must be obtained ultimately from the stator, which is supplied with a corresponding number of ampere-turns from the mains. It is therefore permissible to consider the whole of the leakage flux as coming from the stator, the rotor being non-inductive. The useful flux cutting the rotor and the leakage flux are not in phase, since the leakage flux must have the same phase as the stator current which produces it, whilst the vector sum of the

two component fluxes must be in quadrature with the applied E.M.F. These conditions are represented in Fig. 366, where OE_s represents the applied stator voltage and OI_s the stator current lagging by some angle ϕ. The leakage flux $O\Phi_l$ is in phase with OI_s, and, when added vectorially to the useful rotor flux $O\Phi_r$, gives the total stator flux $O\Phi_s$. The rotor has an E.M.F., OE_r, induced in it, due to the rotor flux $O\Phi_r$ and lagging behind it by 90°. Thus far, the vector diagram is similar to that of the transformer, the difference being that in the present case the leakage flux is very much larger than it is in the transformer. Since the rotor has been assumed to be non-inductive, the rotor current, OI_r, is in phase with the rotor E.M.F., OE_r. The magnitude of the rotor current can be obtained, since the total ampere-turns of the stator are composed of two components, one required to magnetise the system and the other necessary to balance the ampere-turns of the rotor. For the sake of simplicity, it will be assumed that there are the same number of turns on the stator and rotor. If this is not the case, the rotor current must be multiplied by the ratio of stator to rotor turns. The magnetising current may be represented by OI_m in phase with the stator flux.

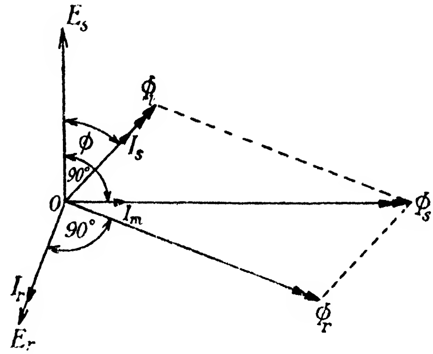


FIG. 366.—Vector Diagram of Induction Motor.

The vector diagram in Fig. 366 has been redrawn in Fig. 367, all the quantities having the same magnitude and phase as before, only this time the three currents form a triangle. Since the stator flux is proportional to the applied voltage, and the latter is main-

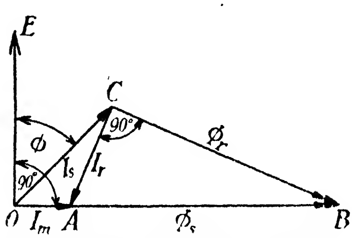


FIG. 367.—Redrawn Vector Diagram.

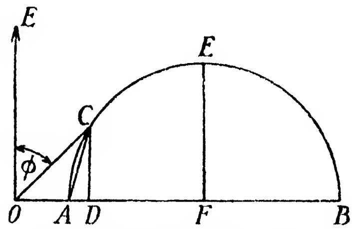


FIG. 368.—Simple Circle Diagram.

tained constant, it follows that the point B is fixed. Again, the no-load magnetising current depends upon the stator flux, and since the latter is constant the former is constant also. Thus the point A is also fixed in a given motor. The line AB is therefore of constant length. But since AC and CB are at right angles, it follows that

the point C moves in a semicircle erected upon AB as a diameter. This construction leads, therefore, to the simple circle diagram shown in Fig. 368, in which OC represents the stator current lagging by an angle ϕ behind the applied E.M.F. The line OA represents the no-load magnetising current, whilst CA is proportional to the rotor current. As the load on the motor is increased, the point C moves round the semicircle away from A and towards B. The power supplied to the motor is proportional to $I \cos \phi$, and can thus be represented to scale by CD. It is seen that this line attains a maximum length when it coincides with EF, which makes it clear why an induction motor breaks down when a certain load is reached. The application of a greater load causes the current to increase, but this results in an actual decrease in the power supplied, with the result that the motor comes to rest. The maximum load which an induction motor can overcome without pulling up is called the *breakdown load*.

It is also seen that the power factor under which the motor operates varies with the load and attains a maximum value considerably before the breakdown load is reached. In fact, the power factor usually attains a maximum value somewhere in the neighbourhood of full load in a commercial motor.

Dispersion Coefficient.—In the theoretical motor the performance of which is represented by the simple circle diagram in Fig. 368, the line OA represents the stator current when the motor is running light. This is a purely idle magnetising current, since the losses are neglected. As no torque is required, the motor will run with no slip (*i.e.* at synchronous speed), because in these circumstances there will be no rotor E.M.F. generated, and hence no rotor current will be produced. The flux set up by the stator is thus free to cut the rotor, because no back ampere-turns are set up in the rotor, with the result that the stator flux passes through the useful rotor paths and the waste leakage paths in parallel. Representing the reluctances of these paths by R_u and R_l respectively, the joint reluctance is given by $\dfrac{R_u \times R_l}{R_u + R_l}$ and the magnetising current is represented by OA.

Next consider the same motor to be clamped so that the rotor cannot move. On applying the same E.M.F. as before to the stator, an idle magnetising current will flow, but this time the motor acts like a transformer with its secondary short circuited. An E.M.F. will be induced in the rotor winding, which will produce a current owing to its being short circuited. This current will set up a number of back ampere-turns which will oppose the passage of the flux through the rotor. The stator will now take a current from the mains capable of setting up just sufficient ampere-turns to balance the rotor ampere-turns together with the ampere-turns necessary to magnetise the system. This really means that the

whole of the stator flux has been deflected into the leakage paths. The magnitude of the flux has not been decreased, since this depends upon the applied voltage, which is maintained constant. It merely means that one of the parallel paths through which the flux passed has been cut off. The flux now being confined to the leakage paths, it follows that a larger magnetising current is required, and this is given by OB (Fig. 368), since the current vector has passed completely round the semicircle.

The magnetising currents in the two cases are proportional to the respective reluctances, since the flux is the same, and therefore

$$\frac{OA}{OB} = \frac{\dfrac{R_u \times R_l}{R_u + R_l}}{R_l}$$

$$= \frac{R_u}{R_u + R_l}$$

$$= \frac{1}{1 + \dfrac{R_l}{R_u}}.$$

But $$\frac{R_l}{R_u} = \frac{\text{Useful flux}}{\text{Leakage flux}}.$$

Therefore $$\frac{OA}{OB} = \frac{1}{1 + \dfrac{\text{Useful flux}}{\text{Leakage flux}}}$$

$$= \frac{1}{\dfrac{\text{Leakage flux} + \text{Useful flux}}{\text{Leakage flux}}}.$$

$$= \frac{\text{Leakage flux}}{\text{Leakage flux} + \text{Useful flux}}$$

$$= \frac{\text{Leakage flux}}{\text{Total flux}}.$$

This ratio, which is a most important one in induction motor design, is called the *Dispersion Coefficient* and is represented by σ, whilst its value largely settles the shape of the circle diagram.

Leakage Factor.—The leakage factor is defined as the ratio

$$\frac{\text{Total flux}}{\text{Useful flux}} = \lambda.$$

There is a direct relation between this ratio and the dispersion coefficient, for

$$\lambda = \frac{\text{Useful flux} + \text{Leakage flux}}{\text{Useful flux}}$$

$$= 1 + \frac{\text{Leakage flux}}{\text{Useful flux}},$$

$$\lambda - 1 = \frac{\text{Leakage flux}}{\text{Useful flux}},$$

$$\frac{\lambda - 1}{\lambda} = \frac{\text{Leakage flux}}{\text{Useful flux}} \times \frac{\text{Useful flux}}{\text{Total flux}}$$

$$= \frac{\text{Leakage flux}}{\text{Total flux}}$$

$$= \sigma.$$

Also since
$$\frac{\lambda - 1}{\lambda} = \sigma,$$

$$1 - \frac{1}{\lambda} = \sigma,$$

$$\frac{1}{\lambda} = 1 - \sigma,$$

and
$$\lambda = \frac{1}{1 - \sigma}.$$

The magnitude of the dispersion co-efficient thus settles the value of the leakage factor, and *vice versa*.

Maximum Power Factor.—The maximum power factor under which an induction motor can operate is directly connected also with the dispersion coefficient. The conditions for maximum power factor are shown in the simple circle diagram in Fig. 369, where OC, representing the stator current, is drawn tangential to the semicircle ACB. The angle of lag, ϕ, is now a minimum, and consequently the power factor is a maximum. But since the angle OCD is a right angle, the angle ϕ is equal to the angle ODC, and the power factor is given by $\frac{CD}{OD}$. But $CD = AD = \frac{1}{2}AB$. Therefore the maximum power factor is equal to

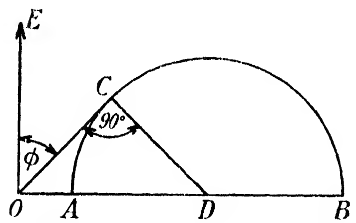


FIG. 369.—Construction for Maximum Power Factor.

$$\frac{AB}{2 \times OD} = \frac{AB}{2 \times OA + 2 \times AD}$$

$$= \frac{AB}{2 \times OA + AB}$$

$$= \frac{AB}{OB + OA}$$

$$= \frac{OB - OA}{OB + OA}.$$

Dividing top and bottom by OB this becomes

$$\frac{\dfrac{OB}{OB} - \dfrac{OA}{OB}}{\dfrac{OB}{OB} + \dfrac{OA}{OB}}$$

$$= \frac{1 - \sigma}{1 + \sigma}.$$

A simple approximation to this formula is given by

$$\text{Maximum power factor} = (1 - 2\sigma),$$

which is near enough for the majority of purposes.

For example, a motor having a dispersion coefficient of 0·05 cannot work on a higher power factor than $(1 - 2 \times 0{\cdot}05) = 0{\cdot}90$.

Power Input.—The applied voltage being maintained constant, the power input is proportional to $I \cos \phi$. But ϕ is equal to the angle OCD (see Fig. 370), and I is represented by OC. Therefore $I \cos \phi$, and consequently the power input, is represented to scale by

$$OC \cos OCD = CD.$$

The power input for any current is thus represented to scale by a vertical line dropped from the point C on to OB. In this way, the maximum power input is represented by EF.

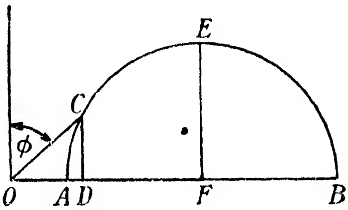


FIG. 370.—Construction for Power Input.

Overload Capacity.—The overload capacity may be defined as the maximum percentage overload that the motor will stand without breaking down and (neglecting all losses) is equal to

$$\frac{\text{Maximum power input} - \text{Full load input}}{\text{Full load input}} \times 100$$

$$= \frac{EF - CD}{CD} \times 100 \text{ (see Fig. 370)},$$

assuming the point C to represent the full load conditions. If the ratio of the full load current to the no-load current be expressed by k, then

$$\begin{aligned} CD &= OC \cos \phi \\ &= k \times OA \times \cos \phi, \end{aligned}$$

and the overload capacity becomes equal to

$$\left(\frac{EF}{k \times OA \times \cos \phi} - 1\right) \times 100$$

$$= \left(\frac{AB}{2k \times OA \times \cos \phi} - 1\right) \times 100.$$

But
$$\frac{AB}{OA} = \frac{OB - OA}{OA} = \frac{OB}{OA} - 1$$
$$= \frac{1}{\sigma} - 1 = \frac{1 - \sigma}{\sigma}.$$

Therefore the overload capacity equals

$$\left(\frac{1 - \sigma}{2\sigma k \cos \phi} - 1\right) \times 100.$$

If the losses of the motor are taken into consideration, the overload capacity is less than is given by the above formula.

Effect of Losses.—The losses to be considered are the iron and friction loss and the stator and the rotor copper losses. The stator iron loss is practically constant for all loads, since it depends upon the applied E.M.F. The rotor iron loss is negligible, since the

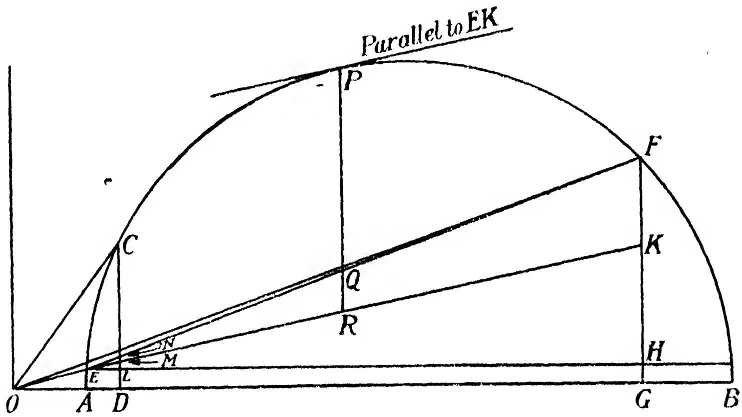


FIG. 371.—Circle Diagram for Commercial Motor.

frequency is only that of the slip. The frictional loss is also practically constant, since for all working loads the speed is approximately constant. All these losses may therefore be regarded as a constant loss, and since the power input is represented by the height of vertical lines such as CD (Fig. 371), a horizontal line may be drawn through E, a little above AB, cutting off a portion of the vertical power lines. The height of this line is obtained from the no-load observation of the motor, OE representing the no-load current and the height of E above AB being determined by the no-load watts. Strictly speaking, the line through E should droop downwards slightly towards the right, since as the current increases the slip increases, thus decreasing the frictional watts slightly, whilst the increased stator IR drop causes a slight reduction in the flux, thus decreasing the iron loss.

Next consider the behaviour of the motor when the rotor is

clamped. The whole of the power input is now wasted in losses, and the stator current vector will take up some position such as OF, the vertical height, FG, representing the total losses. Of these GH represents the constant loss, whilst HF represents the stator and rotor copper losses. But since the stator current is approximately proportional to the rotor current, the total copper losses may be taken as proportional to the (rotor current)^2, *i.e.* to AC^2 (not shown in the diagram for the sake of clearness). Now $\frac{AC}{AB} = \cos CAB$, and since AB is constant, AC is proportional to $\cos CAB = \frac{AD}{AC}$. AD is therefore proportional to AC^2, *i.e.* to the total copper losses. A line joining EF will therefore cut off portions of vertical power lines proportional to AD, *i.e.* to the total copper losses for the current in question. Furthermore, FH may be divided at K, so that HK represents the stator copper loss and KF the rotor copper loss. On joining EK, the copper losses for all currents are separated, the vertical distance between EH and EK representing the stator copper loss and the vertical distance between EK and EF representing the rotor copper loss.

Having accounted for all the losses, the remainder of the vertical through any point, C, represents the output. Thus CD represents the input, DL the constant loss, LM the stator copper loss, MN the rotor copper loss and NC the output.

The efficiency for any current or output can be determined directly from the circle diagram when the loss lines are drawn, it being given by $\frac{CN}{CD} \times 100$. The maximum output is obtained by drawing a tangent to the semicircle parallel to EF. The point of contact represents the point at which maximum output is obtained.

Torque Line.—It was shown on p. 409 that

$$\frac{\text{Rotor input}}{\text{Rotor output}} = \frac{\text{Synchronous speed}}{\text{Actual speed}},$$

and the rotor output is equal to $2\pi nT$, n being the speed and T the torque. The rotor input is, therefore, equal to

$$2\pi nT \times \frac{\text{Synchronous speed}}{\text{Actual speed}}$$

$$= 2\pi T \times \text{Synchronous speed}.$$

For a constant frequency, the torque is therefore proportional to the rotor input, and this is given in Fig. 371 by the line CM, since CN is the output and NM the rotor copper loss. The scale, in lb.-ft., can be obtained from a knowledge of the rotor input, in watts, and the speed at any one point, the watts being converted

in H.P. and then into foot-lb. per minute. The starting torque is given by FK, but by inserting resistance in the rotor circuit the point F is moved round the semicircle towards A, and the starting torque is thus increased up to a maximum value. The maximum torque is obtained by drawing another tangent to the semicircle, this time parallel to EK, the point of contact indicating the required maximum torque. The necessary starting resistance to obtain the maximum starting torque is obtained from the line PQR. QR represents the watts lost in the rotor and PQ those lost in the starting resistance. Since the currents are the same in both, the magnitude of the external resistance per phase is given by

$$\frac{PQ}{QR} \times \text{Rotor resistance per phase.}$$

Determination of Slip from Circle Diagram.—Consider the equation

$$\frac{\text{Rotor losses}}{\text{Rotor input}} = \frac{\text{Speed of slip}}{\text{Synchronous speed}} \text{ (see p. 409).}$$

The slip is therefore proportional to $\dfrac{\text{Rotor losses}}{\text{Rotor input}} = \dfrac{NM}{CM}$. When the motor is clamped, the stator current is OF and the slip is given by $\dfrac{FK}{FK}$ = unity or 100 per cent. For any output such as CN, the slip can be read off the diagram and expressed as a percentage. This construction indicates clearly the effect of an increase in the rotor resistance, which increases the rotor losses and consequently NM, to which the slip is proportional.

Experimental Determination of Circle Diagram.—In order to draw the circle diagram of a motor, it is necessary first to make an observation of the no-load current, watts and power factor at the correct voltage and frequency. Commencing with the main horizontal and vertical lines, the no-load current is set off to scale, the angle of lag being determined from the no-load power factor which is obtained from the voltmeter, ammeter and wattmeter readings. It is convenient to let the current vectors represent the line values, whilst the vertical power lines are made to represent the total power of all the phases. The point E on the circle diagram (Fig. 371) is thus determined. It is now necessary to obtain another point or series of points on the semicircle, preferably far removed from E. For this purpose, the rotor may be clamped and the short circuit current measured, thus obtaining the point F. The height of F above AB is determined either from the angle of lag of the current or from the watts supplied to the motor. Unfortunately, however, the current which would flow if the normal voltage were applied would be such as to burn out both stator and rotor. It is therefore necessary to reduce considerably the voltage applied to the stator

to prevent this occurring. The stator current actually measured in this way must then be increased in the direct ratio of the normal voltage to the actual voltage applied. This assumes that the stator current is proportional to the voltage, and this is not strictly true, on account of the different flux densities set up in the core. To minimise this error as much as possible, it is desirable to pass as large a current through the motor as is safe in order to approximate to the actual conditions. Even then the accuracy of the position of the short circuit point is scarcely good enough to warrant the drawing of the semicircle without further data. In place of the short circuit point a number of readings in the neighbourhood of full load may be employed, including some on considerable overload. The position of each point is marked off from a knowledge of the corresponding values of the current and watts. A semicircle is then drawn through the no-load point, the various load points and the short circuit point if the latter has been obtained, the centre of the semicircle lying on *OB*. The disadvantage of relying on the load points is that they are comparatively close to the no-load point, thus making it difficult to draw in the semicircle accurately, whilst the disadvantage of relying on the short circuit point lies in its dubious accuracy. Obviously, the best effect is obtained by employing both methods, a mean semicircle being drawn through all the points. Points on the upper regions of the semicircle, including the unstable portion to the right of the maximum output point, can be obtained by clamping the rotor and inserting the starting resistance in the rotor circuit. This is equivalent to increasing the rotor resistance and the losses occurring in the rotor circuit. The vertical height, *FK* (Fig. 371), is thus increased, which means that the point *F* is moved round the semicircle towards the left. By putting the handle of the starting resistance in various positions, a number of observations may be made giving points in the unstable region. By still further increasing the resistance of the rotor circuit, points all round the semicircle can be obtained, and these additional points should agree with the load readings already obtained. Care must be taken to obtain these observations rapidly, as otherwise there is a danger of the starter or the motor being burnt out. The difference between these readings with the rotor clamped and those with the motor running and giving an output is that the power which goes to the external load in the one case goes to heat up the starting resistance in the other. From the point of view of the circle diagram, it does not matter whether the watts are dissipated at the pulley or in the external resistance.

Having made the semicircle, a horizontal line is drawn through *E* to represent the constant iron and friction loss.

The lines representing the stator and rotor copper losses are next drawn in. For this purpose the stator and rotor resistances are measured, together with a corresponding pair of values for the

currents in the two circuits. The watts lost in each are now calculated. If there is a different number of phases in the stator and rotor, this must be taken into account. A vertical line through F is now drawn and divided at K so that $\frac{HK}{HF} = \frac{\text{Stator copper loss}}{\text{Total copper loss}}$. If the rotor is of the squirrel cage type, the actual stator copper loss must be calculated for the current OF and HK measured off to the scale of watts already determined. If the short circuit point, F, is not available, the losses for some known load must be measured. Points such as L, M and N corresponding to a current, OC, can thus be determined and the loss lines drawn through these points. Unfortunately, these points lie fairly close together, and a considerable error is liable to be introduced in drawing the diagram.

To obtain the torque scale, the slip is measured from the diagram for one particular output. This gives the speed from which the torque can be calculated, since the output is known.

Importance of σ.—It is seen that the relative shape of the circle diagram is largely dependent upon the value of σ. The no-load current is greatly affected, and this in turn affects the power factor of the motor. An increase in the value of σ results in a decrease in the maximum power factor, thus causing a slightly larger current to flow, which in turn causes a slight drop in the efficiency. An increase in σ also results in a decrease in the overload capacity of the motor. This is seen to be the case, since, for a constant applied voltage, the total stator flux OB (Fig. 371) may be considered constant, and an increase in σ will consequently result in an increase in OA and a decrease in AB. The diameter of the circle is thus reduced, resulting in a decrease in the maximum power output. If the full load is the same as before, this is equivalent to saying that the overload capacity is reduced. It is therefore desirable to aim at obtaining a low value for σ when designing an induction motor.

Effect of Air-gap Length.—The length of the air-gap exerts a great influence on the magnitude of the leakage flux, inasmuch as the reluctance of the paths of the useful rotor flux is largely determined by it. The reluctance of the leakage paths is independent of this dimension, and so the value of σ depends to a very great extent upon the radial length of the air-gap. An increase in this direction, therefore, affects adversely both the power factor and the overload capacity of the motor to a very considerable degree, and to prevent this it should be reduced to the smallest possible limits consistent with good mechanical design. This is the reason why induction motors are designed with very much smaller air-gaps than C.C. motors of the same size, the lengths varying from 0·02 inch for a rotor diameter of 5 inches up to 0·125 inch for a diameter of 4 feet. The ordinary bearings with a spherical seating, commonly adopted

for C.C. motors, are thus unsuitable, a more rigid design being necessary.

Effect of σ on Characteristics.—In order to study the effect of

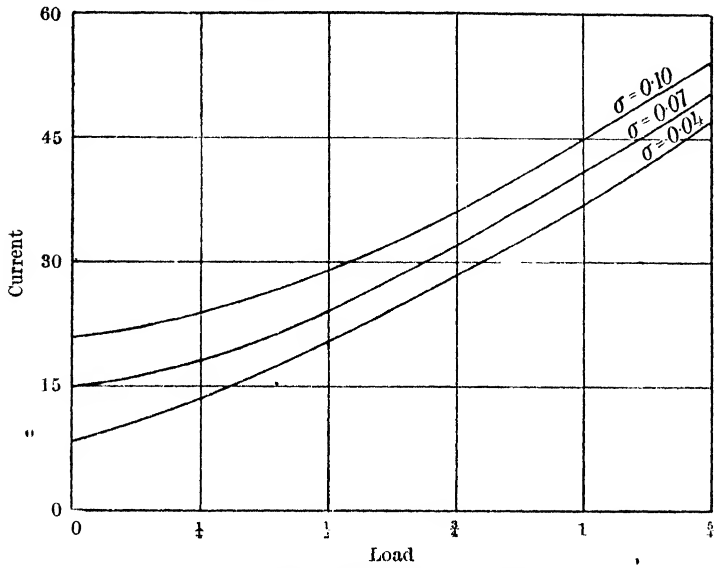


FIG. 372.—Effect of σ on Current.

σ on the characteristics of motors of the same size and output, three cases have been chosen in which the values of σ have been taken

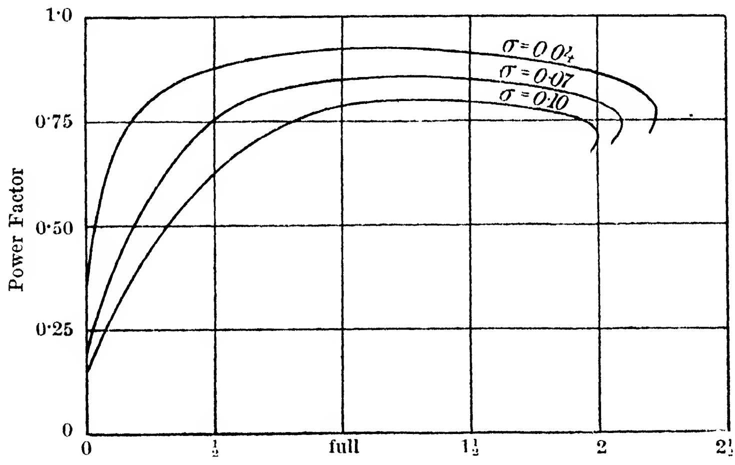


FIG. 373.—Effect of σ on Power Factor.

to be 0·04, 0·07 and 0·10 respectively. From the three circle diagrams curves have been plotted in Figs. 372 and 373, showing the relationships between (*a*) the current and the output and (*b*) the

power factor and the output for each case. The inferiority produced by the larger values of σ is clearly seen from these curves. The no-load currents for the three motors are widely different, whilst the breakdown loads are also affected. The following table gives some comparative data for the three motors, obtained from their circle diagrams.

Motor No.	1	2	3
σ	0·04	0·07	0·10
No-load Current	8·5	15	21
Full Load Current	37	41	45
Full Load Power Factor	0·92	0·85	0·78
Overload Capacity	123%	112%	101%
Ratio $\frac{\text{No-load Current}}{\text{Full Load Current}}$	0·23	0·37	0·47

CHAPTER XXVIII

INDUCTION MOTORS.—DESIGN

Specification.—In order to design an induction motor, it is necessary to know the voltage, frequency and number of phases of the supply, together with the B.H.P. and speed required. With a given frequency, only certain speeds are possible, these being determined by the number of poles. The efficiency and power factor at which the motor is expected to work are specified also in a number of cases.

Number of Poles.—The relation between the frequency, number of poles, and synchronous speed is the same as for alternators, the formula being

$$p = \frac{120f}{n},$$

n being the synchronous speed in r.p.m. The actual speed is about 4 or 5 per cent. less than the synchronous speed, the difference depending on the slip.

Efficiency.—In Fig. 374 (*a*) and (*b*) are shown the average full load efficiencies of modern induction motors, and these figures should be aimed at in designing a new motor.

Power Factor.—In Fig. 375 (*a*) and (*b*) are shown the average full load power factors of modern induction motors, which should also be aimed at in a new design.

Motors having a large number of poles will have a relatively larger leakage flux owing to the reduced pole pitch. This increases the value of σ and reduces the power factor, so that for motors having many poles the lower values on the curve should be taken. In the same way, motors operating on a low frequency will have a small number of poles for a given speed, and, consequently, 25-cycle motors may be expected to operate with slightly higher power factors than 50-cycle motors.

Output Coefficient.—An empirical formula of the D^2L type holds good with induction motors just as with alternators (see p. 274), this formula being

$$\frac{\text{B.H.P.}}{\text{r.p.m.}} = k \times D^2L,$$

D being the air-gap diameter, L the gross length of the core, and k the output coefficient.

The average values of the output coefficient for modern polyphase induction motors operating on a frequency of 50 cycles per second are shown in Fig. 376 (a) and (b). For 25-cycle motors these figures can be increased by about 20 per cent. The values of the output coefficient shown in the curves must not be taken as being rigidly fixed, since various manufacturers differ somewhat in this respect.

Air-gap Diameter.—The air-gap diameter depends upon the B.H.P. and the speed in the same manner as the output coefficient. Of

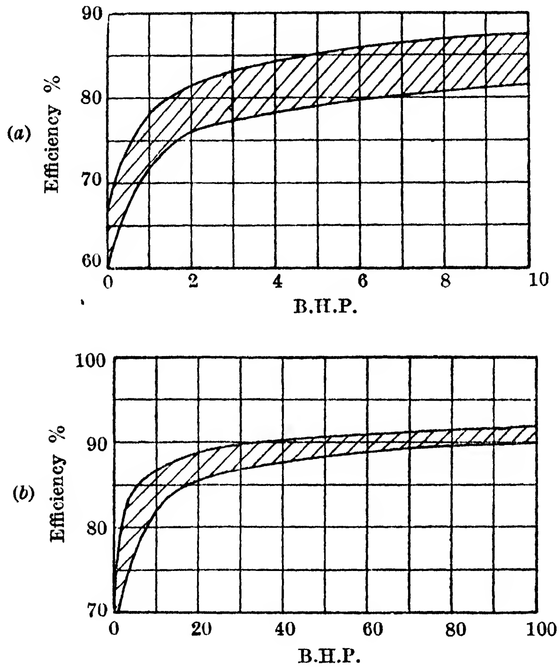


FIG. 374.—Efficiencies of Polyphase Induction Motors.
(a) Up to 10 B.H.P. (b) Up to 100 B.H.P.

course, the diameter increases as the B.H.P. goes up, but since the bulk of the carcase is inversely proportional to the speed, it is also necessary to increase the diameter for the low speeds in order to make room for the larger number of poles. An empirical relation connecting the product of the diameter (in inches) and the speed with the output is shown in Fig. 377, which represents average practice for 50-cycle motors. The values of diameter $\times$ speed should be increased by 15 to 20 per cent. for 25-cycle motors and equivalent modifications made for other frequencies. The diameters obtained in this way will correspond to peripheral speeds of from

2,500 to 5,000 feet per minute. The latter value represents the maximum speed at which the rotor should be run, and the peripheral velocity for any design should be worked out to see that this figure is not exceeded.

Core Length.—When the air-gap diameter has been settled the gross length of the core can be determined approximately, since the value of D^2L is known.

When a complete line of motors is to be designed, it is the usual practice to have two or three core lengths for each diameter. The most economical designs are not obtained in this way, considering each one separately on its own merits, but it leads to a reduction in

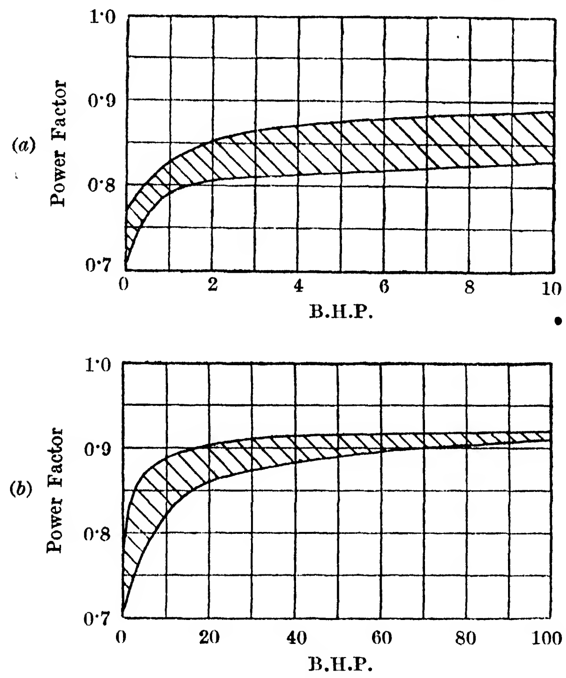


FIG. 375.—Power Factors of Polyphase Induction Motors.
(*a*) Up to 10 B.H.P. (*b*) Up to 100 B.H.P.

the manufacturing costs, since it reduces the number of patterns required. If three motors have the same diameter and differ merely in the length of their cores a large number of their parts are similar. The middle machine is designed for the particular diameter in question, the short length one having rather too large a diameter and the long length one rather too small a diameter than would be expected if they were designed separately.

Ventilating Ducts.—The usual practice with regard to the number and width of the ventilating ducts is similar to that which obtains in the case of alternators. They are spaced usually about $2\frac{1}{4}$ to 3 inches apart and vary from $\frac{3}{8}$ to $\frac{5}{8}$ inch in width.

Flux Densities.—Suitable values for the average flux densities in various parts of the magnetic circuit can be obtained from the

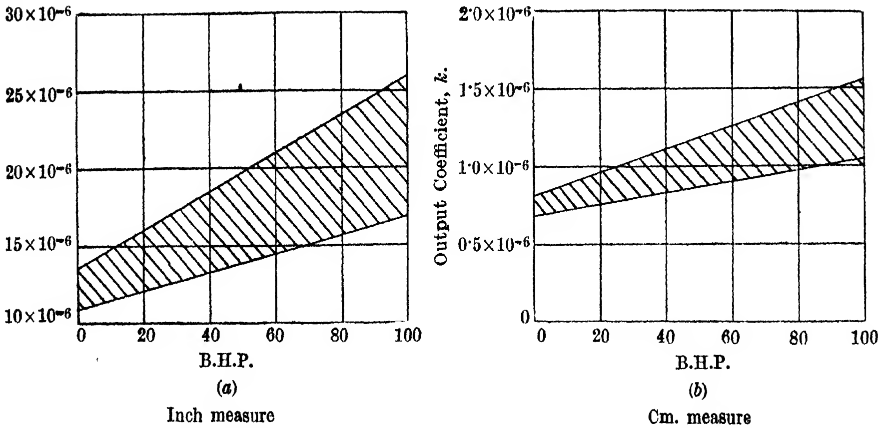


FIG. 376.—Output Coefficients for Polyphase Induction Motors.

following table. The flux distribution in the air-gap and in the teeth is not uniform, however (see p. 389), and the maximum flux

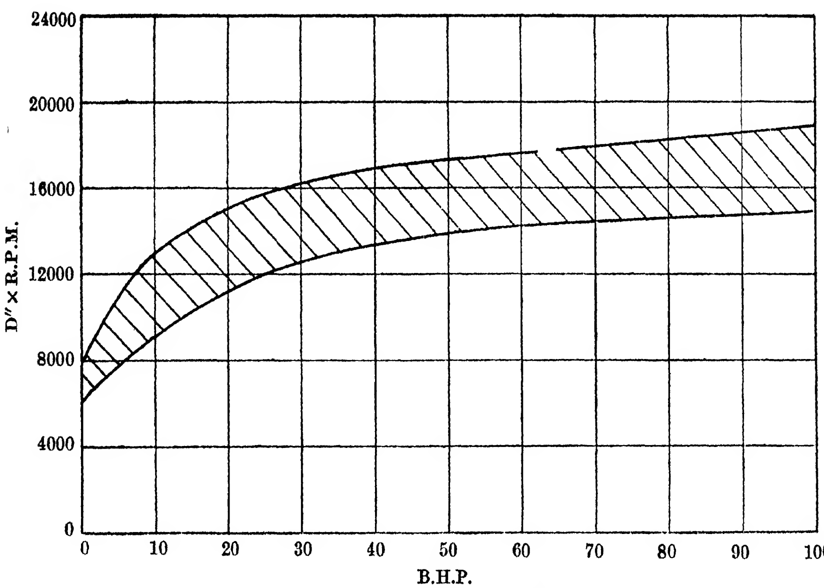


FIG. 377.—Relation between D×R.P.M. and B.H.P.

Polyphase Induction Motors: $f = 50$.
For $f = 25$ increase D × R.P.M. by 15 per cent. – 20 per cent.

density is obtained by multiplying these average values by 1·7 and 1·94 for three- and two-phase motors respectively. These constants

do not affect the flux in the iron at the back of the teeth in either stator or rotor, since this is due to the average effect of all the phases and averages itself out over the whole section.

Part.	Average Flux Density.	
	Lines per sq. in.	Lines per sq. cm.
Air-gap	19500	3000
Stator Teeth	52000	8000
Stator Iron behind Teeth ...	42000	6500
Rotor Teeth	55000	8500
Rotor Iron behind Teeth ...	65000	10000

Flux per Pole.—The total air-gap flux per pole is obtained by multiplying the density in the air-gap by the area of the air-gap per pole. The area of the air-gap may be taken as being the average of the areas of the tops of the teeth per pole in both stator and rotor, adding 20 per cent. to allow for fringing. If semi-closed slots are employed, the iron top of the tooth may be assumed to be 0·8 of the tooth pitch as a first approximation, whilst 0·5 may be taken for open slots.

Another method of obtaining a tentative value for the flux per pole is to consider the teeth. Assuming a slot width equal to the tooth width, the circumferential length of iron per pole is half the pole pitch. The axial length of iron is found by subtracting the total length of the vent spaces from the gross length of the core and then multiplying by 0·9 to allow for the laminations. The area thus found is then multiplied by the desired tooth density to obtain the total flux per pole. A suitable value can now be chosen from a consideration of both methods.

Number of Stator Conductors.—The required number of stator conductors is obtained from the formulæ given on p. 410, viz.,

$$N = \frac{E \times 10^8}{2{\cdot}1 \times f\Phi} \text{ for three phase motors}$$

and

$$N = \frac{E \times 10^8}{2 \times f\Phi} \text{ for two phase motors.}$$

E is the voltage per phase and is $\frac{1}{\sqrt{3}}$ times the line voltage for three phase star-connected motors. A further small allowance may be made if required for the IR drop in the winding.

The final number of conductors chosen must be a multiple of the number of slots.

Ampere-conductors per Inch Diameter.—The number of ampere-

conductors per inch diameter in modern motors generally lies between certain limits which are given in the following table.

Air-gap Diameter.	Ampere-conductors per inch Diameter.
Up to 12″	800—1000
12″—20″	1000—1500
20″—40″	1500—1800

This value is obtained by multiplying the number of stator conductors by the full load current which can be calculated approximately and dividing by the air-gap diameter.

This calculation serves as a check on the design as far as it has gone.

Number of Stator Slots.—The number of slots must be a multiple of the phases times the poles, and it is usual to allow 3, 4 or 5 slots per pole per phase. The proper number can generally be determined by considering the slot pitch and the number of conductors which it will be necessary to put into each slot.

Size of Stator Conductors.—Knowing the current, the size of the stator conductors can be determined by allowing a current density of from 1,500 to 2,000 amperes per square inch, the higher figures referring to the smaller sizes of conductors. For low currents wire will be most convenient to use, whilst for the higher currents former wound coils of bar or strip should be adopted, since anything above about No. 12 S.W.G. is difficult to wind. The use of strip has an advantage, since it brings about a higher space factor in the slot. A circular wire has a cross-sectional area of $\frac{\pi}{4}D^2$, but it occupies a space of D^2 in the slot, and, apart from any insulation at all, this brings the space factor down to 0·78.

The space factors ordinarily employed will vary from 0·25 to 0·4, high voltage machines having the lower values and bar windings being superior to wire windings.

Size of Stator Slots.—When the number of conductors per slot has been obtained, an approximate area of slot can be calculated. The width of tooth and slot should be made about the same, thus giving the depth. The conductors should now be arranged in the slot, putting in the necessary insulation, and the final dimensions of the slot can be decided upon by adjusting it to fit the conductors.

Depth of Stator Iron.—The stator iron behind the teeth carries half the flux per pole, and by using the flux density given in the table on p. 439 the required cross section of iron can be obtained. When this area is divided by the net iron length of the core, it gives

the depth of iron required behind the stator teeth. This enables the outer diameter of the stator stampings to be calculated.

Stator Copper Loss.—The resistance of each winding can be calculated by estimating the length of wire and assuming a specific resistance of 0·8 microhm per inch cube (hot). Knowing the current, the watts lost per phase and the total for all phases can now be determined.

Stator Iron Loss.—The combined hysteresis and eddy current loss may be obtained by reference to Fig. 378, which gives the core loss in watts per unit volume at unit frequency. The value thus obtained must be multiplied by the frequency and the volume of iron. For greater accuracy the iron loss in the teeth and in the iron at the back of the teeth may be obtained separately.

Cooling Surface.—The cooling surface may be determined in just the same way as was done in the case of alternators. The inner

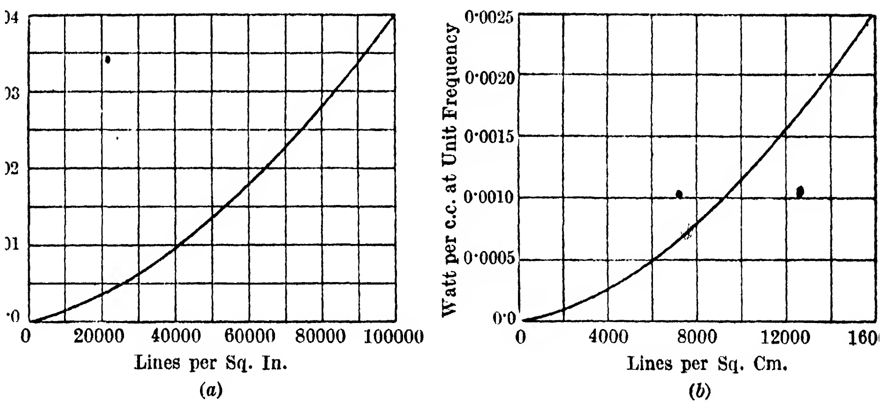


FIG. 378.—Stator Core Losses.

and outer cylindrical surfaces are taken, together with the two ends of the core and one side of each ventilating duct.

Estimated Temperature Rise of Stator.—With normal designs a temperature rise of the stator of about 40° C. should be obtained when the watts to be dissipated are about one per square inch. As a first approximation, the temperature rise may be taken as being proportional to the watts dissipated per square inch, but this rule is by no means accurate, as the temperature rise in a particular motor depends to a very large extent upon its mechanical design.

Air-gap Length.—Very small air-gap lengths are adopted (see p. 432); suitable values may be obtained from Kapp's rule, which is

$$\left.\begin{array}{l}\text{Length of Single}\\ \text{Air-gap in inches}\end{array}\right\} = 0{\cdot}008 + 0{\cdot}0025 \times \text{Rotor diameter in inches.}$$

Number of Rotor Slots.—The same number of slots should not be used on both stator and rotor, since if this is done there are some

positions of the rotor in which the magnetic reluctance is very much less than it is when the rotor is in intermediate positions. The rotor tends to jump into these positions of minimum reluctance and the motor does not start up as well as it otherwise would.

It is usual to put more slots in the rotor than in the stator, the ratio being of the order of 1·25 to 1·5. The number of rotor slots must also be a multiple of the poles × phases.

In the case of squirrel cage rotors, it is usual to adopt a prime number for the number of rotor slots.

Rotor Winding.—The actual number of rotor conductors does not much matter. If there is a large number of turns in series, the open-circuit voltage of the winding is considerable, whilst if fewer turns of larger cross section are employed, the current at the slip rings becomes increased. It is therefore a simple matter to arrange a suitable winding.

Size of Rotor Slots.—The size of the rotor slots can be determined by assuming a similar current density to that in the stator winding and then finding the required slot space. Since the induced voltage per winding will probably be low, a fairly high space factor should be obtained. The dimensions of the slot can then be settled to suit the winding.

Depth of Rotor Iron.—The required depth of rotor iron behind the teeth can be determined in the same way as in the case of the stator, from which the internal diameter of the rotor stampings can be obtained. The question of whether to put in a spider or to key the stampings direct on to the shaft will be settled by this internal diameter of the stampings. If no spider is employed, axial ventilating ducts should be provided to supply the air inlet for the radial ventilating ducts from which the air is thrown out by centrifugal force.

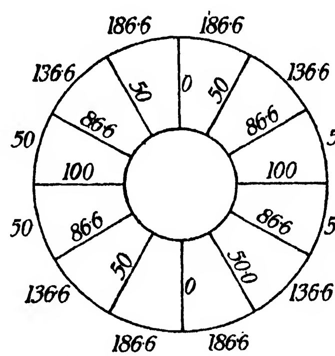


FIG. 379.—Currents in a Squirrel Cage Winding.

Squirrel Cage Rotors.—The number of rotor slots should preferably be chosen a prime number and as high as is practicable. The current per bar is

$$\text{Stator current} \times \frac{\text{Stator conductors}}{\text{Rotor conductors}}.$$

With a density of 3,000 amperes per square inch, this settles the area of the rotor bar and consequently that of the rotor slot as well. The dimensions of the latter should be such as to leave sufficient tooth area to carry the magnetic flux.

To calculate the current in the end ring, consider Fig. 379,

which represents a two pole squirrel cage winding, the two end rings being shown as two circles with the bars joining them. The figures against the bars represent the currents in them at a particular instant, 100 amperes being taken as the maximum. The currents in the other bars decrease according to a sine law. The current in the bar carrying 100 amperes flows into the end ring, 50 amperes in each direction. A little further on this 50 amperes is reinforced by 86·6 amperes from the next bar, making 136·6 amperes in the next section. The next bar adds another 50 amperes, making a total of 186·6 amperes, which is the maximum current carried by the end ring. This maximum current is thus seen to be the sum of the instantaneous values of the currents in half the bars per pole. Assuming a perfect sine law, the average current per bar is $\frac{I}{1{\cdot}11} = 0{\cdot}9I$, and the maximum current in the end ring becomes

$$0{\cdot}9I \times \frac{N}{2p},$$

where p is the number of poles and N the number of bars. The R.M.S. current in the end ring is

$$\frac{1}{\sqrt{2}} \times 0{\cdot}9I \times \frac{N}{2p}$$

$$= \frac{0{\cdot}318IN}{p}.$$

It is seen that not only does the current in any one section of the end ring vary sinusoidally with time, but that there is a sinusoidal space variation as well.

With a current density of 4,000 amperes per square inch in the end ring, its section becomes

$$\frac{0{\cdot}318IN}{4000p}$$

$$= 0{\cdot}00008\frac{IN}{p} \text{ square inches.}$$

Copper Loss in Squirrel Cage Rotors.—If A and L be the cross-sectional area and length, respectively, in inch measure, the total copper loss in the bars is

$$N \times I^2 \times \rho\frac{L}{A}.$$

With a current density of 3,000 amperes per square inch, the total full load copper loss in the bars becomes

$$N \times I \times 3000 \times \rho L$$

$$= N \times I \times 3000 \times 0{\cdot}8 \times 10^{-6}L$$

$$= 0{\cdot}0024NIL.$$

The full load copper loss in the two end rings is

$$2 \times \left(\frac{0{\cdot}318IN}{p}\right)^2 \times 0{\cdot}8 \times 10^{-6}\,\frac{\pi Dp}{0{\cdot}00008IN}$$

$$= 0{\cdot}0064\,\frac{IND}{p},$$

D being the rotor diameter.

The total full load rotor copper loss is therefore

$$0{\cdot}0024NIL + 0{\cdot}0064\,\frac{IND}{p}$$

$$= NI\left(0{\cdot}0024L + 0{\cdot}0064\,\frac{D}{p}\right),$$

and this enables the full load slip to be determined.

Copper Loss in Wound Rotors.—In the case of wound rotors, the copper loss is obtained by estimating the resistance of each phase from its dimensions and assuming that the rotor current

$$= \text{Stator current} \times \frac{\text{Stator conductors}}{\text{Rotor conductors}}.$$

Estimated Temperature Rise of Rotor.—The rotor will have to dissipate only the I^2R loss in the winding, since the iron loss is negligible owing to the very low frequency, viz., that of slip. The frictional loss may be supposed to be dissipated in the bearings.

Estimating the cooling surface in the same way as for the stator, the temperature rise may be calculated by allowing 20° C. rise for every watt per square inch of cooling surface. It must be understood that this calculation is only very approximate.

Friction and Windage Loss.—This will vary from about 3 per cent. in a 5 B.H.P. motor down to about 1 per cent. in a 200 B.H.P. motor, whilst for the largest sizes of all it will drop to something of the order of $\frac{3}{4}$ per cent.

No-load Current.—The ampere-turns per pole for the air-gap

$$= 0{\cdot}313 \times l_g \times B,$$

where l_g is the length of the single air-gap and B is the average air-gap density in lines per sq. in. But the total ampere-turns per pole are twice the ampere-turns per pole per phase for a three phase motor (see p. 390), and also the ratio $\dfrac{\text{maximum flux density}}{\text{average flux density}}$ is 1·7 (not 1·57).

The maximum ampere-turns per pole per phase therefore

$$= 0{\cdot}313 \times l_g \times B \times \frac{1{\cdot}7}{2},$$

and the R.M.S. ampere-turns per pole per phase

$$= 0{\cdot}313 \times l_g \times B \times \frac{1{\cdot}7}{2\sqrt{2}}$$

$$= 0{\cdot}19 l_g \times B.$$

The magnetising current per phase for the air-gap

$$= \frac{0{\cdot}19 \times l_g \times B}{\text{turns per pole per phase}}.$$

For a two phase motor the constant becomes 0·3 instead of 0·19.

The magnetising current for the iron path will be about 10 to 30 per cent. of the above, the value depending largely upon the width of the slot openings. The wider the slot opening the greater is the density in the teeth and the more the magnetising current.

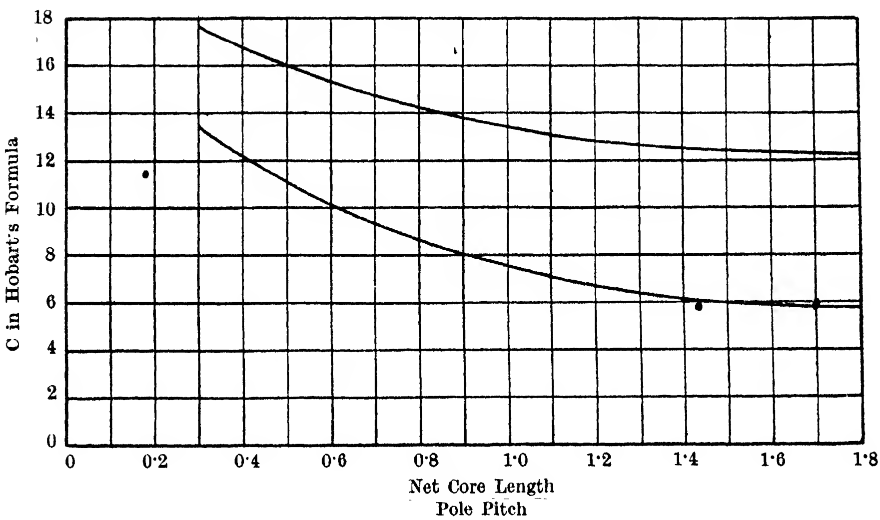


FIG. 380.—Values of C in Hobart's Formula.

The power component of the no-load current

$$= \frac{\text{Power to drive motor light}}{\sqrt{3} \times \text{Line voltage}}.$$

The total no-load current

$$= \sqrt{(\text{Power component})^2 + (\text{Magnetising current})^2}.$$

Estimation of σ.—The value of σ can be estimated fairly closely by a formula due to Hobart, viz.,

$$\sigma = C \times C' \times \frac{\text{Air-gap length}}{\text{Pole pitch}}.$$

The values of C and C' can be read off the curves in Figs. 380 and 381. For squirrel cage motors a third constant, having an approximate value of 0·75, should be introduced.

Another formula for the estimation of σ is due to Behn-Eschenburg, and is

$$\sigma = \frac{3}{N^2} + \frac{l_g}{XN \times (p.p.)} + \frac{6\,l_g}{(p.p.)},$$

where N = average number of slots per pole for stator and rotor,
X = average width of slot opening,
l_g = air-gap length,
and $(p.p.)$ = pole pitch.

Dimensions are in cm.

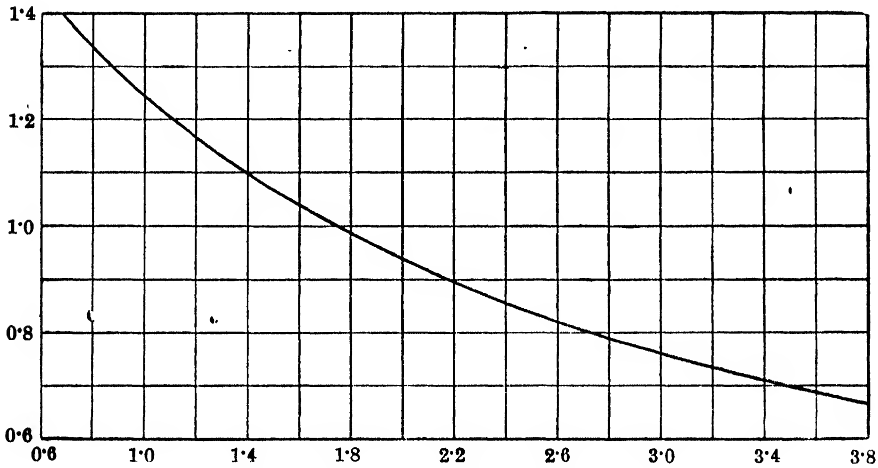


FIG. 381.—Values of C' in Hobart's Formula.

Predetermination of Circle Diagram.—The necessary data for predetermining the circle diagram have now been worked out. Referring to Fig. 371, the no-load magnetising current enables the point A to be determined and $OB = \frac{OA}{\sigma}$. The circle can thus be drawn straight away. The power component of the no-load current fixes the vertical height of the line EH. The resistances of the stator and rotor windings being known, the point F can be obtained by trial, so that FH (to the same scale that HG represents the no-load power to drive) represents the total copper loss for the current OF. FH can now be divided at K, so that

$$\frac{FK}{KH} = \frac{\text{Rotor copper loss}}{\text{Stator copper loss}}.$$

The torque line is obtained by joining EK.

Full Load Current, Power Factor and Efficiency.—These values can now be obtained from the circle diagram in the usual way. The

"apparent efficiency," which is the product of the true efficiency and the power factor, should be calculated also. If desired, the complete performance of the motor can be worked out from the circle diagram.

Example of Design.—As an example, a design will now be worked out for a 10 B.H.P., 3-phase, 50-cycle, 440-volt motor having a synchronous speed of 1,000 r.p.m.

Number of poles = 6.

An efficiency of 0·85 and a power factor of 0·86 will be aimed at, these figures representing average practice. This gives the full load current as

$$\frac{10 \times 746}{\sqrt{3} \times 440 \times 0{\cdot}85 \times 0{\cdot}86}$$

$$= 13{\cdot}5 \text{ amperes.}$$

Choosing an output co-efficient of 11×10^{-6}, we get

$$D^2L = \frac{10}{1000 \times 11 \times 10^{-6}} \text{ (see p. 438)}$$

$$= 910.$$

From Fig. 377, $D \times$ r.p.m. $= 11000$,

so that $D = 11$ in.

The core length $L = \frac{910}{11^2} = 7{\cdot}5$ in.

There will be two ventilating ducts, each 0·375 in. wide.

The flux per pole will now be worked out. The pole pitch $= \frac{\pi \times 11}{6} = 5{\cdot}76$ in. and the net iron length of the core $= (7{\cdot}5 - 2 \times 0{\cdot}375) \times 0{\cdot}9 = 6{\cdot}08$ in. Semi-closed slots will be used in both stator and rotor, and the iron length per pole pitch $= 0{\cdot}8 \times 5{\cdot}76 = 4{\cdot}61$ in. The air-gap area per pole

$$= 4{\cdot}61 \times 6{\cdot}08 \times 1{\cdot}2 = 33{\cdot}6 \text{ sq. in.}$$

Assuming an average flux density of 19500 lines per sq. in., the flux per pole is $19500 \times 33{\cdot}6 = 660000$ lines.

The required number of stator conductors per phase

$$= \frac{440 \times 10^8}{\sqrt{3} \times 2{\cdot}1 \times 50 \times 660000} = 366.$$

Choosing three slots per pole per phase, this gives 54 total slots and 18 slots per phase. There will be, therefore, 20 conductors per slot, giving 180 turns per phase.

The ampere conductors per inch diameter

$$= \frac{13{\cdot}5 \times 1080}{11} = 1300,$$

which is rather high.

Using No. 13 S.W.G. wire for the stator conductors, this gives a current density of $\frac{13 \cdot 5}{0 \cdot 00665} = 2030$ amperes per sq. in. The diameter of this wire $= 0 \cdot 092$ in. bare and $0 \cdot 104$ in. with d.c.c. insulation. With a slot lining of 40 mils of press-spahn the 20 conductors can be arranged in ten layers, alternate layers being staggered, in a semi-closed slot $1 \cdot 375$ in. $\times$ $0 \cdot 375$ in. with an opening of $0 \cdot 125$ in.

The flux carried by the stator iron behind the teeth

$$= \frac{660000}{2} = 330000 \text{ lines,}$$

and with a density of 42000 lines per sq. in. the required iron cross section

$$= \frac{330000}{42000} = 8 \text{ sq. in.}$$

The net iron length

$$= (7 \cdot 5 - 2 \times 0 \cdot 375) \times 0 \cdot 9 = 6 \cdot 08 \text{ in.}$$

Required depth of iron behind teeth

$$= \frac{8}{6 \cdot 08} = 1 \cdot 32, \text{ say } 1 \cdot 5 \text{ in.}$$

Outer diameter of stator stampings

$$= 11 + 2(1 \cdot 375 + 1 \cdot 5) = 16 \cdot 75 \text{ in.}$$

Length per turn of winding

$$= 2 \times \text{pole pitch} + 2(\text{core length} + 2)$$
$$= 2 \times 5 \cdot 76 + 2 \times 9 \cdot 5 = 30 \cdot 5 \text{ in. approximately}$$

Resistance per winding (hot)

$$= 0 \cdot 8 \times 10^{-6} \times \frac{30 \cdot 5 \times 180}{0 \cdot 00665} = 0 \cdot 66 \text{ ohm.}$$

Total stator copper loss

$$= 3 \times 13 \cdot 5^2 \times 0 \cdot 66 = 360 \text{ watts.}$$

To get the stator iron loss, the volume of the stator iron is required. The average tooth width

$$= \frac{\pi(11 + 1 \cdot 375)}{54} - 0 \cdot 375 = 0 \cdot 345 \text{ in.}$$

Volume of stator teeth

$$= 54 \times 0 \cdot 345 \times 1 \cdot 375 \times 6 \cdot 08 = 155 \text{ cu. in.}$$

Volume of iron behind teeth

$$= \frac{\pi}{4}(16{\cdot}75^2 - 13{\cdot}75^2) \times 6{\cdot}08 = 440 \text{ cu. in.}$$

Density in teeth

$$= \frac{660000}{9 \times 0{\cdot}345 \times 6{\cdot}08} = 35000.$$

Corresponding watts per cu. in. per cycle per second

$$= 0{\cdot}008 \text{ (see Fig. 378).}$$

Watts lost in teeth

$$= 0{\cdot}008 \times 155 \times 50 = 60 \text{ watts.}$$

Density in iron behind teeth

$$= \frac{330000}{1{\cdot}5 \times 6{\cdot}08} = 37000.$$

Corresponding watts per cu. in. per cycle per second

$$= 0{\cdot}009.$$

Watts lost in iron behind teeth

$$= 0{\cdot}009 \times 440 \times 50 = 200 \text{ watts.}$$

Total stator iron loss

$$= 60 + 200 = 260 \text{ watts.}$$

Stator cooling surface

$$= \pi \times 16{\cdot}75 \times 6{\cdot}75 + \pi \times 11 \times 6{\cdot}75 + 4 \times \frac{\pi}{4}(16{\cdot}75^2 - 11^2)$$

$$= 1090 \text{ sq. in.}$$

Watts per sq. in.

$$= \frac{360 + 260}{1090} = 0{\cdot}57.$$

The temperature rise of the stator will, therefore, be quite reasonable.

A wound rotor will next be designed.

Air-gap length

$$= 0{\cdot}008 + 0{\cdot}0025 \times 11, \text{ say } 0{\cdot}035 \text{ in.}$$

Choosing four slots per pole per phase, this gives 72 slots in all. A bar winding will be adopted with four conductors per slot, giving 48 turns per phase. The rotor current is approximately

$$= 13{\cdot}5 \times \frac{180}{48} = 50 \text{ amperes.}$$

Choosing a strip with a section $0{\cdot}08 \times 0{\cdot}25 = 0{\cdot}020$ sq. in., a current density of $\frac{50}{0{\cdot}020} = 2500$ amperes per sq. in. is obtained. This winding can be made to fit in semi-closed slots $0{\cdot}23$ in. $\times\, 0{\cdot}75$ in. with a slot opening of 0·10 in.

As in the stator, a depth of iron of 1·75 in. will be allowed behind the teeth, making the inner diameter of the rotor stampings

$$11 - 2\,(0{\cdot}75 + 1{\cdot}75) = 6 \text{ in.}$$

The length of the mean turn of the rotor winding will be approximately the same as in the stator, and the rotor resistance per phase (hot)

$$= 0{\cdot}8 \times 10^{-6} \times \frac{30{\cdot}5 \times 48}{0{\cdot}020} = 0{\cdot}058 \text{ ohm.}$$

Rotor copper loss

$$= 3 \times 50^2 \times 0{\cdot}058 = 440 \text{ watts.}$$

Rotor cooling surface

$$= \pi \times 11 \times 7{\cdot}5 + \pi \times 6 \times 7{\cdot}5 + 4 \times \frac{\pi}{4} \times (11^2 - 6^2) = 670 \text{ sq. in.}$$

Watts per sq. in.

$$= \frac{440}{670} = 0{\cdot}66.$$

The temperature rise of the rotor will also be quite reasonable.

Taking the friction and windage loss at about 3 per cent. of the output, this becomes

$$0{\cdot}03 \times 7460 = 220 \text{ watts.}$$

Total losses

$$= 360 + 260 + 440 + 220 = 1280 \text{ watt}$$

Magnetising current per phase for the air-gap

$$= \frac{0{\cdot}19 \times 0{\cdot}035 \times 19500}{30} \text{ (see p. 445)}$$

$$= 4{\cdot}3 \text{ amperes.}$$

Adding 10 per cent. for the iron path, this becomes

$$1{\cdot}1 \times 4{\cdot}3 = 4{\cdot}7 \text{ amperes.}$$

The power component of the no-load current

$$= \frac{260 + 220}{\sqrt{3} \times 440} = 0{\cdot}63 \text{ ampere.}$$

Actually this would be rather larger, since the no-load copper losses are neglected, but the difference is not noticeable in the final no-load current.

Total no-load current

$$= \sqrt{4{\cdot}7^2 + 0{\cdot}63^2} = 4{\cdot}8 \text{ amperes} = 35{\cdot}5 \text{ per cent. full load current.}$$

The estimated value of σ, obtained from Figs. 380 and 381, is

$$\sigma = 10 \times 1{\cdot}27 \times \frac{0{\cdot}035}{5{\cdot}76} = 0{\cdot}077.$$

According to Behn-Eschenburg's formula

$$\sigma = \frac{3}{10{\cdot}5^2} + \frac{0{\cdot}035 \times 2{\cdot}54}{0{\cdot}1125 \times 2{\cdot}54 \times 10{\cdot}5 \times 5{\cdot}76 \times 2{\cdot}54} + \frac{6 \times 0{\cdot}035 \times 2{\cdot}54}{5{\cdot}76 \times 2{\cdot}54}$$

$$= 0{\cdot}0272 + 0{\cdot}0020 + 0{\cdot}0365$$

$$= 0{\cdot}066.$$

A value of 0·07 will be assumed, taking both methods into consideration. This gives a maximum power factor of

$$\frac{1 - 0{\cdot}07}{1 + 0{\cdot}07} = 0{\cdot}87,$$

and a full load power factor of 0·86 is obtained from the circle diagram of the motor, for which the necessary data is now available. The circle diagram also gives the approximate short circuit current as 65 amperes and the maximum B.H.P. as 25, thus crediting the motor with an overload capacity of 150 per cent.

The full load efficiency is

$$\frac{7460}{7460 + 1280} = 0{\cdot}855,$$

and the full load apparent efficiency is

$$0{\cdot}855 \times 0{\cdot}86 = 0{\cdot}735.$$

Full load current

$$= \frac{7460}{\sqrt{3} \times 440 \times 0{\cdot}735}$$

$$= 13{\cdot}3 \text{ amperes.}$$

Full load slip $= \dfrac{\text{Rotor } I^2R \text{ loss}}{\text{Rotor input}}$

$$= \frac{440}{7460 + 220 + 440}$$

$$\doteq 0{\cdot}054, \text{ say } 5{\cdot}5 \text{ per cent.}$$

Full load speed $= 945$ r.p.m.

Alternative Squirrel Cage Rotor.—Choosing 43 slots, a slot pitch of $\frac{\pi \times 10{\cdot}93}{43} = 0{\cdot}80$ in. is obtained. The current per bar is

$$13{\cdot}3 \times \frac{1080}{43} = 330 \text{ amperes.}$$

Using bars with a cross section of $\frac{5}{16}$ in. $\times$ $\frac{3}{8}$ in., a current density of 2800 amperes per sq. in. is obtained. These bars will go in slots $\frac{3}{8}$ in. $\times$ $\frac{1}{2}$ in. The maximum tooth width is $0{\cdot}80 - 0{\cdot}375 = 0{\cdot}425$ in., and the minimum $0{\cdot}80 \times \frac{11 - 2 \times 0{\cdot}5}{11} - 0{\cdot}375 = 0{\cdot}352$ in.

The R.M.S. current in the end ring is

$$\frac{0{\cdot}318 \times 330 \times 43}{6} \text{ (see p. 443)}$$

$$= 750 \text{ amperes.}$$

The required cross section of the end ring, at 4000 amperes per sq. in.,

$$= \frac{750}{4000} = 0{\cdot}1875 \text{ sq. in.}$$

An end ring with a cross section of $\frac{3}{4}$ in. $\times$ $\frac{1}{4}$ in. is suitable, since this gives the desired cross-sectional area.

The total full load rotor copper loss

$$= 43 \times 330\,(0{\cdot}0024 \times 7{\cdot}5 + 0{\cdot}0064 \times \tfrac{11}{6}) \text{ (see p. 444)}$$

$$= 420 \text{ watts}$$

as compared with 440 watts for the wound rotor.

The full load slip would be about the same in the two cases.

CHAPTER XXIX

MOTOR CONVERTERS

General Arrangement.—The motor converter consists of an ordinary induction motor with a wound rotor and a C.C. generator, rigidly coupled together and arranged either with two or three bearings according to individual circumstances. An illustration of a 1500 k.W. three-bearing motor converter is shown in the frontispiece. In addition to the mechanical coupling, the two rotating elements are also connected together electrically, a hollow shaft being employed for the purpose of carrying the connecting leads.

Connections.—The diagram of connections for a three phase motor converter is shown in Fig. 382, which represents a set having two poles on both A.C. and C.C. sides. The stator of the induction motor is wound for the same number of phases as the supply, but the rotor is usually provided with a twelve phase winding. When running normally, these twelve phases are connected in star, the outer ends being connected to twelve equidistant points on the C.C. armature, from the commutator of which the main continuous current is collected. This end of the set usually is provided with interpoles, and may be either shunt or compound wound. When starting up, only three, or in the largest sizes six, phases of the rotor of the induction motor are used, these being connected to slip rings and thence to a starting resistance in the usual manner with induction motors. In addition, a short circuiting ring is mounted at the end of the slip rings for the purpose of short circuiting the whole of the inner ends of the twelve phases when the normal running conditions have been attained. A synchronising voltmeter, which is used during starting up, is also connected across two of the slip rings, as shown in the diagram.

Principle of Action.—Assuming that the combined set is running at some definite speed, the rotor of the induction motor will have E.M.F.'s induced in it corresponding in phase and magnitude to its slip. These E.M.F.'s will produce currents in the armature of the C.C. generator, which will, in addition, have another series of E.M.F.'s induced in it on account of its rotation in its own magnetic field. The frequency of the E.M.F.'s supplied from the

induction motor is that of slip, whilst the frequency of the E.M.F.'s due to pure generator action is determined by the speed. For these two frequencies to be the same, the induction motor must run with

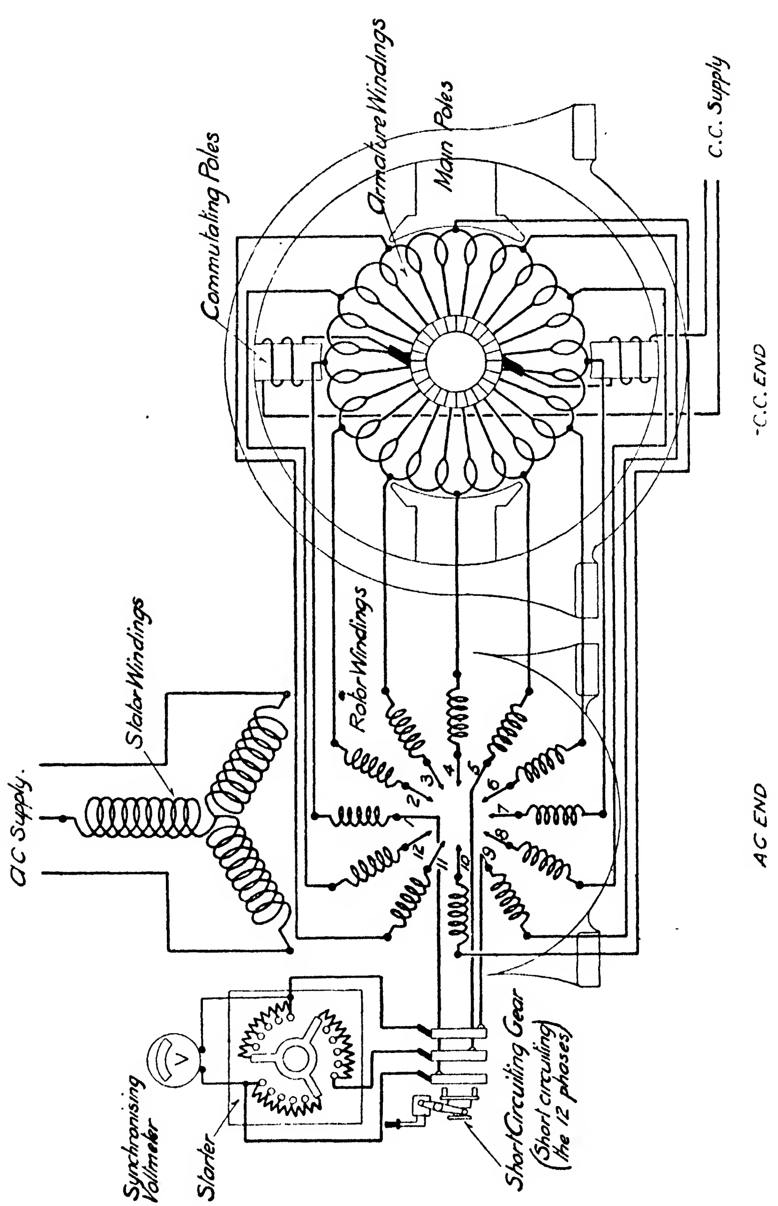


FIG. 382.—Connections of Motor Converter.

a slip of 50 per cent., or, in other words, the set must run at half synchronous speed. If the speed differs from this value by some small amount, one of these frequencies goes up and the other

down, so that circulating currents will flow round the two rotating elements, just as in the case of two alternators, and a large synchronising force is produced tending to make the two frequencies equal. The normal running speed of the combination is thus half the speed of synchronism, and it behaves as a single synchronous machine.

As the rotor of the induction motor is rotating with a slip of 50 per cent., it follows that it is running with an efficiency of 50 per cent. (neglecting the stator losses). Half the power input is transformed into mechanical form by the ordinary motor action, and the remaining half is transmitted to the rotor by transformer action. The mechanical power developed goes towards driving the C.C. generator through the rigid coupling, whilst the electrical power developed in the rotor is supplied to the armature of the C.C. generator and helps to drive it by rotary converter action. The power given out at the commutator in C.C. form is due to the sum of these two effects, one half of it having been converted by motor-generator action, and the other half by transformer and rotary converter action. Thus the A.C. end operates half as an induction motor and half as a transformer, whilst the C.C. end operates half as a generator and half as a rotary converter.

Construction.—The construction of the two machines forming a complete unit differs very little from the standard practice with induction motors and C.C. generators. In the larger sizes three bearings are employed, whilst for the smaller sets only two are used. The induction motor is provided with three or six slip rings, for the purpose of starting, and a short circuiting ring to short-circuit the twelve phases when the speed of synchronism is attained. The air-gap is made much larger than is customary with induction motors of similar size. The object of employing the small air-gaps usually adopted is to keep the magnetising current down as much as possible, thereby raising the power factor, but this does not apply in the case of a motor converter, since the magnetising current is drawn from the armature of the C.C. end. Larger clearances can therefore be used without introducing any harmful effects.

On the C.C. end the currents are fed into the rotating element through the hollow shaft direct from the rotor of the induction motor.

Speed, Frequency and Number of Poles.—It has been shown that the speed of a motor converter is half the synchronous speed of the A.C. end when both machines have only two poles each. This is also true for multipolar sets when both machines have the same number of poles. The general formula connecting the speed and the numbers of poles is obtained by considering the frequency of slip of the induction motor and the frequency of generation of the C.C. generator and equating them. If n be the speed in

r.p.m., f and f_s the frequency of supply and slip respectively, and p_a and p_c the number of poles on the A.C. and C.C. ends respectively, the speed of the induction motor is equal to

$$\frac{f-f_s}{p_a} \times 120$$

and
$$f_s = f - \frac{np_a}{120}.$$

The frequency of the C.C. generator is equal to

$$f_s = \frac{np_c}{120}.$$

Therefore
$$f_s = f - \frac{np_a}{120} = \frac{np_c}{120},$$

$$np_a + np_c = 120f,$$

and
$$n = \frac{120f}{p_a + p_c}.$$

The set, therefore, runs at a speed equal to the synchronous speed of an induction motor having as many poles as both A.C. and C.C. ends together. An increase in the number of poles at either end causes a reduction in speed.

The frequency of the currents in the C.C. generator is

$$f_s = \frac{120f}{p_a + p_c} \times \frac{p_c}{120}$$

$$= f \times \frac{p_c}{p_a + p_c}.$$

The C.C. end is thus always operating on a frequency considerably lower than that of the supply, which is advantageous, since the performance of rotary converters is always better on low frequencies.

In an ordinary induction motor, the number of rotor turns is not rigidly fixed, but in the present case the C.C. generator must be supplied with a definite voltage, depending upon the value of the C.C. line voltage required and the number of phases. The rotor at the A.C. end must therefore have the correct number of turns per phase to generate this voltage when running with its normal slip.

Power converted Mechanically and Electrically.—Considering the A.C. end, the output of the induction motor is given by

$$\frac{\text{Mechanical rotor output}}{\text{Electrical rotor input}} = \frac{\text{Actual speed}}{\text{Synchronous speed}},$$

whence

$$\text{Mechanical rotor output} = \text{Rotor input} \times \frac{\text{Actual speed}}{\text{Synchronous speed}}$$

$$= \text{Total input} \times \frac{\text{Actual speed}}{\text{Synchronous speed}},$$

neglecting the stator losses.

But the actual speed is $n = \dfrac{120f}{p_a + p_c}$, and the synchronous speed of the induction motor by itself is $\dfrac{120f}{p_a}$. Therefore the mechanical output of the rotor is

$$\text{Total input} \times \frac{120f}{p_a + p_c} \times \frac{p_a}{120f}$$

$$= \text{Total input} \times \frac{p_a}{p_a + p_c}.$$

The power received by the C.C. end in electrical form is equal to the power developed electrically in the rotor of the induction motor, and this is obtained from the equation

$$\frac{\text{Electrical power developed in rotor}^{1}}{\text{Rotor input}},$$

$$= \frac{\text{Speed of slip}}{\text{Synchronous speed}} = \frac{f_s}{f}.$$

But $\qquad f_s = f \times \dfrac{p_c}{p_a + p_c}$ (see above).

Therefore the electrical power developed in the rotor

$$= \text{Rotor input} \times \frac{f \times \dfrac{p_c}{p_a + p_c}}{f}$$

$$= \text{Total input} \times \frac{p_c}{p_a + p_c}.$$

Thus $\dfrac{p_a}{p_a + p_c}$ is the fraction of the total power converted by motor-generator action, and $\dfrac{p_c}{p_a + p_c}$ is the fraction of the total power converted by transformer and rotary converter action.

Starting.—Motor converters can be started up from either the C.C. or the A.C. end.

In starting up from the C.C. end, an ordinary motor starter is

[1] In an ordinary induction motor this is the rotor loss.

employed, the A.C. end being synchronised like an alternator when the correct speed is attained.

Starting up from the A.C. end is very simple in the actual operations gone through. Three, or, in the largest sizes, six, of the twelve rotor phases are brought out to slip rings, the other ends of these phases being permanently connected to the C.C. armature. The slip rings are connected to a rotor starter in the usual manner, and a synchronising voltmeter, V (see Fig. 382), is connected across two of the rotor slip rings, this comprising the whole of the synchronising gear. The C.C. end being unexcited, the rotor starting switch is closed, and the set commences to run up to speed as an induction motor. Since the synchronous speed of the motor converter is only half the synchronous speed of the induction motor acting by itself (assuming the same number of poles at both ends), the rotor starter allows the set to run up to a higher speed than that of synchronism. When a speed of about 10 per cent. above the normal running speed is obtained, the shunt regulator at the C.C. end is adjusted until the rotary converter begins to excite as a C.C. dynamo. When this occurs, the speed commences to fall and approaches that of synchronism. The voltmeter, V, now begins to be affected by two different voltages of slightly different frequency, viz., the voltage induced in the induction motor rotor having a frequency equal to that of slip, and the voltage induced in the rotary converter armature which is connected to the voltmeter through the induction motor rotor, the frequency of this voltage being determined by the speed. The voltmeter pointer now commences to pulsate after the manner of the synchronising lamps in the case of alternators, the pulsations gradually becoming slower as the synchronous speed is approached. The correct speed of synchronism is reached when the pulsations cease and the voltmeter is at zero. The starter is now short-circuited and the remaining rotor phases are all connected together by the short-circuiting ring mounted near the slip rings. The set is now ready for running on load.

The starting current varies from one-quarter to one-third of the normal full load current, and depends upon the magnetising current of the A.C. end.

Efficiency and Power Factor.—The efficiency of a 500 k.W. motor converter should vary from, say, 85 per cent. at one-quarter load up to 92 per cent. at full load, whilst the power factor should vary from, say, 0·96 up to unity from half to full load, the current being sometimes lagging and sometimes leading.

Application to Sub-station Work.—Motor converters now form the great rival to motor-generators and rotary converters in sub-station work. No external transformer need be used, since the stator of the induction motor can be wound for any voltage, irrespective of the voltages in the rotating parts, which must be designed

to suit the C.C. line pressure. There are therefore no H.T. connections in the rotating elements to cause possible trouble. Motor converters can also be used to supply a three wire system, the balancers being done away with, since the set can take care of the unbalanced load by itself as shown below.

Motor Converters for Lighting Load.—Motor converters intended to supply a lighting load may have the C.C. end either shunt or compound wound. Shunt machines can be designed to work with a voltage drop of about 5 per cent. from no-load to full load, whilst compound machines may have either a rising or a drooping characteristic. A power factor of unity may be obtained at practically all loads with a high efficiency, particularly on light loads.

Three Wire Motor Converters.—When motor converters are used to supply a three wire system the C.C. outers are connected to the brushes on the commutator of the rotary converter, whilst the

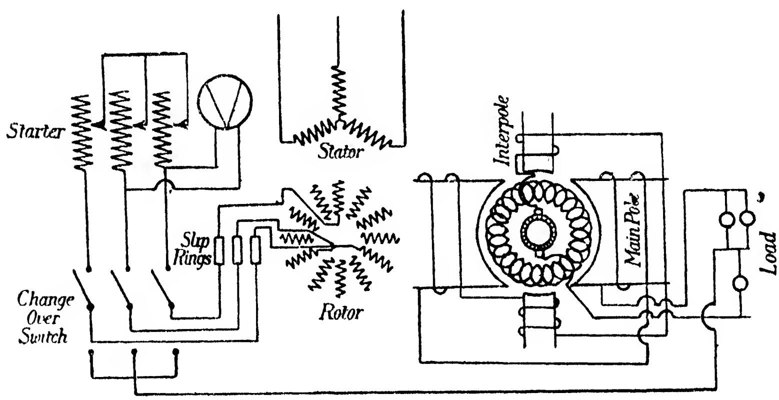


FIG. 383.—Motor Converter for Three Wire System.

middle wire is connected to the neutral point of the induction motor rotor. The potential of this point is midway between the potentials of the outside ends of any two diametrically opposite phases, and consequently is midway between the potentials at opposite points on the C.C. armature. The potential of the middle wire is therefore midway between the potentials of the brushes on the commutator, and any out-of-balance current finds its way back to the armature of the rotary converter by way of the neutral of the induction motor rotor. The actual connection is made on the A.C. starter, as shown in Fig. 383, where three leads are brought out to the starter through a triple pole change-over switch. When starting up, the middle wire of the three wire system is open-circuited, and is only connected to the motor converter when the starting switch is thrown over to the running position after the set has been brought into synchronism.

The out-of-balance current is dealt with in the same way as in

the case of three wire rotary converters, where the middle wire is connected to the star point of the transformer secondary.

Motor Converters for Traction Load.—Motor converters for traction work are compound wound, usually with a rising characteristic. The power factor reaches unity at about three-quarter full load, the current lagging slightly for lower loads and leading slightly for higher loads. Occasionally, sets are installed to deal with both traction and lighting load, but this usually necessitates greater voltage regulation and leads to slightly increased dimensions.

Single Phase Motor Converters.—In the case of a single phase motor converter, the A.C. end must run as a single phase induction motor, and consequently must be provided with an auxiliary winding on the stator to produce the starting torque. The split phase is obtained in this instance by putting an inductance in series with the auxiliary winding for the purpose of getting the required phase displacement. It is usual, also, to provide the C.C. end with an amortisseur to damp the pulsation of the field produced by the rotor currents.

Inverted Motor Converters.—Motor converters can be used to convert C.C. to A.C. in the same way as rotary converters, in which case they are called *inverted motor converters.* They must be synchronised in the same way as alternators, and have similar voltage drops on full load at various power factors. A shunt regulator is necessary at the C.C. end to maintain constant speed at all loads for the purpose of keeping the frequency constant.

Use as a Power Factor Improver.—When a motor converter is used for this purpose, it is started up and synchronised from the A.C. side, whilst the C.C. side is over-excited and left disconnected from the bus bars. The over-excitation necessitates a leading current in the C.C. armature, as in the rotary converter, thus causing the current to lead in the induction motor rotor. The stator and rotor currents of the induction motor are linked together like the primary and secondary of a transformer, and so the leading current in the rotor causes a leading current to flow in the stator, this latter being drawn from the supply.

Comparison with Motor-generators and Rotary Converters.—Since the size of the induction motor depends upon the speed of the rotating field rather than on that of the rotor, the A.C. end is somewhat smaller than in the case of an induction motor-generator of the same output. In comparison with a rotary converter, however, there are two rotating machines against one and a static transformer. As regards efficiency, the motor converter is superior to the motor-generator and is equal to the rotary converter on full load, but is slightly poorer on light load. The power factor is better than that of an induction motor-generator, equal to that of a synchronous motor-generator, and, where variation in the C.C. voltage is required, it is slightly better than in a rotary con-

verter. The starting is not quite so simple as with an induction motor-generator, but is simpler than with a rotary converter or a synchronous motor-generator that has to be synchronised. Rotary converters operate better on low frequencies, and in the motor converter the frequency at the C.C. end is considerably lower than that of supply, which is advantageous from the point of view of commutation. Motor converters also are not liable to reversal of polarity, as is the case with rotary converters.

CHAPTER XXX

SERIES MOTORS

Simple Series Motor.—The simple single phase A.C. series motor is very similar to the C.C. series motor in its general construction and arrangement, and consists of a rotating armature with a commutator and a stationary field system, these two elements being connected in series with one another and fed from the line voltage. The diagrammatic arrangement of the circuit is shown in Fig. 384, where A represents the armature and brushes and F the field winding. As in C.C. series motors, the brushes are placed on the commutator so as to make connection with conductors in the neutral zone. Since these motors are largely used for traction work where reversible rotation is required, the brushes are not given any lead, but are set in the no-load neutral position. One of the main points of difference between the A.C. and C.C. series motor lies in the field system. Since the current is alternating in the A.C. case, the magnetic field set up is also alternating, and with the ordinary solid magnet system this would lead to prohibitive iron losses, and so the whole of the magnetic field system is laminated throughout. Also, it is not necessary to employ salient pole pieces as in the C.C. motor, since the poles may be produced by a stator similar to that employed in the case of induction motors, where the position of the poles is decided by the windings. In this way, a uniform air-gap is obtained all the way round the rotor. This method of construction is very similar, theoretically, to that adopted in the case of turbo-alternators with a cylindrical rotor, except that the field system is the internal element in the turbo-alternator and the external element in the series motor.

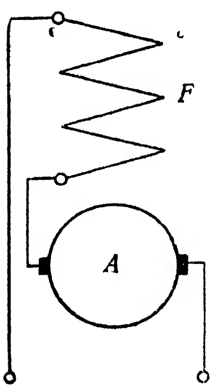


FIG. 384.—Connections of Simple Series Motor.

E.M.F.'s induced in the Armature.—In addition to the E.M.F.'s induced in the armature due to rotation in the magnetic field, there are also E.M.F.'s induced by transformer action, the field winding acting as the primary and the armature winding as the secondary. These E.M.F.'s will now be considered in detail.

The resultant magnetic field is due to the combination of the fluxes set up by the main field and the armature winding respectively. Actually these fluxes do not exist separately, but it is convenient to consider them as two distinct fluxes superposed on one another for the purpose of investigating their effects.

First, consider the effect of rotation in the main field. All the conductors from A to A' down the left-hand side of the armature (Fig. 385) will have E.M.F.'s induced in the same direction, and all the conductors from A to A' down the right-hand side of the armature will have E.M.F.'s induced in the opposite direction. The E.M.F. per conductor will be a maximum at B and B' and zero at A and A', but it is across these latter two points that the maximum p.d. is set up in the armature due to this cause, and it is at these two points that the brushes are situated. When the current reverses the magnetic field reverses, and thus the back E.M.F. between the brushes is reversed, since the rotation is unchanged. This E.M.F. is therefore permanently opposed to the applied E.M.F., no matter what value the speed may have. The frequency of this back E.M.F. is equal to that of the field and therefore to that of the supply, and is independent of the speed of rotation. Its magnitude, however, depends upon the strength of the field and the rate of cutting lines of force. With an ordinary drum wound armature the magnitude of this E.M.F. is given by the same formula as used for C.C. machines, viz.,

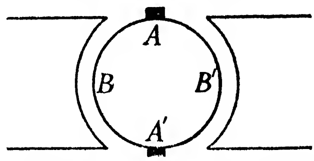


FIG. 385.—To illustrate E.M.F.'s induced in Armature.

$$E_{av} = \frac{n\Phi_{av}N}{60 \times 10^8} \times \frac{p}{q},$$

where E_{av} is the average value of the E.M.F., n is the speed in r.p.m., N is the number of conductors, Φ_{av} is the average value of the flux per pole, p is the number of poles, and q is the number of parallel circuits on the armature. With sinusoidal distribution the average value of the flux per pole, Φ_{av}, is equal to $\frac{2}{\pi}$ times the maximum value of the flux per pole, Φ, and if T be the number of turns in series, then

$$T = \frac{N}{2q}$$

and

$$E_{av} = \frac{n \times \frac{2}{\pi}\Phi \times 2qT \times p}{60 \times 10^8 \times q}$$

$$= \frac{\frac{4}{\pi} \times n\,\Phi T p}{60 \times 10^8}.$$

The R.M.S. value of the voltage is given by

$$E = \frac{\pi}{2\sqrt{2}} E_{av} \text{ (see p. 12)},$$

and hence

$$E = \frac{\pi}{2\sqrt{2}} \times \frac{4}{\pi} \times \frac{n\Phi Tp}{60 \times 10^8}$$

$$= \frac{\sqrt{2} n\Phi Tp}{60 \times 10^8} \text{ volts.}$$

This E.M.F. is in phase with the flux which produces it. As will be shown later, the power factor of the motor is less than unity, and consequently the back E.M.F., which is in phase with the flux, and, approximately, the current, is not in phase with the applied terminal voltage.

The second E.M.F. induced in the armature is due to rotation in the armature reaction field, and the axis of this flux is a line joining the brushes. This flux is geometrically at right angles to the main field in a bipolar motor, and consequently the maximum p.d. occurs between the points B and B'. As far as this E.M.F. is concerned, the points A and A' are equipotential, and therefore this E.M.F. has no effect upon the total back E.M.F. induced. Unfortunately, however, the conductors at A and A' are situated so as to have the maximum voltage induced in them, and since the turns in this region are short-circuited by the action of the brushes on the commutator, this results in a tendency to spark.

There are next the E.M.F.'s set up by transformer action. Considering the armature as stationary for the time being, the maximum E.M.F. is induced in those turns which embrace the greatest area in a plane at right angles to the flux. These turns are situated at AA', and consequently this effect will also tend to produce sparking at the brushes. The conductors between which the maximum p.d. exists are situated at BB' (at right angles to AA'), and consequently the cumulative effect of the whole winding is neutralised at the brushes, since the E.M.F. induced in the conductors between B and A is balanced by the E.M.F. induced in the conductors between B and A'; similarly on the other side of the armature. The phase of this induced voltage is 90° behind the flux, since it is proportional to minus the rate of change of flux, and its frequency is the same as that of the flux, which is the frequency of supply. Since the magnitude of this voltage is only dependent upon the rate of change of the flux and the number of turns, it follows that it is independent of the speed of rotation, and consequently the same E.M.F. is induced due to this cause, whether the armature is rotating or not. The ordinary formula for the induced E.M.F. in a transformer is

$$E = \sqrt{2}\pi f\Phi T \times 10^{-8} \text{ volts (see p. 172)},$$

but in this case a factor $\frac{2}{\pi}$ must be introduced to account for the fact that all the turns do not link with all the flux. Therefore

$$E = \frac{2}{\pi} \times \sqrt{2}\pi f \Phi T \times 10^{-8}$$
$$= 2\sqrt{2} f \Phi T \times 10^{-8} \text{ volts.}$$

If the field is not uniform, or if the armature winding is differently arranged, the value of the constant $\frac{2}{\pi}$ is affected somewhat. In the case of a multipolar wave wound armature, T will be the number of turns in series between the brushes.

The ratio of this transformer E.M.F. to the rotation E.M.F. is

$$\frac{\text{Transformer E.M.F.}}{\text{Rotation E.M.F.}} = \frac{2\sqrt{2} f \Phi T \times 10^{-8}}{\dfrac{\sqrt{2} n \Phi T p}{60 \times 10^{8}}} = \frac{120 f}{np}.$$

But the synchronous speed is

$$n_s = \frac{120 f}{p}.$$

Therefore

$$\frac{\text{Transformer E.M.F.}}{\text{Rotation E.M.F.}} = \frac{n_s}{n} = \frac{\text{Synchronous speed}}{\text{Actual speed}}.$$

The fourth E.M.F. induced in the armature is one due to transformer action set up by the armature reaction cross flux. This is produced in the same way as the transformer E.M.F. due to the main field, except that it acts along an axis AA' instead of BB', since the armature reaction cross flux is at right angles to the main field. The phase of this induced E.M.F. is again 90° behind the flux, and will require the application of an E.M.F. at the brushes 90° ahead of the flux to overcome it. This is only another way of saying that the armature possesses reactance, since the E.M.F. induced is an E.M.F. of self-induction, and the voltage required to overcome it, together with the E.M.F. required to overcome the resistance of the armature and brushes, forms the impedance voltage of the armature.

Torque.—The torque developed depends upon the instantaneous product of the armature current and the field. The field is proportional to the current, neglecting the effect of change of permeability, so that the torque is nearly proportional to the square of the current. When the current reverses in direction the field does so at the same instant, since they are practically in phase with each other, and so the torque is always developed in the same direction. Owing to the variation in the strength of the current,

the value of the torque is constantly varying between zero and a maximum value at a frequency equal to double that of the current, so that although the torque is unidirectional in character it is pulsating in magnitude.

Speed.—Assuming for the moment that the power factor and efficiency remain constant for all loads, then the output is proportional to the current for a constant applied voltage. But since the torque is proportional to the square of the current, the speed, which is proportional to $\frac{\text{output}}{\text{torque}}$, is inversely proportional to the current and to the output. Actually this proportionality does not hold good exactly, but it is sufficient to show that the speed decreases considerably with increase of load and that the speed-load characteristic is of the same general shape as in the case of the C.C. series motor.

Vector Diagram.—In drawing the vector diagram of the series

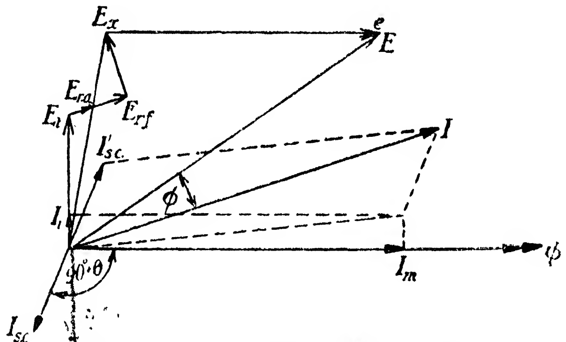


FIG. 386.—Vector Diagram of Series Motor.

motor, it is convenient to start with the flux vector. In Fig. 386, Φ represents the magnetic flux entering the armature and linking with the winding. The magnetising current necessary to produce this flux is represented by I_m in phase with the flux. The iron losses necessitate a further small current, I_i, in phase with the terminal voltage, the phase of which is not yet known. As a first approximation, this current may be drawn at right angles to the magnetising current, which will not make any appreciable error, since it is relatively small. A third component of the current is introduced on account of the current flowing round the turns short circuited by the brushes on the commutator. These turns have an E.M.F. induced in them by the transformer action of the main field. This E.M.F. lags behind the flux by 90°, and since these turns possess both resistance and reactance, the short circuit current will lag behind the E.M.F. by some angle θ. The phase of this current is therefore $90° + \theta$ behind the flux Φ and is represented in the vector diagram by I_{sc}. To balance the ampere-turns set up in this

way an equivalent number must be supplied by the main current exactly opposite in phase, and this results in a third component of the current, I'_{sc}. On adding these three components together, vectorially, the resultant current, I, is obtained.

The terminal voltage will consist of components representing the armature transformer E.M.F. and armature resistance drop, the voltage drop due to the resistance and reactance of the field winding, and the rotation E.M.F. The armature transformer E.M.F., which brings about the armature reactance, lags behind the armature flux by 90° and requires the application of a counter E.M.F. leading the flux by 90°; this is represented by E_t. There is next the voltage drop due to the armature resistance in phase with the total current, I, represented by E_{ra}. In the case of a simple impedance the resistance and reactance components of the voltage differ by 90° in phase, but in the present instance this is not so, since the induced back E.M.F. is in quadrature with the flux cutting the armature, and this is not in phase with the armature current. This alters the phase relationship somewhat. The voltage drop due to the resistance of the field winding is again in phase with the current and is represented by E_{rf}. The field winding also sets up a certain leakage field, which appears as a reactance and necessitates a voltage, E_x, leading the current by 90°. Lastly, there is the rotation voltage, e, in phase with the main flux and having a magnitude proportional to the speed. Adding all these components together, the resultant terminal voltage, E, is obtained. The angle of phase difference, ϕ, between the terminal voltage, E, and the current, I, gives the angle of lag in the whole circuit, and $\cos \phi$ is the power factor of the motor.

It is thus seen that for a high power factor, the reactances should be small and the rotation E.M.F. large. The latter is obtained by running the motor at a high speed, above that of synchronism. The motors previously dealt with have not been able to run at hypersynchronous speeds, but there is no fundamental reason why this should be impossible in the present case.

At the moment of starting the rotation E.M.F. is zero, and this advances the phase of the terminal volts with respect to the current, thus making the power factor worse. The power factor gradually improves as the speed increases. For heavy loads, the power factor comes down again, since the speed falls as the load goes up.

Circle Diagram.—Considering the motor to have a constant voltage applied to its terminals, this voltage can be split up approximately into two main components 90° out of phase with each other. The component in phase with the current consists of the resistance drops in the armature and field windings and the rotation voltage. The latter is not strictly in phase with the current, as shown in the vector diagram, but will be assumed so for the purposes of the simple circle diagram. The voltage in quadrature with the current

consists of the reactance drops in the armature and field windings. This again is only an approximation, since the vector diagram showed the voltage overcoming the leakage reactance of the armature 90° out of phase with the flux instead of the current. These assumptions are made, however, in order to split up the constant applied voltage into two components at right angles to each other. In Fig. 387, if OB represents the constant applied voltage, OC represents the voltage overcoming the total reactance of the motor and CB represents the total voltage in phase with the current, and since OB is constant and the angle OCB is a right angle, it follows that the point C will move in a semicircle erected on OB as a diameter

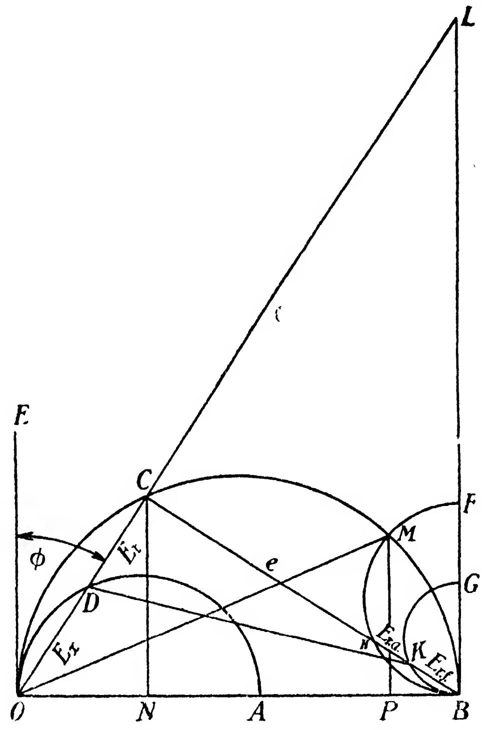


FIG. 387.—Circle Diagram of Series Motor.

(see Fig. 48). Since the reactance of the motor may be assumed constant, the current is proportional to the voltage across the reactance of the motor, and is represented to scale by OC. The phase of the current, however, is represented by CB and the angle of lag is given by the angle CBO, which is equal to the angle EOC. The power factor, therefore, is given by $\frac{CB}{OB}$ or $\cos EOC$. OB is now divided at A, so that

$$\frac{OA}{AB} = \frac{\text{reactance of field}}{\text{reactance of armature}} = \frac{E_x}{E_t} \text{ (Fig. 386),}$$

and another semicircle is erected on OA as a diameter. Now wherever the point C may be, OC is divided by this second semicircle at D in the same ratio as OB is divided at A, since the triangles OBC and OAD are similar. (The line AD is omitted in the diagram for the sake of clearness.) OD therefore represents the voltage drop in the reactance of the field and DC the voltage drop in the reactance of the armature. As in Fig. 53 (which see), a vertical line is erected at B and a length BF is measured off along it such that

$$\frac{BF}{OB} = \frac{\text{Total resistance}}{\text{Total reactance}} = \frac{E_{rf} + E_{ra}}{E_x + E_t}.$$

Another semicircle is now erected on BF as a diameter and this is cut by the line CB at H. Now the angle HBF equals the angle COB, and therefore

$$\frac{BH}{BF} = \frac{OC}{OB}$$

and $$BH = OC \times \frac{BF}{OB}$$

$$= \text{Total reactance voltage drop} \times \frac{\text{Total resistance}}{\text{Total reactance}}.$$

$$= \text{Total reactance} \times \text{Current} \times \frac{\text{Total resistance}}{\text{Total reactance}}$$

$$= \text{Current} \times \text{Total resistance}$$

$$= \text{Total resistance voltage drop}.$$

The line BF is now divided at G, and another semicircle erected on BG as a diameter, in the same way that OB was divided at A. In the present instance $\frac{BG}{GF}$ is made equal to $\frac{\text{Resistance of field}}{\text{Resistance of armature}}$, and the line CB is now split up at H and K, so that BK represents the voltage drop due to the resistance of the field, E_{rf}, and KH represents the voltage drop due to the resistance of the armature, E_{ra}. The remaining part of the voltage in phase with the current, HC, must therefore represent the voltage, e, induced in the armature due to its rotation in the magnetic field. The voltage drop across the brushes on the armature is given by the vector sum of e, E_{ra}, and E_t, and is represented by DK in the circle diagram. Since e is proportional to the flux $\times$ speed, the speed is proportional to $\frac{e}{\text{flux}}$ or $\frac{e}{\text{current}}$, assuming the flux to be proportional to the current. In the circle

diagram, therefore, the speed is represented by $\frac{CH}{OC}$. Let OC and BF be produced until they meet at L. Now $OC =$

$$OB \cos COB = OB \cos CBL$$

$$= OB \times \frac{CB}{BL} = OB \times \frac{CH}{FL}.$$

Therefore the speed is proportional to

$$\frac{CH}{OC} = \frac{CH}{OB \times \frac{CH}{FL}} = \frac{FL}{OB},$$

and since OB is constant, the speed is represented to scale by FL.

As in the case of the induction motor, the input is measured by CN to a scale of watts. The copper losses can be obtained by drawing vertical lines from the points H and K, the vertical height of H representing the total copper loss and the vertical height of K the copper loss in the field only. The output (neglecting iron and friction loss) is thus obtained by subtracting the vertical height of H above the base line from the input CN, from which the electrical efficiency can be obtained. The torque is proportional to the instantaneous product of current and flux, and, neglecting the phase difference between these two quantities and also the effect of magnetic saturation, the torque may be taken as being approximately proportional to the (current)^2. Now in the circle diagram

$$\cos COB = \frac{OC}{OB} = \frac{ON}{OC}$$

and therefore $$OC^2 = ON \times OB.$$

The torque being proportional to OC^2 and OB being constant, it follows that the torque is represented to scale by ON.

At the moment of starting e is zero, and if the full voltage be applied the starting current will be represented by OM and the starting torque by OP.

Power Factor.—From the circle diagram in Fig. 387 it is seen that the power factor is better on light loads than on heavy loads, since the vector OC is swung round to the right as the current increases, thus increasing the angle of lag. When the point C reaches the top of the semicircle, the power input has reached a maximum and the power factor has dropped to $\frac{1}{\sqrt{2}} = 0{\cdot}707$. Further increase of load reduces the power factor still more, so that when M is reached the power factor is at its worst. This, however, is the condition at starting when the back induced voltage, e, is zero, so

that when the motor is first switched into circuit the power factor is very poor, but it gradually improves as the motor gains speed. Also, the higher the speed the greater does e become, thus improving the power factor. If reference be made to the vector diagram in Fig. 386, which is rather more accurate than the circle diagram, it will be seen that if e be made sufficiently large it is possible for the motor to draw a leading current from the supply. This is due to the action of the short circuited coils under the brushes, which set up a number of ampere-turns and cause a phase displacement between the flux and the main armature current. It is not desirable to magnify this effect deliberately, however, although it would improve the power factor, since it would do it at the expense of the commutation and the efficiency.

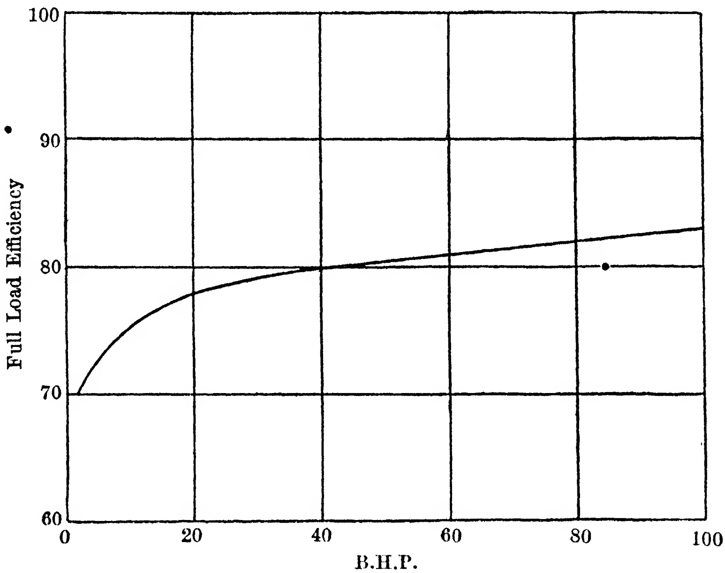


FIG. 388.—Efficiencies of Series Motors.

The best way of obtaining a high power factor is to neutralise the armature reactance (which will be dealt with in the case of the compensated series motor) or to reduce the working field strength. This reduces the reactance of the field winding and brings about the high speeds which are desirable.

It is also seen that a low frequency is advantageous, since this results in relatively low reactances and high power factors.

Full load power factors of 0·8 to 0·95 are met with in practice.

Efficiency.—The efficiency of a single phase series motor generally is lower than that of the corresponding C.C. motor. For medium sized motors it reaches a value of about 0·8, whilst for large sizes it may rise as high as 0·85. Fig. 388 shows the approximate values

of the full load efficiencies which may be expected in modern motors of this type.

Performance.—The approximate performance of a series motor can be derived from the circle diagram shown in Fig. 387. It is usual to plot the various quantities as a function of the B.H.P., although in some cases the torque is chosen instead. Fig. 389 represents the performance of a 25 B.H.P. motor obtained in this manner.

Starting.—The usual starting device for a single phase series motor consists of an auto-transformer with a number of tappings on the winding, so as to enable the voltage across the motor terminals to be gradually raised. This arrangement allows a reduced voltage to be applied to the motor and also relieves the line to a large extent

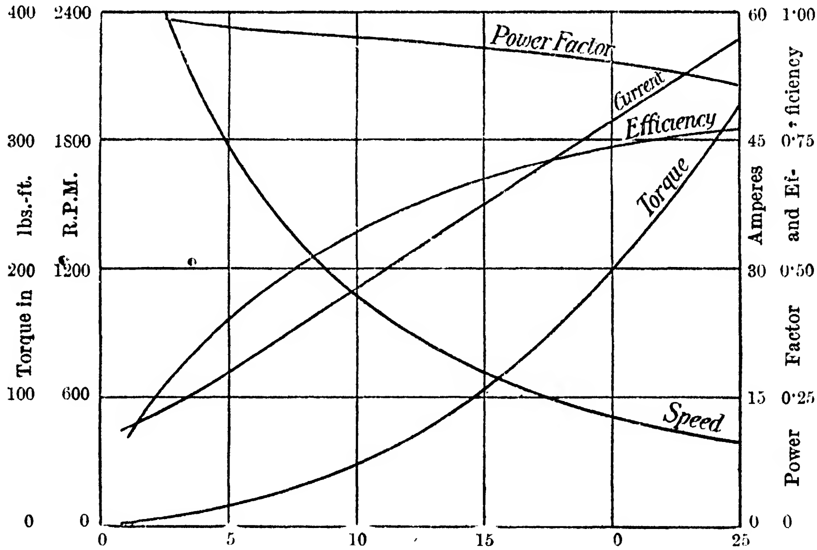


FIG. 389.—Performance Curves of 25 B.H.P. 500 Volt Series Motor.

of the excessive current which the motor is allowed to take at the moment of switching on. The connections are shown in Fig. 390, where *M* represents the motor, *A* the auto-transformer, and *C* a choking coil known as a *preventive coil.* One terminal of the motor is connected to one end of the auto-transformer and the other to the middle point of the choking coil. The two ends of the choking coil are connected to two moving fingers which in some positions lie on the same contact and in others bridge over two contacts. When both fingers are on the same contact, the currents flow in opposite directions in the two halves of the choking coil, thus rendering it practically non-inductive, so that the drop in volts is inappreciable. When bridging two contacts,

the impedance of the choking coil is sufficient to prevent the short circuiting of the particular section of the auto-transformer concerned. The choking coil thus appears non-inductive to the motor circuit

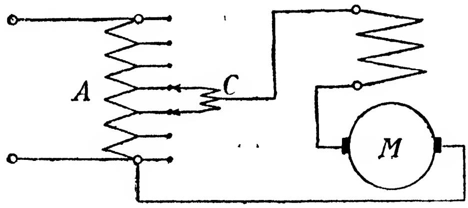


FIG. 390.—Auto-Transformer for starting Series Motor.

and yet acts like a highly inductive resistance to the local circuit across the contacts.

Compensated Series Motor.—One of the methods of improving the power factor of the series motor consists in counteracting the leakage reactance of the armature. The latter tends to set up a distorting field, the axis of which is a line drawn through the brushes in a bipolar case. These cross ampere-turns can be neutralised by means of an auxiliary field winding placed midway between the main field windings in much the same way that interpoles neutralise the cross ampere-turns of the armature in a C.C. motor. The general arrangement is shown in Fig. 391, where A represents the armature, F the main field winding, and C the compensating winding. This

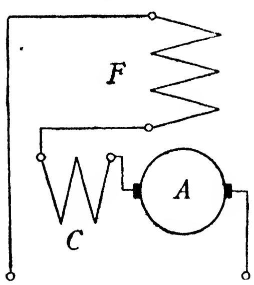


FIG. 391.—Compensated Series Motor.

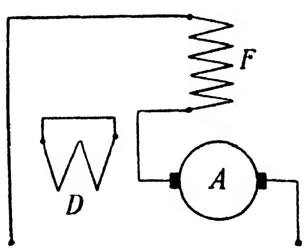


FIG. 392.—Compensated Series Motor with Damping Coil.

compensating winding is arranged to set up as many ampere-turns as the armature, and since it acts in direct opposition it practically eliminates the leakage cross flux and renders the armature non-inductive. The addition of the compensating winding also suppresses to a large extent the rotation E.M.F. which is induced by the armature cross flux in the short circuited coils under the brushes. The compensating winding, therefore, has a beneficial effect on the commutation as well as effecting an improvement in the power factor.

An alternative arrangement to that shown in Fig. 391 consists

in short circuiting the compensating winding on itself and disconnecting it from the main circuit altogether. The auxiliary winding is now often called a *damping coil,* and the connections of this type of motor are shown in Fig. 392, where D represents the damping coil and A and F have the same significance as before. This produces the same result as the former method of connection, although the action is. somewhat different. The damping coil acts like the short circuited secondary of a transformer the primary of which is formed by the armature winding, whilst the flux linking the two consists of the armature cross leakage flux. The E.M.F. induced in the short circuited damping coil causes a current to flow which sets up ampere-turns sufficient to neutralise the ampere-turns of the armature, thus suppressing the cross flux and rendering the armature tolerably non-inductive.

The difference between the two methods of compensation lies in the fact that in one case the neutralising ampere-turns are obtained directly from the main circuit, whilst in the other they are obtained by transformer action from the armature.

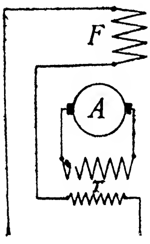


FIG. 393.—Series Motor with Transformer.

Series Motor with Transformer.—As series motors are usually limited by their design to line pressures of about 300 volts,. it is necessary to supply a transformer when higher voltages are dealt with. There is, however, no objection to the field winding being connected to the H.T. supply, and so only the armature is supplied from the L.T. side of the transformer, which is connected in series with the field winding as shown in Fig. 393, in which T represents the transformer. This transformer can also be used for starting purposes by providing the secondary with a number of tappings, so that the increase in the apparatus is not as much as would appear at first sight.

Commutation.—In addition to the usual process of reversing the current which takes place in C.C. motors, there are the extra E.M.F.'s induced in the short circuited coils to be dealt with. This adds to the difficulty of commutation and leads to special features in the design of the commutator. This is made of larger diameter than in the corresponding C.C. motor, so that it more nearly approaches the diameter of the armature. A large number of commutator bars is also essential, the maximum number possible for the particular armature winding being put in. Narrow brushes are also employed, so as to avoid short circuiting more turns than absolutely necessary at a given instant. Since the current is large at starting, commutation will be more difficult at this time and sparking may be anticipated, but as the motor gains speed the current decreases and the commutation improves. As tolerably good commutation can be obtained over a wide range of speeds, it follows

that the series motor is suitable for variable speed work. The addition of interpoles also exerts a very great effect on the commutation as in C.C. motors, and they are now very largely employed.

Interpoles.—In C.C. motors interpoles fulfil a double purpose. They act as a compensating winding for the purpose of neutralising the armature cross flux, and they also aid commutation by introducing a reverse flux into the armature so as to cut the conductors coming under the brushes. In A.C. motors these two functions are separated. The action of the compensating winding has already been discussed, but it must be remembered that this winding must embrace a complete pole pitch, since the M.M.F. set up by the armature acts over the whole of this region. The commutating winding, on the other hand, is only required to produce a local effect on those conductors undergoing commutation. For this reason, the interpole (or commutating pole) winding is concentrated over a narrow area, and one large tooth is provided on the stator core to receive this winding. The main field winding and the compensating winding are wound in the same sized slots, displaced half a pole pitch with respect to each other, whilst the interpole winding is situated on the centre line of the compensating winding and is wound on a single tooth of larger dimensions than its fellows.

Sphere of Application.—Owing to its variable speed properties, the series motor, in common with the other types of single phase commutator motors, is particularly adapted to traction work. For this class of work a single phase system has obvious advantages over a polyphase system in the transmission and collection of the current, since an earthed return can be adopted and only one live wire need be used. Apart from this consideration, the polyphase induction motor is unsuitable for variable speed work and the synchronous motor is obviously out of the question. The various types of single phase commutator motors have to compete, therefore, with the single phase induction motor and have now practically displaced the latter from the market in this class of work.

CHAPTER XXXI

REPULSION MOTORS

Simple Repulsion Motor.—The simple repulsion motor consists of a field system and an armature with a commutator just like a series motor. The difference between the two motors lies in their connections and action. The field system or stator does not possess salient poles, but is built up of toothed laminations as in the series motor, thus producing a uniform air-gap. The field winding is wound in the stator slots, the position of the poles being determined by the winding. The armature possesses an ordinary distributed winding connected to the commutator in the usual way, but the brushes are set at an angle of about 70° from the position of maximum p.d. on the armature. These brushes are short circuited and are not connected electrically to the main circuit at all. The stator winding producing the main field is connected across the supply. The connections of a simple repulsion motor are represented diagrammatically in Fig. 394.

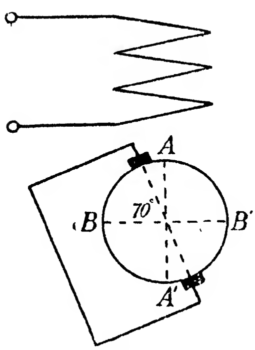


FIG. 394.—Connections of Simple Repulsion Motor.

Theory of Operation.—Considering the motor as a transformer in which the stator winding forms the primary and the rotor winding the secondary, the maximum p.d. is set up between the points AA'. If the brushes be placed on this line the maximum current will flow in the armature, but no torque will be produced, since the axis of the main field coincides with the axis of the flux set up by the armature. To obtain the maximum torque these two fluxes should be at right angles to each other in a bipolar case. The position corresponding to this is along the line BB', but if the brushes be placed in this position they short circuit two equipotential points on the armature, and thus no armature current is produced. As it is essential that both an armature current should flow and a phase displacement should exist between the axes of the two fluxes, an intermediate position between AA' and BB' is chosen for the

circuit current nor the phase displacement is a maximum, but their instantaneous product has a definite value, whilst in the two extreme positions it is zero.

In the series motor, the rotation is set up by the armature current producing poles along the axis of the brushes, these poles being attracted to those of the main field of opposite sign. In the present instance, however, the armature tends to set itself in such a position that it expends the minimum amount of electrical energy, just as a sheet of copper suspended in the field of an alternating electromagnet will endeavour to set itself so as to produce the minimum eddy currents. This means that the armature shown in Fig. 394 will tend to rotate in a counter-clockwise direction. Owing to the apparent repelling action instead of the usual attracting action, the motor is called a *repulsion motor*. This action does not take place in the series motor, although the transformer E.M.F. is present, since the brushes are set in the neutral position.

If the brushes had happened to lie on the other side of the vertical line AA', the induced rotation would have been in the opposite direction, so that in order to reverse the direction of rotation all that is necessary is to move the brush rocker backwards through 40° against the original direction of rotation.

Owing to the fact that the armature is not connected to the main circuit, it is possible to design these motors to work directly on H.T. systems without the help of a transformer, since only the stationary stator winding is connected to the line. In this respect the simple repulsion motor has the advantage over the series motor, but this advantage is lost when the repulsion motor becomes compensated, as will be seen when the connections are studied (see Fig. 398).

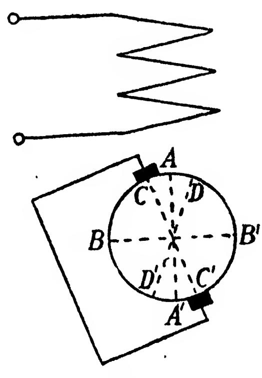


FIG. 395.—To illustrate E.M.F.'s induced in Armature.

E.M.F.'s induced in the Armature.—The flux set up by the stator winding cuts the armature in a vertical direction in Fig. 395. The resulting short circuit current of the armature sets up a field of its own, the axis of which is the axis of the brushes. This latter field can be resolved into two components at right angles to each other. One of these components may be considered as acting in a vertical direction along the line AA' so as to oppose the main stator flux, whilst the other component produces a horizontal flux along the line BB'. This horizontal component of the armature flux can be regarded, therefore, as a cross flux in quadrature with the now reduced main flux. The armature ampere-turns can also be split up into the two components which produce the back and the cross flux. The ampere-turns producing the back flux are obtained from

the conductors lying between C and D' and between D and C', whilst the ampere-turns producing the cross flux are obtained from the conductors lying between C and D and between D' and C'. Imagining the armature winding to be divided in this manner, each portion has two voltages induced in it. The back turns CD' and DC' have one E.M.F. induced in them by the transformer action of the main field and another E.M.F. due to rotation in the cross field. The transformer action of the cross field and the rotation in the main field produce no resultant E.M.F. in these turns. The cross turns CD and $D'C'$ have a transformer E.M.F. induced in them by the cross field and a rotation E.M.F. due to the main field. In these turns the transformer E.M.F. of the main field and the rotation E.M.F. due to the cross field have no resultant value. The transformer E.M.F.'s lag behind their respective fluxes by 90° whilst the rotation E.M.F.'s are in phase with them.

In addition to these E.M.F.'s in the armature, there is also the back E.M.F. induced in the stator winding, due to the transformer effect of the original stator flux.

Torque.—The torque developed by the armature conductors is due to the fact that the armature tends to set itself in such a position that the currents flowing through it are reduced to a minimum, since in this way it reduces its losses to a minimum. In the motor shown in Fig. 394 this torque is acting in a counter-clockwise direction. But the brushes on the commutator short circuit a number of turns independently of the main short circuiting lead, and these turns will tend to set up a flux of their own at right angles to the plane of their coils, *i.e.* at right angles to the brush axis. This flux will interact with the main stator flux and will tend to produce rotation in a *clockwise* direction. This torque therefore opposes the main driving torque, which is weakened by this differential effect.

Atkinson Repulsion Motor.—The stator field can be imagined to be split up into two components at right angles to each other, one of these acting along the axis of the brushes and the other in quadrature with it. The former component is the one producing the armature current by transformer effect, whilst the latter produces that component of the field which may be said to develop the torque. The actual stator winding can thus be replaced by two windings at right angles to each other and connected in series, as shown in Fig. 396, F being called the field winding and T the transformer winding. This modification of the simple repulsion motor is known as the Atkinson repulsion motor. In reality, of course, the two stator windings do not produce separate fluxes, but only one resultant, so that there is no theoretical

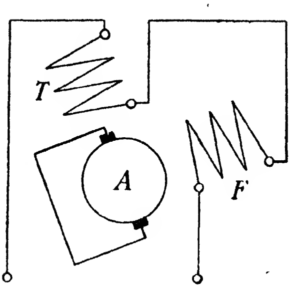


FIG. 396.—Atkinson Repulsion Motor.

difference between the simple repulsion motor and the Atkinson motor. One advantage possessed by the latter, however, lies in the fact that the direction of rotation can be reversed by reversing one of the stator windings without touching the brush rocker.

Vector Diagram.—The approximate vector diagram can be most easily studied by referring to the Atkinson modification on account of the splitting up of the stator flux, but it also applies to the simple repulsion motor, since there is no theoretical difference between the two. The transformer winding, T', in Fig. 396 will produce a flux parallel to the brush axis, which is represented by Φ_t in the vector diagram in Fig. 397. This induces a voltage, E_{t_2}, in the armature by transformer action, this voltage lagging behind Φ_t by 90°. The stator winding, T, acting as the primary of the transformer has a corresponding voltage, E_{t_1}, induced in it, this voltage leading

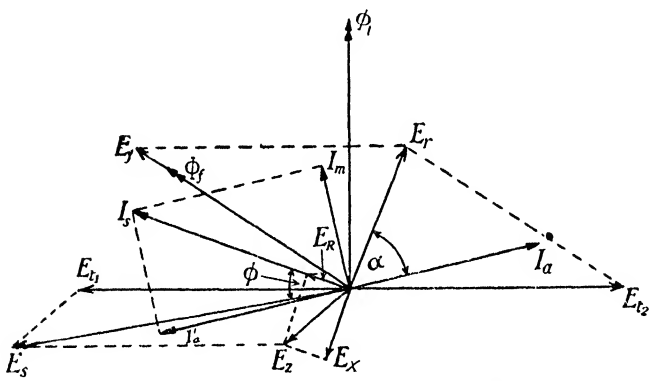


FIG. 397.—Vector Diagram of Repulsion Motor.

the flux by 90°. Now the stator current will lag behind this E.M.F. by a small angle, and this stator current has to produce the flux Φ_f when flowing through the field winding, F. Owing to the hysteresis and eddy current loss, however, the stator current leads this flux by a further small angle, since it includes a small power component, with the result that the flux Φ_f is not in quadrature with the flux Φ_t, but leads it by rather less than 90°. The armature now has a second E.M.F. induced in it, due to its rotation in the flux Φ_f. This E.M.F. is in phase with the flux Φ_f, and is represented by E_f. The resultant armature E.M.F. acting along the line of the brushes is the vector sum of E_{t_2} and E_f, and is represented by E_r. The secondary or armature current, I_a, will lag behind this resultant voltage, E_r, by a definite angle, α, which will depend upon the relative magnitudes of the armature resistance and leakage reactance. The primary or stator current, I_s, will consist of two components. The first is the magnetising current necessary to produce the flux Φ_t. In an ideal case this would lag behind the voltage, E_{t_1}, by 90° and would be in phase with Φ_t, but owing to

the presence of hysteresis and eddy current losses this current will lead the flux by a small angle and is represented by I_m. The second component of the stator current is that required to balance the ampere-turns of the secondary or armature circuit and will be exactly opposite in phase to I_a. This component is represented by I'_a. The resultant stator current, I_s, is obtained by adding vectorially the two components I_m and I'_a. At first sight it may appear as if another magnetising component of the stator current is required for the field winding, F, but this is not so, since the phase of the current is settled by the above two components and the field winding, F, has to take whatever current the transformer winding permits. It is seen from the vector diagram that the flux Φ_f lags behind the current, I_s, by a small angle, this being due to the iron losses caused by this flux. Although the field winding, F, does not introduce any additional magnetising current, yet it will bring about a certain voltage drop in the stator, and this will be dealt with next. The resistance of the complete stator circuit will bring about a voltage drop, E_R, in phase with the current, I_s, and the reactance of the field winding, F, will bring about another voltage drop, E_X, leading the stator current, I_s, by 90°. The voltage associated with the flux Φ_t in the transformer winding, T, has already been dealt with and is represented by E_{t_1}. Combining E_R and E_X, the impedance voltage, E_Z, is obtained, and combining this with E_{t_1} the total applied stator voltage, E_s, is obtained. The power factor of the motor is given by $\cos \phi$, where ϕ is the angle of phase difference between the applied stator voltage, E_s, and the stator current, I_s.

Starting.—Repulsion motors can be started up by means of a starter like that used for a C.C. series motor, or it may be started up by means of an auto-transformer. The latter method is superior, inasmuch as it relieves the line to a certain extent of the excess current which flows during the starting period.

When the motor is first switched into circuit the rotation E.M.F., E_f (Fig. 397), is zero. The resultant E.M.F., E_r, acting in the rotor circuit is now coincident with E_{t_2}. This has the effect of retarding the rotor current in phase considerably, which in its turn causes the stator current to lag behind the stator voltage by a larger angle, thus reducing the power factor. As the rotor gains speed the rotation E.M.F. is gradually increased and the rotor current is gradually advanced in phase. This reacts on the stator circuit so that the power factor improves as the speed increases.

Owing to the very large current taken at starting, the starting torque is high, notwithstanding the poor power factor. This can be seen from Fig. 397, for the torque is proportional to the instantaneous product of the rotor current and the resultant flux. The phase of the resultant flux lies in between that of Φ_t and Φ_f, and at

the moment of starting the rotor current is very nearly in phase opposition. As the rotor current is also large at starting, the resulting torque is very great. As the motor gains speed the rotor current swings round in phase to the position shown in the vector diagram, and this produces a drop in the torque, apart from the reduction in the actual magnitude of the current.

Commutation.—The commutator of a repulsion motor is characterised by its relatively large diameter and high number of bars, just as is the series motor. The brushes also are made very narrow. In the short circuited coils there are two distinct E.M.F.'s induced in addition to the usual reactance voltage due to the reversal of the current. The two E.M.F.'s are due to the transformer action of the field winding, *F* (Fig. 396), and the generator action of the transformer winding, *T*. The transformer E.M.F. lags behind the flux due to *F* by 90°, whilst the rotation voltage is 180° out of phase with the flux due to *T*. Since the two fluxes are very nearly in quadrature, it follows that the two E.M.F.'s are very nearly in phase opposition and tend to neutralise each other. This neutralisation is most complete at the synchronous speed, so that the commutation is best at speeds in this neighbourhood.

Compensated Repulsion Motor.—In the Atkinson repulsion motor, the stator winding was split up into two components, one acting along the line of the brushes and the other at right angles to it. The flux due to the latter component can be produced directly from the armature, by transformer action, without the aid of this second winding, which can be done away with altogether. The flux due to the winding, *F*, in Fig. 396 is proportional to the stator current. If this stator current were passed through the armature by means of two additional sets of brushes on the commutator arranged along the axis of the winding *F*, the resulting armature ampere-turns set up along this axis would act in the same direction as the ampere-turns of the stator winding *F*. The diagram of connections of this new variation, which is called the *compensated repulsion motor*, is shown in Fig. 398. An important point to notice is that there are four brush spindles, although it is only a two pole machine. One pair of brushes, *AA'*, is set along the axis of the single stator winding. These brushes are short circuited and carry what is called the *armature* or the *short circuit current.* The other pair of brushes, *BB'*, is set along a line at right angles to the first pair and is connected in series with the stator winding. The current passed through these brushes is the stator, or *exciting current* as it is termed. This current, by passing through the armature, sets up a number of ampere-turns which produce a flux in the direction

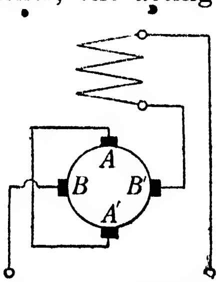


FIG. 398.—Compensated Repulsion Motor.

BB'. This flux combines with the flux set up by the stator winding directly, producing a resultant flux which acts along an axis in between *AA'* and *BB'*. If the motor be regarded in this manner, it is seen to be the equivalent in principle of the simple repulsion motor.

One result of this modification is that the second stator winding of the Atkinson repulsion motor is done away with, and as this winding produced the main field flux, its removal leads to the elimination of the reactance associated with it. In the simple repulsion motor, the one stator winding fulfilled the same functions as both the stator windings of the Atkinson repulsion motor, so that the stator reactance of this motor is reduced as well. The effect on the behaviour of the motor can be studied by means of the vector diagram in Fig. 397. The voltage drop due to the reactance of the field winding, *F*, is there represented by E_X, and if this component be eliminated the stator voltage, E_s, is brought more nearly into phase with the stator current, I_s. It is thus seen that the compensation of the motor leads to an improvement in the power factor which is maintained at a high value over a wide range of hypersynchronous speeds.

If the brushes *AA'* be removed, it is seen that the motor changes into a simple series motor, but this alteration causes a profound change in its working, since the two motors are quite different in their action.

Winter-Eichberg Motor.[1]—One of the advantages possessed by the simple repulsion motor over the series motor lay in the fact that the armature was not connected to the stator winding, which could therefore be wound for high voltages. This advantage is lost in the compensated repulsion motor, since the armature is connected in series with the stator across the brushes *BB'*. The improvement in the power factor is thus obtained at the expense of limiting the voltage on which the motor may be used. If the armature brushes, *BB'*, are fed from the secondary of a transformer, however, this advantage may be regained, and this is the arrangement adopted in the Winter-Eichberg motor which is illustrated diagrammatically in Fig. 399. By providing the transformer with a number of tappings it may be used for starting the motor in addition to its normal use, the stator being switched straight on to the supply.

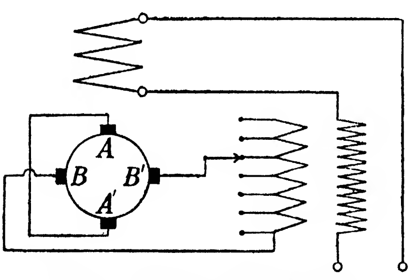


FIG. 399.—Winter-Eichberg Motor.

[1] This motor is sometimes termed the Latour-Winter-Eichberg motor, since it was invented by Latour, working independently of Winter and Eichberg, about the same time.

Both the stator winding and the transformer supplying the armature have good power factors, since they act like loaded transformers, so that the resulting power factor of the motor is very good. In common with the simple repulsion motor, its power factor is poor at the moment of starting, but this improves rapidly as the motor gains speed.

Other Commutator Motors.—There are a number of other commutator motors on the market at the present day, these being mostly combinations of those already described and induction motors. The two main parent groups are, however, the series motor and the repulsion motor groups.

Special forms of three phase commutator motors are also manufactured, but their description lies outside the scope of this book, and for information regarding these special machines reference should be made to more advanced text-books.

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